ELECTRICAL ENGINEERING ECONOMICS

*

VOLUME TWO

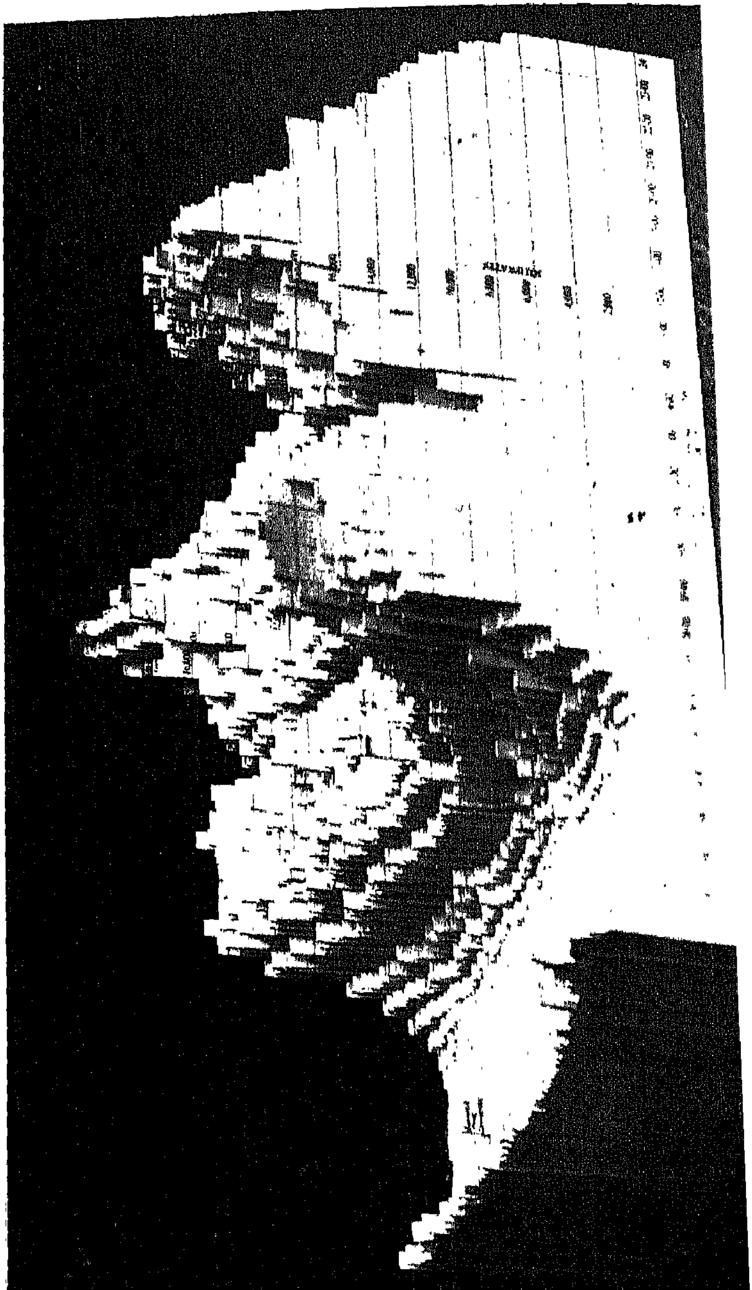
COSTS AND TARIFFS IN ELECTRICITY SUPPLY

By the same author

ELECTRICAL ENGINEERING ECONOMICS
Volume One : General Principles and Choice of Plant.

ECONOMIC TABLES FOR ELECTRICAL ENGINEERS
To Facilitate the Economic Choice of Electrical Plant.

ELECTRICITY TARIFF TYPES



ELECTRICAL ENGINEERING ECONOMICS

VOLUME TWO

Costs and Tariffs in Electricity Supply

By
D. J. BOLTON
M.Sc.,M.I.E.E.

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Extract from
PREFACE TO FIRST EDITION

There has never been any lack of interest in the subject of electricity tariffs. Like all charges upon the consumer, they are an unfailing source of annoyance to those who pay, and of argument in those who levy them. In fact, so great is the heat aroused whenever they are discussed at institutions or in the technical press, that it has been suggested there should be a " close season " for tariff discussions. Nor does this interest exaggerate their importance. There is general agreement that appropriate tariffs are essential to any rapid development of electricity supply, and there is complete disagreement as to what constitutes an appropriate tariff.

A book on such a subject can, therefore, hardly fail to be of service. So many papers and discussions have dealt (fully or in part) with electricity costs and charges, that a mere collection and codification of all this material would of itself be well worth while. The present aim, moreover, is a somewhat more ambitious one, namely to present a fundamental and critical review of the underlying principles of rate construction, together with a quantitative statement of the tariff situation in Great Britain.

. . .

The present tariff position in this country is little short of chaos. Even the terminology has not been standardised, and the tariffs themselves appear to be the unbridled whim of the particular undertaking. To quote only one example—taking a single load group (industrial power) and a single type of tariff (the block rate), and considering only the larger undertakings (one quarter of the whole), there were found to be 102 different tariffs! At this rate, the block-rate tariffs alone would muster about 400 different specimens. Kipling might well have said of electricity :—

There are nine-and-sixty ways in which the user pays
And every single one of them is right.

. . .

I believe that those who merely desire a plain statement of facts and an explanation of present tariff operations will have no difficulty in finding what they want, unencumbered by any embarrassing theory. At the same time, I am not one of those who hold that tariff construction (unlike every other branch of engineering whatsoever) has nothing

to gain from theory, and must for ever pursue its present hand-to-mouth empiricism. I have therefore spared no trouble in trying to furnish a philosophy of tariffs as well as a mere collection of working rules and body of practice.

March 1938.

PREFACE TO SECOND EDITION

SINCE the above was written, tariff terminology has been standardised, and whilst we are no nearer to unification than we were in 1938, the nationalisation of the industry has at least smoothed the path. Meanwhile, another Electricity Commission Committee has sat (though without publishing its conclusions) and the Electricity Boards have made a start on their task of standardisation.

In this second edition the nomenclature has been revised in accordance with the British Standard Glossary. New chapters have been added and most of the remainder re-written. Where possible, the statistics and magnitudes have been brought up to date, but frequently this was impossible or would have involved too drastic an overhaul. The excuse for pre-war figures must then be that they are used largely for illustration purposes and do not affect the principles involved or distort the conclusions arrived at.

As explained elsewhere, one-third of my book on *Electrical Engineering Economics*, first published in 1928 and revised in 1935 dealt with supply economics, and that part of the ground was subsequently re-traversed in this book on Costs and Tariffs. War destruction gave an opportunity to revise the two books simultaneously, and they are now issued as two volumes under the general title of *Electrical Engineering Economics*. Volume I deals with general principles (including depreciation and capital charges) and with the economic choice of electrical plant, whilst the present book is Volume II.

The book is divided into four parts, namely Tariff Theory, Costs, Retail Tariffs, and Power-Factor Costs and Tariffs. The first part (Chapters I to III) deals with the general theory of pricing in relation to supply and demand, and the particular characteristics of electricity demand and utilisation. Costs are dealt with in Chapters IV to VIII whilst the remaining chapters cover the actual tariffs. This last part is descriptive and explanatory in character, and deals primarily with types rather than magnitudes. It concludes with a section on the costs and tariffs aspect of power factor.

It remains to be seen how soon the new authority will succeed in so ironing out the tariff creases (a bulldozer would be a more appropriate instrument) as to relegate the descriptive portion to the status

of a museum catalogue. In the meantime it will be useful both to those who have to do the new work of "simplification and standardisation" and to those consumers still suffering under the vagaries of the old.

Similar ground to this descriptive tariff section was covered by the issue in 1942 of my booklet *Electricity Tariff Types*, and this is still available for those desiring a short statement on the subject. The glossary-index from this has been adapted for the present volume.

D. J. BOLTON.

May 1950.

Thanks are due to the various authorities mentioned in the text for permission to reproduce material, and to Messrs. W. A. Lewis and P. Schiller for help in proof-reading and for many valuable suggestions.

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CONTENTS

PART I

THEORY OF PRICE FIXING

Chapter I treats generally of price in relation to supply and demand, and Chapter II is an elementary study of electricity demand and its elasticity. Chapter III relates this to the marginal-cost pricing theory.

CHAPTER I

PRICE: SUPPLY AND DEMAND*

Dual Aspects.—The price of electricity, like the price of anything else, is governed by the twin considerations of supply and demand. There is a certain willingness to supply, based on the cost at which the electricity can be generated and delivered to the consumer's premises; and there is a certain need or demand for electricity, expressed in the figure which consumers are willing to pay. In general, the price which results must reflect and satisfy each set of considerations —both producer and consumer must receive a "fair deal."

There are, it is true, non-economic factors on both sides which obscure the issue and deflect the natural course of events, temporarily or even permanently. On the production side, national or municipal actions may encourage or subsidise electricity supply for the sake of its incidental effects on health or industry. Conversely, they may bleed it through differential rating or by employing the profits to the advantage of other services. On the consumption side, habit, prejudice or excessive advertising may unduly favour either electricity or one of its rivals. Also, within the localised area in which a particular undertaking functions, political and special influences sometimes operate.

Ultimately, however, it is doubtful whether any of these extraneous factors will greatly affect the price of a thing such as electricity, which does not normally arouse violent personal feelings. In the long run, the total receipts for electricity must pay for the total costs, and in the long run electricity must represent value for money to the consumer. The present chapter is therefore a treatment of tariffs as arising from the two factors of supply and demand, both of them governed by rational economic considerations. An attempt will be made to show firstly how these two curves interact, and secondly how they are composed, the one from costs and the other from utilities. The treatment will be descriptive rather than quantitative, but in the next chapter some attempt will be made at an evaluation of demand.

These two considerations, of what the service costs to produce and what it is worth to consume, must underlie all tariff construction. On both counts, moreover, electricity supply presents a particularly difficult problem. On the one hand, production costs vary with time and

* The word "demand" in this and the next chapter refers to the action of the consumer in "demanding" so many units at such-and-such a price. It must not be confused with the electric-power "demand" or maximum demand, *i.e.* the load in kilowatts measured in a specified manner.

place, and stand in no simple relation to the amount produced. On the other hand, consumption values vary because of the wide range of utilisation, and the differing competitive power of the various uses. It seems probable that there must always be a variety of electricity tariffs, differing in kind as well as in size.

Electricity, in fact, is so exceptional that most writers on tariffs have treated it as though it existed in vacuo, and without any assistance from the science of economics. The author believes that in order to understand the exceptions one must know the rules, and that, however special a case electricity may be, there is always something to be learnt from the general principles of price-fixing. The first step is therefore to see how supply and demand normally operate, and how the price of a commodity or service results therefrom.

General Laws of Supply and Demand.—These can best be understood in terms of the buying and selling of goods in the market-place. The demand for an article is typified by the number of persons desiring it and coming to the market with money in their pockets. The supply is typified by the number of producers of that article coming to the market and willing to sell. It is necessary first to consider a simplified or "ideal" state of things which may be called the condition of pure competition. Such a state exists when there is a standardised product and so many buyers and sellers that the actions of no single one of them have any appreciable effect upon the whole market.

The engineer should be warned that whilst dignified by the name of "laws," most of the principles here laid down are no more than general tendencies. Although (in the absence of anything else) they can always be expected to operate, they may at any time be upset or obscured by local or special factors. They are the regular tides upon which float a whole fleet of divergent travellers. Moreover, one can never expect the same uniformity and reliability in the realm of sociology that one meets in the realm of physics. Human beings are necessarily more complicated than materials and machines, because they have all the variables of the latter plus a number of their own. They are affected by physical and chemical changes just as much as a cable or a dynamo, and in addition they possess a psychology and can be swayed by non-material considerations.

Finally, it is rarely possible to isolate a variable and to study the effect of one change at a time. Every "law" enumerated below should therefore be qualified by the proviso "other things being equal" —in practice a well-nigh impossible requirement.

Demand.—It is an axiom that people will buy more of an article when it is cheap than when it is dear. Demand goes up as price goes down and vice versa—*i.e.*, there is an inverse relationship between them.

If a curve is plotted connecting the price of an article (ordinates) and the number changing hands (abscissæ) the curve will follow the general shape of the line *DD'*, Fig. 1. At any given price *pn*, the number changing hands will be *on*, and a price movement up or down will check or stimulate the sales.

It is conceivable that in a particular case there might be a simple mathematical relationship between the two quantities, *e.g.*, the number bought might be exactly inversely proportional to the price. The curve would then be a hyperbola, the ends *D* and *D'* being asymptotic to the two axes. Usually the relationship would be a less simple one, and more easily expressed by a curve than by a formula.

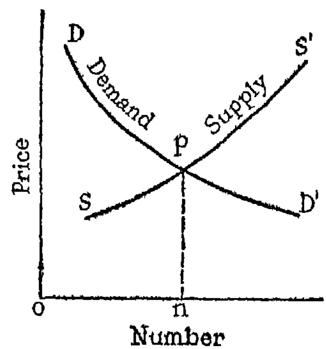


FIG. 1.—Supply and Demand.

In any case, if the end *D* goes continually upward this means that, however high the price rises, a few articles will be demanded—possibly as souvenirs or because of some essential or unique property. But if *D* meets the *Y* axis this means that at some particular upper price the sales stop entirely, probably because alternatives are obtainable. If the curve continues to infinity at *D'* this implies an endlessly increasing market as prices go down, probably because other uses are then found for the article. If it drops down to meet the *X* axis, this indicates complete saturation, no more being required however cheaply offered. Thus the steepness of the demand curve may be said to express the necessity or uniqueness of the article, whilst a more horizontal shape implies elasticity and possibilities of substitution.

Although expressed in terms of commodities, the demand curve applies almost equally to all other forms of wealth. It applies to services, to labour, to capital (whose price is the rate of interest) and even to such specialised forms of capital as land.

Supply.—In the case of ordinary commodities the law of supply is just the opposite of the law of demand. A rise in price which would

depress demand has the effect of stimulating supply, whilst a drop in price reduces the supply. A curve plotting price to a base of quantity supplied* slopes upwards as shown at SS' in Fig. 1. At any given price pn, the number offered for sale is on; any lower or higher price will result in a smaller or bigger number being produced and brought to market. The point p, where the demand and supply curves intersect, gives the price at which production will exactly balance sales.

The law of supply (as represented by a rising curve of price against quantity) is much less universal than is the law of demand. It applies to freely produced commodities and services but not necessarily to those of a monopolistic character or those in which there is government action either central or local. It is only partially applicable to land, capital and labour.

The time element is important in connection with both the above curves. In general, it may be said that time is required for a price change to achieve its full effect, but this time-lag is usually less with demand than it is with supply. In the case of demand, skilful publicity will greatly assist in shortening the time.

If, however, any considerable period of time is involved there will be other complications. In this and the next chapter the relationship of price and numbers is treated as far as possible in isolation, as at a single time and place. It postulates "other things being equal," which over a period of years they cannot be. The demand, for instance, is affected not only by price and income levels but by the availability of alternatives, by technical developments, changes in consumers' habits, and modes of life, etc., and with such a thing as electricity these will alter greatly over the years.

Quantity and Price Reactions.—It will be seen from the curves that any change in price has opposite effects on supply and demand. At one particular price, pn, the two exactly balance, and production equals consumption. If the price rises to a value oh (Fig. 2) there will in due course be a larger quantity, os, produced and offered for sale. But at this higher price the demand is a smaller quantity, od. The result will be a surplus, ds, and this will cause the price to fall until the demand overtakes the supply. Thus supply and demand react on each other, the mechanism of communication being the price. Price is the "cutting edge" of the whole machine, being both the means and the measure of the interaction which takes place.

* A note should be made regarding the order followed in the variables of these curves. The usual practice in graphing the relationship between two quantities is to plot the cause or independent variable as abscissæ along the base, and the effect or dependent variable as ordinates. In economic matters, cause and effect are often difficult to disentangle, but it would appear that in this case the standard order is not followed, *e.g.*, in the demand curve it is the low price that induces the large sales, not *vice versa*.

It is impossible to say where this set of reactions starts or finishes, since it forms an endless chain of processes, each of which is both a consequence and a cause. Probably the best starting point for study is the surplus or deficiency of goods, since this can most easily be visualised. Reverting to our imaginary market, directly a shortage appears in any commodity the stall-keeper will raise his price or he will be left without any to sell. The higher price will discourage purchasers and stimulate supplies, and in due course the stall-holder will find himself with a surplus, and the price will fall. But owing to the varying time-lags and human reactions the whole process is much more complicated and more uneven than would appear from the above.

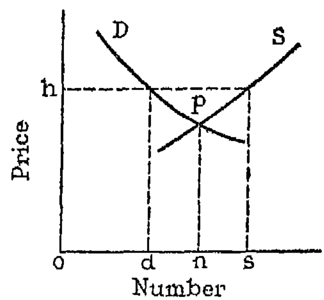


FIG. 2.—Price Reactions.

It will be seen that this chain of mechanism, consisting of surplus or deficiency, price, supply and demand, maintains a sort of precarious balance which it would be flattering to call equilibrium, although it is true that any change sets up forces which tend ultimately to remedy that change (and usually to produce the opposite change). Rather than use the word "stability" it would be more correct to compare the whole thing to the riding of a bicycle—self-adjusting although in unstable equilibrium—always in the act of falling, yet never (or hardly ever!) suffering a total spill. There would, in fact, appear to be a peculiar appropriateness in the use of the term "trade cycle"!

Factors composing Supply and Demand.—The preceding sections have treated the operations of demand and supply as though these quantities had an arbitrary existence of their own instead of being merely the expression of human needs and human productivity. The next step therefore is to examine the motive power behind this price machine, and to see how goods come to be demanded and supplied. This involves thinking in terms of people rather than machines, since it is human inclinations that prompt the demand and human disinclinations that limit the supply.

The driving power behind demand is clearly utility, and that behind supply is evidently costs. But a certain difficulty arises here. Price has been described as resulting from the interaction of supply and demand—it is like the handkerchief on the rope in a tug-of-war between two roughly equal teams. But when we look at the motive powers behind the teams no such approximate equality appears to exist. Price has a very close correlation to costs but it seems to have no correlation whatever to utility. On the contrary, the really useful things like bread and water, cotton and paper are cheap, whereas the things we can easily do without, such as champagne and silk dresses, are very expensive. It would therefore appear that the symmetrical disposition of supply and demand as equal arbiters in price-fixing breaks down as soon as we look at the underlying factors. Demand can only arise from utility, yet utility seems to have almost no effect upon price.

This difficulty can be explained by means of the conception of "marginal utility." When the housewife decides to buy 6 lb. of bread a week instead of five at a total cost of a shilling, this does not mean that its utility to her is only represented by 2*d.* a lb. It means that the utility of the extra loaf is worth that, in comparison with other possible purchases. If she were without bread the utility of a single loaf might be £1, and if the price were that high she might still be willing to pay it. But only when the price comes down to 2*d.* is it worth her while to buy a sixth loaf. If the baker were able to charge each individual household on a sort of two-part or sliding-scale tariff, it would be possible for him to charge heavily on the first loaf; but under normal conditions of competition, each loaf fetches only the price of the last one. Price is therefore not affected by the mean utility of the whole purchase but only by the marginal utility of the last increment that is just worth purchasing.

It is evident that the marginal utility of a commodity to an individual diminishes with every increase in the amount he has. But what is true of the single purchaser is true of users in the mass; so that, whatever the price, total purchases always approach the saturation value at which the last increment is just worth this much. In the case of luxuries, such as fountain pens, the margin may refer to extra purchases by existing users or to a "marginal purchaser." At a certain price it will just be worth while for a rich man to buy his third pen or a poor man to buy his first.

On the supply side the position is very similar, and somewhat easier to unravel. The dependence of price upon costs is more evident than is the dependence of price upon utility, but even in this case it is not the mean costs but the marginal costs that are important. Since the worst-advantaged producer is presumably making a living, the price must be such as just to pay his costs. Producers who are better placed secure a rent or surplus which may be retained or be "creamed off"

by the owners of the plant. (This "economic rent" therefore corresponds to the difference between mean utility and price on the demand curve.) Apart from this rent it will be seen that the height of the supply curve at any point expresses the cost of producing the incremental unit at that point.

Summary for Free Competition.—The general laws of supply and demand may now be summed up as follows (expressed, for convenience, in terms of commodities). The price of a commodity tends always to that figure at which supply equals demand. But supply is governed by costs and demand by utility, both these being marginal. Hence a commodity tends to be produced on a scale at which its marginal cost of production is equal to its marginal utility (each measured in money) and both are equal to its price.

The equation which governs the number of units changing hands can therefore be put into mathematical form as follows: Marginal cost = mean price = marginal utility (all in pence per unit).

Monopoly: Supply Curve.—Up to this point the treatment has supposed a state of pure competition. The next step is to consider the effect of monopoly, and the particular case of electricity supply. Monopoly may occur on either the production side (single seller) or on the consumption side (single buyer); but the former is more likely, and the only one to be considered here. Pure monopoly on the production side exists when the commodity or service is a well-defined and uniform one, and when there is only one producer of this particular commodity. There is hardly such a thing as a water-tight monopoly, since there are alternatives to almost everything. Even where there is a legal monopoly of public electricity supply, private generation is possible, and there are non-electrical methods of supplying the same needs. But technical, even more than legal, considerations limit this freedom in practice; and in the domestic field at least, however high the price may rise, it is rarely feasible for the individual householder to generate for himself. Nor are there effective alternatives to electricity for certain household purposes.

The starting point, as before, is to trace the demand and supply curves, of which the former, at least, will not be affected by a monopoly on the production side. It will be recalled that the demand curve shows the relationship between the number that will be bought at any given price. Its height gives the mean price per unit of all the units bought, but is governed by the *marginal* utility of the last one. The supply curve shows the relationship between the supply price and the number produced. Its height gives the mean price per unit of all the units supplied (or which tend to be supplied) but is governed by the *marginal* cost of the last one. If, in order to increase production from 1,000 to 1,001 units, total costs go up from £100 to £101, the

price paid for all units must be high enough to induce this extra production, namely £1 per unit, even though the average cost of all production is only a tenth of this.

With a freely produced commodity this supply curve is a rising one, since a greater quantity means the coming into operation of inferior land or less-efficient plant and undertakings, set in motion by the higher price offered. The price must therefore be sufficient to cover the cost of this incremental production, *i.e.*, it is marginal cost. Better situated land and producers will then earn a rent or surplus (called " economic rent "). But under monopoly production there is no open market on the supply side, and therefore no supply curve representing the amounts which the producers stand ready to sell at the given prices. The *S* curves in Figs. 3 and 6 represent merely the costs of the one producer, and the height at any point represents his incremental cost.

The cost of an incremental unit in monopolist production, such as electricity supply, is a somewhat academic conception, since while the annual output in kWh may change in small amounts it is difficult to think of the station size altering by tiny increments. If, however, one can imagine this to happen, then assuming all the supply characteristics to remain constant (such as load factor and spare-plant proportions), a change in the units supplied will mean a proportional change in every item in the system. Under these circumstances there is no reason why the incremental cost should be greatly different from the average cost for all the units, although it will probably decline slightly owing to the generally greater economy of large-scale production. Since the consideration of costs forms no part of the present chapter, the simplest likely assumption is best ; and in what follows, the incremental cost per unit is assumed to be constant. The supply curve is then horizontal and represents both mean and marginal costs. (It will be seen later that difficulties of allocation may make the *identifiable* marginal cost, at steady price levels, considerably below the mean cost.)

Price under Monopoly.—Fig. 3 shows the supply and demand curves (*S* and *D*) for a monopoly such as electricity. The point where they intersect (*p*) is a possible price solution, since it gives a value which would satisfy both parties to the transaction. Under free competition, the price would tend to settle at this point, the number sold being *on* at a price *np*. But under monopoly supply, the producer can fix a higher price than this, *and maintain it*, since there is no one to undercut him. He can, in fact, fix any higher price he likes (up to the maximum of *oa*) ; it is true he will sell less units, but he may make a bigger profit.*

* Since only economic factors are in question, any legal or other maxima are omitted from the consideration.

In order to see the economic limits to price-raising under monopoly it is necessary to draw a third line, the incremental or marginal revenue curve (I), shown chain-dotted in the figure. This curve plots the amount added to the total revenue by the sale of each additional unit.† Its relationship to the demand curve is given by the fact that for any number of units *om* and price *mq*, the area under the I curve (*akmo*) equals the rectangle *bqmo* (= total revenue).

If the monopolist wishes to maximise his profits he will fix the price from the intersection (k) of the I and S curves not the D and S ones

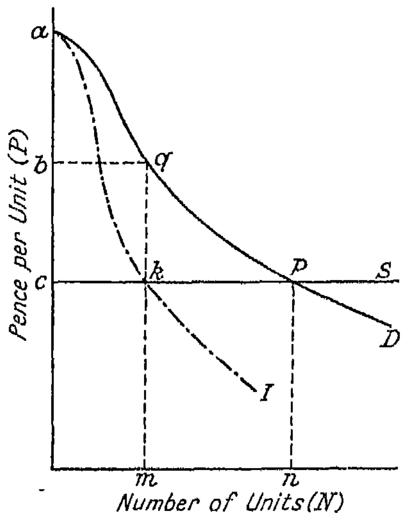


FIG. 3.—Price under Monopoly.

(*i.e.*, at the point q). His greatest surplus occurs when the last addition to the revenue just pays the cost of the extra unit sold. At this point the rates of change of income and expenditure (with reference to N) are equal and opposite, so that the surplus is a maximum. On the diagram, the income and expenditure are represented by the areas under the I and S curves respectively, so that the surplus is the area lying between them, *i.e.*, the segment *cka*, which is clearly a maximum at the point shown. (Another way of looking at it is to consider the

† To plot the I curve, multiply each ordinate of the demand curve (P) by its corresponding abscissa (N) to give the total revenue ($R = P \times N$). The slope of R will then give the height of I. R is not shown in the above figure, nor is the geometrical construction (known as graphical differentiation) by which I is obtained from R. (*N.B.*—All these graphs are drawn to the same base, namely the number of units N, and all except R have the same ordinates—pence per unit. The ordinates of R are scaled in pence.)

case shown in the figure where the supply curve S is horizontal, representing both mean and marginal cost. Then any given number of units om will sell for a mean price per unit of mq, but will have a mean cost of mk, giving a profit per unit of kq. In this case the product kq, $\times$ om represents the total profit on the transaction, and this product is a maximum at the point shown.)

Consider a monopolist supplier who has a free hand in price-fixing, and see what happens as the price is gradually lowered from the maximum of oa. Each successive price reduction means a bigger sale at a lower price per unit. So long as the I curve is positive (*i.e.*, for the whole of the portion shown in the figure) this swells the gross revenue. From a to q it swells the net revenue also, since gross revenue is increasing faster than costs are increasing. At the point q the profits are a maximum.

From q to p a profit is still being made on the transaction as a whole, but the profit gets less as the point moves down the curve. The demand curve is still above the supply curve so that the mean revenue per unit is greater than the cost. But the extra units sold in this portion add less to the revenue than they do to the cost, because in order to sell them the price has to be lowered on the existing sales. From the strictly profit-making point of view the expansion qp is therefore not worth while. On the other hand, from the public utility point of view it *is* worth while—only if the point p is passed will a loss be incurred. (It is understood that the supply curve covers all the essential costs and charges, including interest and repayment at fixed rates. The only item omitted is the fluctuating surplus or profit, *e.g.*, the amount by which the dividend on the ordinary shares exceeds the market rate of interest or the rate paid on the debentures.)

In fitting electricity supply into the above framework it must be borne in mind that, in the past, nearly two-thirds of the field has been occupied by local authority undertakings not working for pecuniary profit. Their chief interest was, or should have been, to extend the use of electricity as widely as possible within the limits imposed by running expenses and essential capital charges. The remaining part of the supply has been provided by companies, whose aim, or one of whose essential aims, was to make a profit. If each type of organisation achieved what it set out to do, there should have been some difference observable in the two results.

The above is a simplication of what is, in fact, highly complicated. The curves are often irregular and only partially known; theoretical generalisations are therefore extremely dangerous. Moreover, personnel counts for much, and many a man will work contrary to rational economic motives through sheer interest in doing a job well. But in spite of all these exceptions and cross currents it may be said that, granted sufficient economic enlightenment, the general tendency will be for a company to work towards position q and for a public body to

work towards p.* In what follows, the former will be referred to as a "profits" basis and the latter as a "costs" basis.

As in so many other cases, a warning must be sounded against taking these distinctions too rigidly. The points p and q are the two extremes between which actual practice may be expected to range; and they have been defined and labelled in order to clarify the issue rather than to indicate particular operating points.

It might appear that much of the present chapter is irrelevant to price-fixing if a purely costs basis is taken. It is then merely a simple question of whether a certain price reduction is possible, not whether it is worth while. For, provided the cost curve is flat or nearly so, it would seem the duty of the public authority engineer to fix the price at this height regardless of consequences. The demand curve is then only of interest in telling him what load to expect—it does not affect his (price-fixing) actions in any way. Actually (as is shown in Chapter III) there are other expenses, not comprised in the marginal cost, whose distribution should depend upon the demand curve; and consumer response is an element in pricing whether the undertaking is a public corporation or a company.

Two-part Modification: Summary for Electricity Supply.—There are several respects in which the actual facts are frequently more complicated than the above theory would indicate. It has so far been assumed that all the electricity consumed by any particular group is taken at the same price, *i.e.*, a simple flat-rate tariff is implied. This, however, is the exception rather than the rule, even within a single group, and the next step is to adapt the theory to a complex tariff.

Consider the case in which the tariff is in two parts, a standing charge of f pence per annum plus a running charge of r pence per kWh (f will usually vary either with the consumer's demand or his house size). The actual consumption must then be split up into two portions, the first block of n kWh representing the primary or high-yield portion whilst all the remainder forms the secondary or follow-on portion. Payment for the primary portion, including the service or convenience of having electricity for some particular purpose, is covered by a price of $f/n + r$ pence per kWh, whilst the subsequent units are priced at r pence per kWh. (A variable-block tariff, particularly if it has only

* It might be thought that a simple check on this would be to compare the mean prices that were charged by companies and by local authorities respectively. Apart, however, from the different character of the areas they usually served, there are a number of other complicating factors. For example, the amounts put by in depreciation reserves, loan repayment, etc., were considerably higher in the case of municipal undertakings, and this alone would be sufficient to level up present prices though it might accentuate future ones. Moreover, even the company undertakings, although their *raison d'être* was, of course, to make profits, acknowledged certain responsibilities as public utilities and did not necessarily operate merely as commercial enterprises.

two blocks, can obviously be fitted into the same framework, and so can a multiple tariff consisting of lighting and heating flat rates.)

On the principles here outlined, the price ruling in the first block ($f/n + r$) cannot be higher than the marginal utility of the block, *i.e.*, the usefulness (valued in pence) of the last unit consumed in that block. It must not be lower than the marginal cost of the same, *i.e.*, the cost of supplying the last unit; and it may be anywhere in between. Exactly the same remarks apply to the second block price (r).

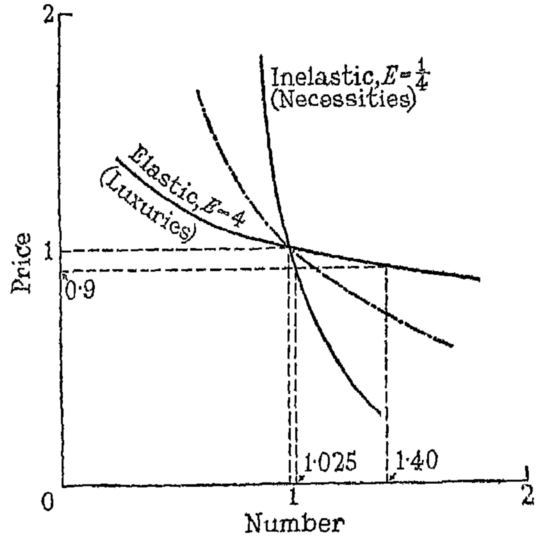


FIG. 4.—Demand Curves.

Provided the loads and prices can be sectionalised in this way, these principles will be found of general application to electricity tariffs. The price for any section of load must not be lower than the incremental cost and cannot be higher than the incremental utility of the particular service or market which it is desired to fill. (The word "cannot" means that if the price is fixed any higher the last unit will not be purchased.) Nor is it necessary to have the complete range of figures. Provided portions of the supply (incremental cost) and demand (incremental utility) curves for each load section are known round about the working point, the price can be fixed—either where these curves cross or somewhat higher.

Another important practical conclusion which emerges is the absolute necessity for complex tariffs. If all units are sold at the same price, this cannot be higher than the utility of the last unit sold. Mean price will be governed by marginal utility. But subsequent results will show that when the load is sectionalised the various demand curves prove to

be quite different, so that a price which approaches saturation on one curve is throttling another. If we are to sell, say, 1,000 units a year to a householder at a single flat rate, this must be no higher than the utility to him of the thousandth unit. If the price were a penny it is not enough that the total value of his electricity is a thousand pence or £4 3*s*. 4*d*. a year. At such a rate he might be very glad indeed to satisfy his more urgent requirements, but before he will plug in a bedroom fire which will bring up his consumption from 900 to 1,000 he must be satisfied that the extra comfort is worth the extra 8*s*. 4*d*.

Necessity and Luxury Demands : Elasticity.—Before applying the foregoing theory of price-fixing quantitatively to the case of electricity supply, it will be well to study the demand curves again. This will be done in terms of goods, but it should be stated that, in matters of demand, precisely the same laws apply to services, capital and land as to ordinary commodities. The first step in this study is to distinguish between necessities and luxuries, although it must not be forgotten that these are relative terms.

Necessities are those goods (or services) which are urgently needed by large numbers of people, but usually only in strictly limited quantities. Thus food and clothing in certain amounts are essential to each individual, but no one needs (and very few would want) large quantities of meals or clothes at any one time. The demand curve is then a very steep one, as shown in Fig. 4. Even a high price will not deter people from buying their irreducible minimum, and conversely even a very low price will not tempt them to buy in much larger quantities. Since the whole population is presumed to be already buying the article in question, there is no new market to draw upon, so that what is true of the individual applies also to the sales as a whole.

Luxuries are those goods or services which can easily be foregone, and in fact are foregone by large numbers of people even of those who desire them. It follows that if the price of a luxury article goes down, the sales are likely to go up very considerably, since there are large numbers of potential users to draw upon. The demand curve in this case is relatively flat, as shown in the figure, since a decrease in price causes a large sales increase whilst a rise in price may put the article almost out of the market.

Even within a single class of commodity, *e.g.* foodstuffs, it is evident that bread is, relatively speaking, a necessity whilst fowl and turbot are luxuries. If the price of bread went up, some people would no doubt eat less food whilst others would consume more potatoes and other cereals, but even so, the bread sales would be only moderately reduced. Conversely, if the price fell to one-half or one-tenth, the increase in consumption would not be enormous. But if the price of chicken were to fall in this ratio there would be many times as much consumed as there is at present.

It is usual to employ the word "elasticity" to denote the quality possessed by the flat curve, and "inelasticity" the quality of the steep curve. These are loose terms and a precise mathematical definition is given later. For the moment it will be sufficient to define them in terms of the intermediate (chain-dotted) curve by saying that when the demand curve is flatter than this the demand is said to be elastic and when it is steeper it is inelastic. It will be seen that elasticity is characteristic of luxury articles and inelasticity of necessities.*

Before leaving the subject it will be well to tabulate the various causes of sales elasticity, all of which will be found to be strictly relevant to electricity supply. When the price goes up, consumption may go down owing to (*a*) abstinence, (*b*) use of substitutes, (*c*) individual production. Thus, if the price of bread goes up, one may eat less food, one may consume more of the other cereals, or one may bake for oneself. In the case of a service such as electric light, one may use less light, one may use other illuminants or one may generate for oneself. Conversely, when the price goes down it is possible (*i*) to use more in the original capacity, and (*ii*) to develop other uses. Anything tending to make any of the above easier will increase the elasticity of the demand, and the flatness of the curve.

There is a further distinction which may be made. Except in the case of absolute necessities there are two distinct channels through which elasticity may operate, no matter which of the above causes it is due to. It may operate through additional consumption by existing consumers or through the connection of fresh consumers. It is often desirable to distinguish between these two channels, and with electricity supply it is a simple matter to do so. Curves plotted to a base of units per consumer show the former, whilst those showing consumers per head of population show the latter. The overall elasticity, shown by the units per head, is the resultant of the two.

Price-Fixing of Necessities and Luxuries.—In the discussion on price reactions under free competition it was implied that the price goes up or down automatically as a shortage or surplus reveals itself. But in almost all cases the actual price-fixing is done deliberately by the producer, although it is true that with freely produced commodities this balance of stock does furnish the motive power for price changes. Since the producer naturally tries to get the highest price he can, it will be well to consider just what limits this price-raising.

* Warning should again be given that all these terms—luxury, elasticity, and the like—denote relative quantities and not absolutes.

The demand curve is never completely vertical or completely flat: there are always alternatives or substitutes. It is easy to quote extreme cases, but most articles occupy some mean position in the chain which ranges from loaves to lipstick or from a water supply to a valeting service. And it is a commonplace that the luxuries of one age become the necessities of the next.

A rise in price normally produces two effects, both tending to check this rise, namely, a reduction in consumption and an increase in production. But in the case of necessities the former element is barely present, and only the latter is operative. Referring again to Fig. 4 the demand curve for an absolute necessity is almost vertical, so that a price change hardly affects the sales, and a rise in price cannot be relied upon (on the demand side) to produce a surplus of goods. Furthermore, when there is any check upon free production, and still more when there is a monopoly, the supply curve also fails to operate; and a rise in price cannot be relied upon (on the supply side) to stimulate production and produce a surplus. A private monopoly in an essential article or service is therefore deprived of both the natural checks on price-raising, and unless strictly controlled it is almost certain to result in exploitation.

There is therefore an important difference between necessities and luxuries in respect of the inducement they offer to price reductions. In the case of a necessity, with its inelastic sales curve, there is no check upon the price except through competition between producers, or by legislative or communal action. The price of bread is low because there is a choice of bakers or because the price is fixed by the State—an uncontrolled monopoly in private hands would here be a very great danger. But with luxuries, having a flat sales curve, there is another strong inducement to keep prices down, namely that of increasing the sales. Even with a single seller there is little to fear, because the article sold can so easily be done without.

It follows that the monopoly of supply in a necessitous service forms a very powerful weapon for ill or good. In private hands it tends inevitably to abuse unless checked by strict external control. In disinterested hands and wisely directed it can be an effective instrument of public service. For when both necessity and luxury consumptions are comprised in a single supply, it will often be possible to charge more highly on the necessitous consumption and less on the luxury consumption; thus developing the latter without materially damaging the former. There are, however, important limits which must be set to this process, and these are discussed in Chapter III.

The above can be more precisely studied from the curve and theory of Fig. 3. Under free competition the price is not determined by any one producer, and it tends to gravitate to the position p. But the monopolist can fix it at any higher figure he chooses, and up to the point q there is an actual inducement for him to do so. Now the difference between q and p is fixed by the difference between the mean revenue or demand curve (solid line) and the incremental revenue curve (chain-dotted line). This difference will become greater the steeper the demand curve.* Conversely, with a horizontal demand

* An exact proof is given in the Appendix to the author's paper, *Journal I.E.E.*, 1938, **82**, p. 206, from which the present chapter is extracted.

curve the two lines coalesce. Hence the steeper the demand curve, and the more necessary or unique the article, the greater is the power and risk of monopoly supply.

Mathematics of Elasticity.—It now becomes necessary to define elasticity in more precise terms. If we are speaking of any one point such as that where the curves cross in Fig. 4, there is no doubt as to what is meant—a small slope indicates a big change in sales for a given change in price, and *vice versa*. This change may be denoted by the general term "response"—the two thick lines then indicate big and small responses to price change, while the chain-dotted line shows a moderate response.

It might appear from this that elasticity could best be defined from the slope of the demand curve. This, however, would be misleading because almost all demand curves vary greatly in slope, being relatively steep at first and flat later. In some cases they bend into one or other of the axes: more often they bend away and become asymptotic to both axes. It is therefore desirable to define elasticity in relation to the normal, inversely related, load curve.

If the y and x variables of the demand curve are denoted by P (price per unit) and N (number of units) respectively, then the actual response of the curve is the rate-of-change of numbers with price, *i.e.*, the reciprocal of the slope dN/dP. But the elasticity is the *proportional* change of numbers with price: $= \delta N/N$ divided by $\delta P/P$. Numerically, the elasticity is measured by the percentage increase in sales resulting from a 1 per cent. decrease in price.

A criterion of elasticity can then be established as follows: If at all points in the demand curve the quantity sold is inversely proportional to the price $\left(N \propto \frac{1}{P}\right)$, *i.e.*, if the curve is a rectangular hyperbola, the elasticity is uniform throughout and may be regarded as unity. The total aggregate of expenditure on the commodity ($=$ price per article $\times$ number of articles) is then a constant. Such a curve is shown chain-dotted in Fig. 4.

To test the elasticity of a demand curve at any point, the method therefore is to draw a portion of the rectangular hyperbola through that point, and see whether the demand curve is steeper or less steep than the hyperbola (elasticity less or more than unity). In words, if a larger total amount is spent on a thing when its price is low than when its price is high (curve less steep than hyperbola) the demand may be said to be elastic, and if a smaller amount, inelastic. If the curve is horizontal anywhere, the elasticity at that point is infinity, and if vertical it is zero. Unit elasticity occurs when a 1 per cent. drop in price gives a 1 per cent. increase in sales so that the total revenue is unaltered.

It will be seen that the above description of elasticity refers not to the absolute response or slope of the curve but to its percentage slope. The elasticity may then be precisely defined as the limiting value (when the changes are made infinitely small) of the ratio of the proportional change in sales to the proportional change in price or $\delta N/N$ divided by $\delta P/P$, where N and P denote the number and price respectively. But since an increase in price brings a decrease in sales it is necessary to insert a negative sign; thus the elasticity is given by

$$E = \left\lfloor - \frac{\delta N}{N} \times \frac{P}{\delta P} = - \frac{P}{N} \times \frac{dN}{dP}\right.$$

When the total revenue is a constant, $N \times P =$ Constant (C) or

$$N = \frac{C}{P}, \text{ so } \frac{dN}{dP} = - \frac{C}{P^2}.$$

The elasticity, from the above formula, is then given by

$$E = - \frac{P}{N} \times \frac{dN}{dP} = \frac{P}{N} \times \frac{C}{P^2} = \frac{C}{NP} = 1.$$

It will therefore be seen that the formula agrees with the description in so far as it gives a unit value for the case described. (*N.B.*—The solid lines in Fig. 4 are portions of the curves plotting elasticities of 4 and $\frac{1}{4}$. A 10 per cent. drop in price—shown dotted—increases the sales by approximately 40 per cent. on one curve and $2\frac{1}{2}$ per cent. on the other.)

It is worth noting that all these sales changes, particularly the increases which follow from price reductions, are not immediate and automatic—their full effect only comes as the result of time and publicity.

Electricity Uses and Demand.—It is easy to see examples of the above in electricity supply, particularly in the domestic sphere. Owing to the wide range of uses, the demand curve may be anything from the very steep to the very flat, since electricity is a necessity for some purposes and a luxury for others.* One cannot drive the domestic cleaner with an oil engine, one cannot have gas plating, or coal-driven telephones, nor can one run the all-mains set off the water mains. For such purposes electricity is indispensable, and the same thing is now almost true of lighting. Such a demand is inelastic, and, however high the price, one must pay it or generate for oneself. Conversely, if the price is reduced, this will not mean that the vacuum cleaner or the wireless will be used any more than they were, or even that the lamps will be always left burning.

* In using the words "necessity" and "inelastic" to describe the type of demand which is insensitive to price changes, the word refers to the electricity and not to the service. An electric washing-machine may be a luxury, but once the apparatus is installed electricity is a necessity for its operation, and the price of supply has little effect on the usage.

A fall in the price of electricity for purposes such as the above will produce comparatively little increase in load. Very few present users would *double their lighting consumption* if the price were suddenly halved, and the same applies even more to such things as cleaners and irons. It is true that in order to get the overall elasticity one must add to the elasticity of existing consumers the elasticity of new connections. But the growth in connections depends on the proportion of unwired houses in cabled streets, the cost of and facilities for wiring, and so many other factors besides the price of energy, that the elasticity on this score is not likely to be very great. Even allowing for this, it is probable that a drop in price to one-half would not increase the lighting load by more than 30 per cent.

Compare all this with the consumption, again in the home, for room heating, cooking or water heating. In each case there is an alternative, not so good perhaps, but quite effective. Electricity still has its peculiar merits : worth much, but hardly priceless, and unless the tariff is low the cheaper alternatives are likely to prevail. Such a load is extremely responsive and elastic, and under these circumstances a reduction in price to one-half might increase the consumption by several hundreds per cent.

Power loads lie somewhere between the two extremes just mentioned, since they are more closely competitive (and therefore more elastic) than lighting loads, but less so than heating loads. Electric power from the mains is usually very much better and cheaper than anything else available (particularly to the small-scale user), but not overwhelmingly so. Electricity then is valuable, but not indispensable, and the demand curve has an intermediate slope. The magnitude as well as the type of use is important here, and a price which will satisfy a baker for driving his dough mixer will not tempt him for heating his oven, nor will it secure the power load of the large industrialist. Another special feature of the power-load elasticity is that the time-lag is much greater than with the domestic load, since the total power requirements are almost unaffected by the cost of energy whilst the change-over from other drives takes time to effect.

The above notes have referred more particularly to the shape of the demand curve, but similar remarks apply to its height. For whilst the slope of the curve indicates inflexibility of demand (based on uniqueness), the height indicates intensity of demand (based on utility). Now there is evidently no intrinsic measure of the relative utility of, say, boots and books or light and heat, and if there were it would vary from person to person. It is often said that the " effective yield " of a kilowatt-hour is far greater when used in lighting than when used in heating—it will light a room for ten hours on a dark winter's night whereas it will only warm the same room for about twenty minutes. But this is because we have already formed some opinion as to what should be the relative costs of the two services. Ultimately there is

no measure of the value of the yield except what people are willing to pay for it, and this in its turn is a sort of balance between the intensity of the need and the cost of alternative methods of satisfying it. In electricity supply therefore the height of the demand curve as well as its slope is chiefly dependent on what the electricity is used for, and what are the costs of other sources or substitutes.

The following is a list of uses of electricity, roughly in the order of their rigidity of demand—itself a measure of their competing power with the alternatives available. In all cases there is the alternative of private generation which becomes more feasible the larger the size.

Small specialised uses: timekeeping, wireless reception, small-scale communications, plating, battery charging, etc.
Very small power: vacuum cleaners, grinders, drills, slicers, etc.
Lighting.
Medium and large power: industrial, traction, etc.
Heating.

The value to an individual of the different services could be roughly assessed by supposing the energy price to be gradually lowered in the manner of a Dutch auction. A typical householder, for example, might decide that electricity was *just* worth while at 1*s*. a unit for operating a wireless set, that it was just worth 8*d*. a unit for vacuum cleaning, 6*d*. a unit for lighting, 1*d*. for cooking, and so on. It will be noted again that the size as well as the purpose of the application is an important element in demand elasticity. When only small amounts of power are required, as in most refrigerators and washing machines, the cost of the apparatus is usually the determining factor, and the price of energy hardly affects the use at all.

Without unduly anticipating what follows, the moral of the above may be briefly pointed out. The more nearly essential and unique the services conferred by electricity, the higher and steeper will be the demand curve, and the higher can be the price (in so far as this is allowed to be governed by demand). When the service rendered is a strictly competitive one the price must be so too, since the load will then be sensitively dependent upon the tariff figures. When both sorts of consumption occur within a single household or factory, the situation can be met either by separate metering or by a fixed charge corresponding to the rigid portion of the consumption.

CHAPTER II

ELECTRICITY DEMAND

Demand Curve Analysis.—The previous chapter described the general theory of price-fixing. It dealt with relationships rather than magnitudes, and such electrical examples as were given employed either hypothetical figures to illustrate the theory or else *a priori* arguments as to what might reasonably be expected to happen. The present chapter is a slight attempt at a quantitative evaluation of electrical demand, and a fuller account of the work will be found in the paper already referred to.*

The main difficulty in obtaining suitable data is the difficulty common to all economic problems, that of isolating the variables. The first step is evidently to separate the various loads into their different groups. The two principal groups are the industrial power and the lighting, heating and cooking, which between them account for over 95 per cent. of both units and revenue. (Fig. 32 on p. 177 shows the results graphically.) The industrial group is relatively uniform and homogeneous : the electricity is chiefly put to one purpose, namely, power production, and the competitive differences within the group are a function of size rather than of kind.

The second group, consisting largely of domestic consumption, includes a much greater variety of uses which compete at a number of very different price levels. The utmost differentiation that one can hope to achieve is to split the domestic consumption into two portions, the high-yield or necessity portion comprising lighting, cleaning, wireless, etc., and the low-yield or luxury portion comprising heating and cooking. Lighting is the type and forms the bulk of the first portion, and when there are separate tariffs it is frequently the only use included. It will therefore be convenient to use the words "lighting" and "heating" to indicate the two portions.

A third type of consumption included under the heading "lighting, heating and cooking", and therefore difficult to separate out, is the commercial—shops and office lighting and heating. Even when all these difficulties have been overcome, and a pure single-use demand curve is obtained, it is desirable to split this up still further into its two components.

Demand and Elasticity Components.—The total demand or number of units changing hands can be regarded as the resultant of

* *Journal I.E.E.*, 1938, **82** p. 185.

two components: number of consumers and number of units per consumer. Each of these components has its corresponding elasticity, since consumption may change for two different reasons:

(*a*) change in the number of consumers,

(*b*) change in the consumption per consumer.

(For simplicity, it is assumed that these two factors operate independently and that (*a*) has no effect upon (*b*)—in other words, that the average new consumer has the same consumption as the average existing consumer.)

These must be studied separately, since certain variations affect them differently. It will be obvious that under a two-part tariff both parts will affect (*a*), but only the running charge will affect (*b*). Again, the time-lag will be much greater for (*a*) than for (*b*); cheaper electricity can only result in more consumers over a period of years, whereas it may result in immediate increase in consumption by existing consumers. Again, wiring facilities will affect (*a*) but not (*b*). Finally, the (*a*) component of elasticity is almost irreversible whereas the (*b*) component may work either way. Consumers may vary their consumption up or down as the running charge goes down or up, but householders who become consumers because the tariff is low are unlikely to disconnect when the price goes up.

It will be noted that whilst the total consumption is the product of its two components, the overall elasticity is the *sum* of the component elasticities. Thus, if a 1 per cent. drop in price gives a ½ per cent. increase in consumers and a 2 per cent. increase in units per consumer, the increase in units will be 2½ per cent. and the overall elasticity will be 2½.

This can be proved as follows:

Let the number of consumers or connections be denoted by N_a and the corresponding elasticity by E_a.

Let the number of units per consumer be denoted by N_b and the corresponding elasticity by E_b.

Then the number of units $N = N_a \times N_b$

The "connection" elasticity E_a is by definition $\dfrac{P}{N_a} \times \dfrac{dN_a}{dP}$

(The negation sign is omitted in this and subsequent expressions.)
Similarly, the "consumption per consumer" elasticity is

$$E_b = \frac{P}{N_b} \times \frac{dN_b}{dP}$$

The overall elasticity $E = \dfrac{P}{N} \times \dfrac{dN}{dP} = \dfrac{P}{N}\left\{ \dfrac{d}{dP}(N_a \times N_b) \right\}$

$$= \frac{P}{N_a \times N_b}\left\{ N_b \frac{dN_a}{dP} + N_a \frac{dN_b}{dP} \right\} = \frac{P}{N_a} \times \frac{dN_a}{dP} + \frac{P}{N_b} \times \frac{dN_b}{dP}$$

$$= E_a + E_b$$

Domestic Demand Curves.—In order to lessen the labour involved, only those undertakings selling ten million units a year were considered; but as these accounted for 88 per cent. of all the domestic units sold in the country they may be regarded as fairly representing the whole field. Target diagrams were then constructed, plotting energy price p against domestic consumption N, each separate undertaking being represented by a point on the diagram (crosses for local authorities and circles for companies). Three such diagrams were constructed, plotted against units per consumer (a), consumers per head of population (b), and units per head (c). The last of these is reproduced in Fig. 5; and it will be evident that this overall demand is the product of the two components, or $c = a \times b$. In plotting the component curves, the number of domestic consumers was taken as 90 per cent. of the total number of consumers in the undertaking's area as given in the Electricity Commissioners' Statistical Return.

In each case a middle curve was drawn as nearly as possible between the various points, as indicated by the dotted line in the figure. Considerable latitude is possible in plotting this curve, and only the general trend can be pronounced upon with any certainty. There is, however, one check, since any abscissa on the (c) curve must be the product of the corresponding abscissæ on the (a) and (b) curves. A few of the points lie right off the curve (as indicated by letters in the figure), usually consisting of wealthy residential or shopping areas in which a high price has not prevented a high consumption. Some other points lie off the scale of the graph but not off the curve direction, and are indicated by arrows.

In Fig. 6 is shown a portion of the demand curve (D) reproduced from Fig. 5. On the same graph is shown the incremental revenue curve (I) derived from D in the manner already described (see footnote, p. 11). In order to illustrate how demand economics can be utilised in price-fixing, let it be supposed that the cost of supply is constant, namely, 1·2*d.* per unit (line S). This line would cut the D curve to the right of the figure at a point representing about 420 units per head. On a "costs" basis the price would be fixed at this point, since it covers all expenses and brings in the maximum possible load.

At any higher price and smaller number of units there would be a surplus or profit per unit represented by the vertical height of D above the horizontal line 1·2. If this intersection be multiplied by the number of units it will give the total profit obtained, and this total is a maximum at the point k. On a "profits" basis the price would be fixed here, namely, at 1·6*d.*, and would bring in a load of 200 units per head. The total profit would then be 0·4*d.* × 200 = 80*d.* per head.

It will be noted that the I curve becomes negative on the left (elasticity below unity) and has a peak in the middle. (The actual I curve showed two distinct peaks which have been merged into one in the figure in order not to complicate the foregoing explanation.) In fact,

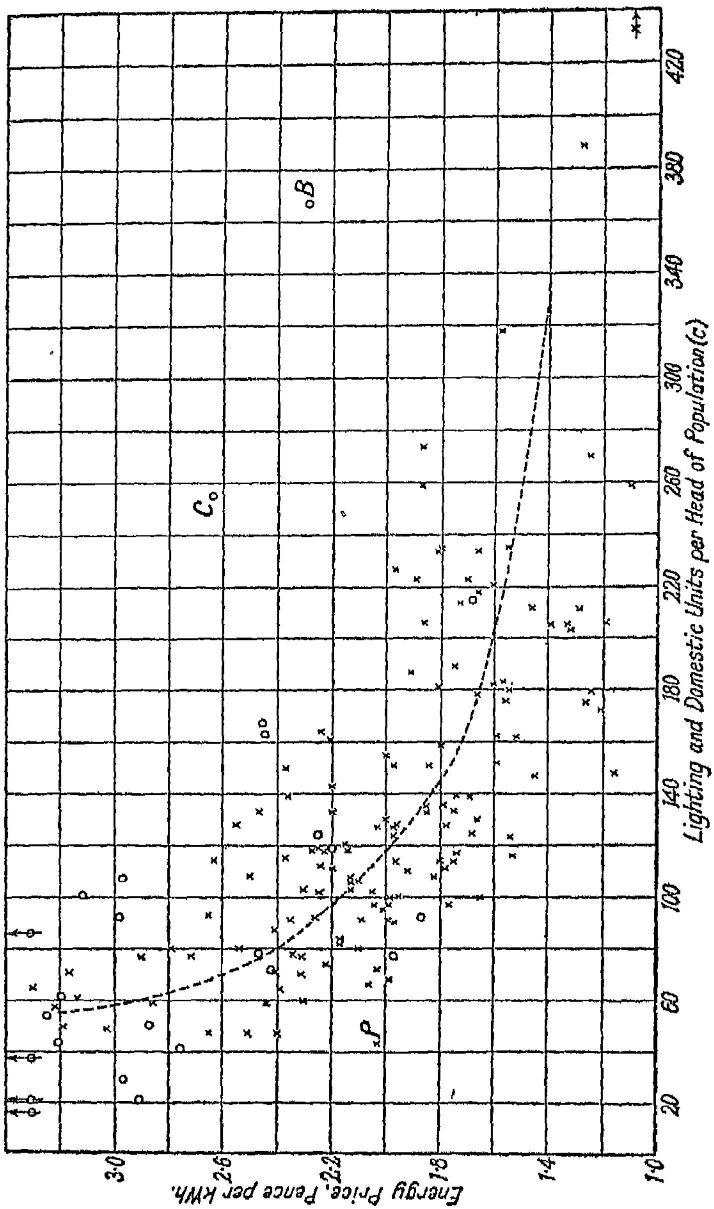


FIG. 5.—Domestic Demand Curve. P = Poplar C = Chelsea. B = Brompton and Kensington.

the downward sweep of the *I* curve is part of our old friend the vicious circle—high prices and low consumption. To the left of this point it does not pay to reduce the price but rather to increase it and contract the sales still further. The only way to break the circle is to make a big price reduction on to the other side of the peak. It is evident that when the price is high and the consumption per head is low the use is generally for " necessity " purposes and within a restricted range of consumer-incomes. Undertakings thus placed must be willing to plunge or they had better stay where they are ; a half-hearted advance would be, financially, a retrogression.

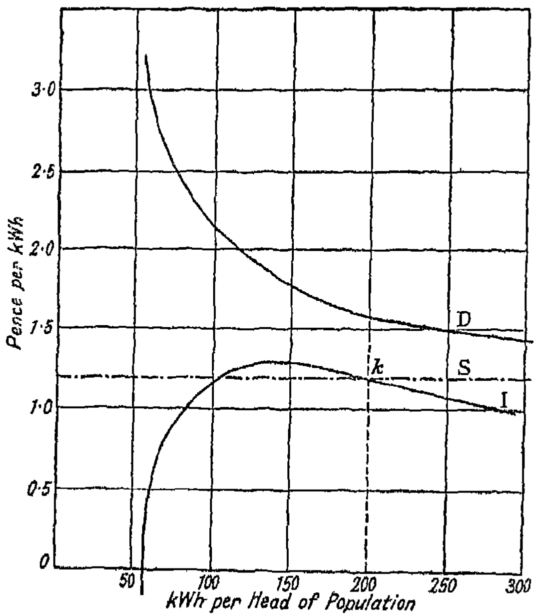


FIG. 6.—Incremental Revenue.

Domestic Elasticities.—In the previous section the correct price under either a " costs " or a " profits " basis is shown by the intersections of the various curves. The same thing can be expressed algebraically and can be extended to include the concept of elasticity. Let *P* be the price or height of the demand curve for any given sales *N*, *S* the (incremental) supply cost, *I* the incremental revenue and *E* the elasticity $\left(=\frac{P}{N} \times \frac{dN}{dP}\right)$. Then on a " costs " basis $P = S$, whilst on a " profits " basis $I = S$, and $P = S \frac{E}{E-1}$.

This last expression (which is proved in the paper to which reference

has already been made) provides an alternative method of arriving at the correct price on a "profits" basis. Provided the elasticity and the supply cost are known in the neighbourhood of the projected price, it is possible to calculate the right figure in a single operation. Moreover, the plotting of elasticity rather than incremental revenue is a useful method of exhibiting the fluctuations of load response and the desirability of changing an existing tariff. Price reductions without loss of profits can then be carried out down to the point at which P equals $S\dfrac{E}{E-1}$.

It will be noted that all the graphs so far plotted have the same base, namely, the number of units changing hands. At this point, however, a change is advisable, since in studying revenue and elasticity it is more useful to know what energy price they are associated with, than what number of units. Furthermore, in the great majority of cases it is a drop in price that is contemplated rather than a rise, and the question is, what sales increase or what revenue change can be anticipated when the price is reduced from this to that? The next figure therefore has a base of energy price, and this is scaled with the zero on the right.

By reading off values from the domestic demand curves and taking differences $\left(\dfrac{P}{N}\times\dfrac{dN}{dP}\right)$ the curves of Fig. 7 were plotted. The top curve c' shows the elasticity of units per head, taken from the overall demand curve of Fig. 5. The lower curve and the difference show the two component elasticities, namely, those of units per consumer (a') and consumers per head (b'). In the case of elasticity, the overall figure (units per head) is the *sum* of the other two, or $(c') = (a') + (b')$.

Taking the top curve showing overall elasticity, the following points should be noted. Above 3d. per unit the elasticity is actually less than unity. There is then no possible object from the profit-making point of view in lowering the price, for even if the additional units thus sold cost nothing at all to supply, the extra business would still not be worth having. For example, consider the case in which 100 units per head of the population served are being supplied at *3d.* a unit. An elasticity of unity means that in order to increase the sale by 10 per cent., the price must be dropped by 10 per cent. By selling 110 units a head at *2·7d.* will bring in the same gross revenue as before* and therefore a smaller amount of profit since the extra 10 units cannot be supplied for nothing.

Even with an elasticity of $1\frac{1}{2}$ the selling price should not be less than three times the cost if profits are to be maintained. Hence it may be said that unless the overall price in the above case can be made less

* Since the changes are finite and not infinitely small the results are only approximate.

than $2\frac{1}{2}d.$ it is more profitable to raise the price rather than lower it. At $3d.$ or over, even the gross revenue will increase with increase of price, and of course profits will increase still more since less units will have to be supplied.

Considering next the more desirable portion of the curve, at prices below $2\frac{1}{2}d.$ the elasticity rises to 2 and price reductions start to become profitable. Selling price here need only be double the cost for profits to be a maximum. Between $1{\cdot}8d.$ and $1{\cdot}5d.$ the elasticity has a further and very marked rise, though it shows a tendency to

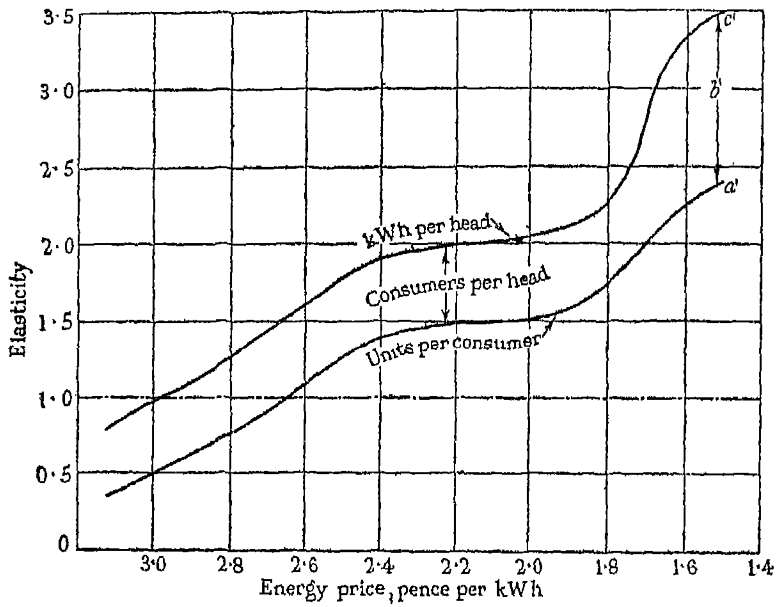


FIG. 7.—Domestic Elasticities.

flatten out somewhat at $1.5d.$ (the data are barely sufficient to establish this point). Elasticities of 3 and over are fully obtainable in the neighbourhood of $1{\cdot}5d.$, and until the figure drops to 2 or less there is no appearance of anything like saturation. Taking the maximum figure, namely, $3\frac{1}{2}$, and assuming for the sake of example a supply cost of $1{\cdot}2d.$, the correct price on a profits basis would be given by :

$$P = S\frac{E}{E-1} = 1{\cdot}2 \times \frac{3\frac{1}{2}}{2\frac{1}{2}} = 1{\cdot}6d. \text{ (approx.)}$$

and this agrees with the construction already shown in Fig. 6.

There is one important point regarding the price scale in Figs. 5 to 7. These show the *overall* price per unit for domestic consumption,

whereas the economic determining factor is the *incremental* price. This means that the curves really apply to considerably lower energy prices than those shown. Thus an overall price of 1·5*d.* will include undertakings in which the heating or follow-on rate of a two-part tariff is, say, ½*d.* and in which the majority of the consumers are taking full advantage of these rates.* The overall price of 2·5*d.* will represent, say, ¾*d.* follow-on rate and a rather smaller proportion of fully-developed consumers. Since the economic choice is determined by the marginal price of the additional units, these should be taken as of the order ¾*d.* at the 2·5*d.* mark and ½*d.* at the 1·5*d.* mark.

There are certain broad conclusions which may be drawn. Above 3*d.* the consumption is almost entirely in the "necessity" lighting group, elasticity is low and small price-reductions are unprofitable. As the price is reduced below 3*d.* there are two marked rises in elasticity occurring at about 2¾*d.* and 1¾*d.* The first of these probably represents the beginnings of non-lighting consumption. The "follow-on" or heating rate is here about 1*d.* or ¾*d.* and electric heating becomes a possibility. The second big rise (at 1¾*d.* or, say, ½*d.* marginal) probably refers to the full development when electric heating becomes a really attractive proposition.

The component parts of the overall elasticity are also of some interest. The elasticity of consumers per head of the population is small and fairly constant at a figure of about ½. This means that a 1 per cent. reduction in price will produce about a ½ per cent. increase in the number of consumers. The elasticity of units per consumer makes up much the greater part of the total elasticity and is, moreover, the element that varies with the price position. That is to say, the variations in elasticity at 2¾*d.*, 1¾*d.*, etc., discussed above, are almost entirely due to the consumption response of existing consumers rather than to any growth in their number.

Single Undertaking (Domestic).—It will be perceived that the above analysis suffers from two serious flaws. It is unable to distinguish between lighting and heating consumption, and it is worked out on a basis of overall price. Unless the tariff is a flat-rate one, the overall price is a composite figure and does not represent the marginal price which is really the determining factor.† Thus, while much useful information can be obtained from these curves, it is difficult to apply them quantitatively.

One way of getting over both these difficulties is to take a single

* In a house with a fixed charge of £4 p.a. and a running charge of ½*d.* the overall price would be 1·5*d.* per unit when the consumption was 960 units p.a.

† The two-part tariff difficulty is a very real one, and is now much more serious than it was when these illustrations were prepared because of the bigger proportion of consumers on a two-part tariff. Under it, the type of curve shown in Fig. 5 (especially if plotted for individual consumers) can be regarded as representing, not a consumer at all, but the automatic operation of a two-part tariff.

undertaking, giving lighting and heating on separate meters and having sufficient history to illustrate a number of price changes. Such an undertaking is Hampstead, and by the courtesy of the Engineer and Manager, H. Brierley, the following results were obtained. Until the end of the period illustrated below, all the domestic electricity in Hampstead was sold on a multiple tariff with two flat rates, and figures are available not only of the numbers and prices of the units but also of the number of separate connections of each sort.

The total period covered is fourteen years, from 1920–1934, an inter-war period during which there were substantial changes in the general price level. Another difficult variable in a time-period is the number of separate households, representing the possible clientele for electrical development. In order to allow for this, the total number of separate assessments for local taxation was obtained for each year, and the electricity sales were divided by this figure. The overall consumption, instead of being units per head of population served, thus becomes units per assessment, while the components of this are units per consumer and consumers per assessment. The last figure slightly exceeded 100 per cent. in the lighting connections in 1934, indicating that there were rather more lighting consumers than separately rated premises.

It is impossible to express the elasticity pure and simple because of the change of date. In this period of fourteen years there have been extensive changes in housing and domestic habits, use of wireless, etc., and probably some change in income-level, to say nothing of the steady pressure of publicity, the increased prestige of electricity and the rising standard of living. Superimposed on this was a big drop in the general price level. A growth in consumption over the period is therefore only partially attributable to the elasticity *per se*—that is to say, the response to price changes.

What the figures can bring out, however, is the difference in the relative elasticity of lighting and heating. Instead of giving the total price change over the period, the fourteenth root of this has been taken so as to show the average percentage change per annum, *i.e.*, the mean ratio of each year's figures to the previous year. The results are displayed in the table on the next page.

In the case of lighting there was an average price reduction per annum of 6·8 per cent. This resulted in a total sales increase per annum per household of 7·2 per cent., made up of a 3·1 per cent. rise in units per consumer and a 4 per cent. rise in consumers per household. In the case of heating, an almost equal price reduction ratio gave increases of just about double those of lighting in each case. If the percentage change in overall units (*i.e.*, per household) be divided by the percentage change in price, a figure of 1·06 is obtained for lighting and 1·80 for heating. This figure might be termed the relative elasticity or elasticity-plus-time factor, since it is the combined result of pure elasticity and the various time elements enumerated above.

LIGHTING

	Price.	Units per Consumer (*a*).	Consumers per Assessment (*b*).	Units per Assessment. (*c*) = (*a*) × (*b*).
At 1920 . . .	8*d*.	368	0·66	243
At 1934 . . .	3*d*.	560	1·15	643
Mean annual change	− 6·8%	+ 3·1%	+ 4·0%	+ 7·2%*

HEATING

	Price.	Units per Consumer (*a*).	Consumers per Assessment (*b*).	Units per Assessment. (*c*) = (*a*) × (*b*).
At 1920 . . .	2*d*.	916	0·14	127
At 1934 . . .	0·6*d*	2,130	0·41	880
Mean annual change	− 8·2%	+ 6·2%	+ 8·1%	+ 14·8%*

Comparison Figures.—For purposes of comparison the following figures may be cited, referring to the total domestic consumption of all authorised undertakings. The first line refers to the fourteen years ending 1935, *i.e.*, approximately the same period as in the Hampstead case. As before, the fourteenth root of the change ratio has been taken, so as to show the mean annual percentage change. It was not until 1927 that the Commissioners started to list the number of consumers, and the second line of the table refers to the last nine years only. This shows not only the overall consumption per head (*c*) but also its component parts (*a* and *b*).

Length of Period (ending 1935).	Price (*p*).	Mean Annual Change in :			Ratio of Changes (*c*/*p*). (Relative Elasticity).
		Units per Consumer (*a*).	Consumers per head (*b*).	Units per head (*c*).	
14 years . . .	− 7·5%	—	—	16·5%	2·2
9 years . . .	− 6·9%	− 0·4%	15·4%	15·5%	2·2

It may be thought that the Hampstead figures show up poorly by comparison with these last, since Hampstead gave a relative elasticity of 1·06 and 1·80 for lighting and heating, whereas these latter give 2·2 for domestic supplies as a whole. When the overall figure is analysed, however, it is found to be due entirely to a very high ratio for consumers per head of population, namely, 15·8 per cent. increase for 6·0 per cent. decrease in price, giving an elasticity ratio of 2·3. The units per consumer have actually dropped some half per cent. per annum compared with a rise in Hampstead of 3 per cent. (lighting) and 6

* $c' = a' + b'$ approximately.

per cent. (heating) for about the same price change. In fact, the elasticity throughout the country was almost entirely due to the taking on of fresh consumers, which is far from being the most profitable type for the undertaking.

Later Domestic Figures: Two-part Tariff Difficulties.—An investigation into the electricity-demand : price : income-level relationship in three large Dutch towns and in the whole of Holland is described in a Dutch paper.* (It will be noted that the effect of changes in income level has been neglected in the other examples of this chapter.) In this study it was not found possible to break down the price under complex tariffs into its component parts. A comparison was therefore made between mean price per unit and consumption per head of population, in which a significant correlation was observed. The results appeared to be roughly in agreement with those of a previous investigation in which an elasticity of demand of about 1·8 was obtained. (It will be noted that this was the figure obtained for heating supplies in Hampstead.)

Whilst not presuming to criticise these particular results without fuller knowledge, and realising that some degree of correlation may be traced between even the most distantly related variables, one may express scepticism as to the value of studying the relationship between quantities having no logical connection. There is no rational reason why a two-part tariff consumer should vary his consumption because of changes in the standing charge, and even if he does so his action might be in either direction. If the standing charge goes up he may argue that he might as well use electricity freely so as to get as much value as possible from an overhead cost which cannot be avoided. Alternatively, owing to income shortage or the endeavour to work to a fixed budget, he may consume less so as to keep his bill nearer to what it was. Whichever way he argues, the results cannot be said to give a measure of elasticity.

Not only is the mean price made up of several components but so is the overall consumption (consumption per consumer, consumers per head of population, and population of area of supply). If for example the number of consumers is approaching a saturation point equal to the number of households its elasticity may be almost zero at a time when the elasticity of units per consumer is very high. Unless the two can be separated the results may be just a blur. To plot overall consumption against mean price per unit is to use a blunderbuss where a precision rifle is called for.

The difficulty should therefore be avoided by breaking the problem down into every possible component. In the first place the domestic consumption must be isolated from all other groups, or the results may

* *Statistical and Economic Researches of the Central Office of Statistics*, Vol. 3, No. 1, March, 1948.

be swamped by industrial or commercial loads. Consumption per two-part consumer should then be plotted against running charges whilst consumers per head of population should be plotted against standing plus running charges, suitably weighted, or failing this against mean price per unit. The addition of the two elasticities, plus the proportionate increase in the population, will then give an idea of the change in total consumption with change in price.

Industrial Demand.—The method of a target diagram of separate undertakings is little use for the industrial load, owing to the absence of any common denominator. In the case of the domestic load, the units were divided by the population served or by the number of consumers, whilst the number of consumers was examined on the basis of the number of householders. All of these are rational bases for domestic load computation because everyone has to live somewhere, and every house is a potential electrical market. Moreover, the basic needs per head as regards heating and lighting are fairly constant and uniform, and the only serious complicating factor is the varying degree of wealth available for satisfying these needs.

But though we all live in houses we do not all work in factories, so that the potential power load is very unevenly distributed. Some undertakings serve largely the wealthy or retired and others the business population, and in such areas the industrial load will be small whatever the price. There can be no useful comparison in this matter of industrial load response between residential areas such as Hampstead or Harrogate and manufacturing centres like Huddersfield or Birmingham; however low the power rate might be in the former places, the load would be insignificant. Population, in fact, means houses and families, but it does not necessarily mean mechanisable industries. Thus it follows that while domestic units per head form a tolerable comparison between different areas, industrial units per head are entirely useless.

Equally useless is any splitting up of the demand into units per consumer and consumers per head. The industrial consumer (*e.g.*, factory manager) does not represent an electrical or population entity like the householder does, and such an analysis would have very little meaning. As a result of these difficulties, power comparisons must be carried out over a period of years and a succession of price changes, either for a single undertaking or else for an aggregate, the area served being the same for each point considered. In order to eliminate a further variable, namely, the state of employment, the units in the following case are divided not by the population, but by the average number of insured persons in employment during the year. (This omits all agricultural workers.)

The table below refers to all authorised undertakings, and it covers the last fourteen years for which figures were obtainable. As in the

previous instance these fourteen values may be regarded as a series of thirteen change ratios. Taking the thirteenth root of the total change will give the average ratio which each year bears to the previous year. This gives the following results :—

Mean annual change in price (p) . . .	—6·7 per cent.
Mean annual change in units per worker (c)	9·5 „ „
Relative annual elasticity $\left(\frac{c}{p}\right)$	1·42 „ „

This last figure may be compared with the Hampstead figures of 1·06 for lighting and 1·80 for heating. The industrial relative elasticity appears to lie about midway between these other two.

Inference from Trading Results: Other Factors.—It has been noted that, if the elasticity is less than one, a decrease in price must result in a loss on the operation even though the extra units sold cost nothing at all to produce. With an elasticity greater than unity there is a possibility of a profit resulting from price reduction, provided the increased units are not expensive to produce. These facts, in conjunction with past history of electricity supply in Great Britain, enable some estimate of elasticity values to be made indirectly.

It is clear that, in the past, electricity undertakings in this country (companies as well as local authorities) have reduced domestic electricity prices not only without incurring serious loss but often with notable increase of profits. Frequently the altruistic intentions of public authority undertakings and the hardheaded business acumen of farseeing company managements have led to the same price reduction procedure even though for different reasons. Generally the changes have been made on evolutionary lines without any abrupt alteration in policy, but there have been cases where the revision was so drastic that an immediate comparison can be made between the position before and after the changes were effected.

An example was quoted by Professor Miles Walker* referring to the Oxford undertaking which changed hands in 1931, following which the new management introduced a drastic revision of the previous tariff policy. The result was that as regards domestic prices it changed almost suddenly from being one of the highest to one of the lowest in the country. After two years of technical preparation, intensive domestic development was set on foot in 1933, and by 1935 an increase in consumption of 79 per cent. was registered over the corresponding quarter a year previously.

Speaking of the earlier tariff, the author says : "It was probably thought at the time that these tariffs were arranged to ensure the largest possible dividend. It is certain that a small reduction in the prices would not have been beneficial from the shareholders' point of

* "The Prices for Electric Supply," *Journal I.E.E.*, 1936, 79, p. 511.

view, because nearly all the consumers who could afford to pay high prices were probably connected and a small reduction would not have brought in enough extra consumers to pay for the reduction in the bills of those already connected. It required some courage to reduce the prices to less than one-third for large domestic users."

Financially the results, so far from spelling red ruin, were that the total net income earned was a greater percentage of the total capital involved than it was when the prices were high. In other words, had the management retained its company structure whilst carrying out the development policy it could have actually increased its dividends slightly. The change could therefore have been justified, not merely as a piece of municipal benefaction, but purely as a commercial venture yielding higher profits than under the old tariffs.

The above result was exceptional only in the suddenness with which it was brought about, thus making a useful economic comparison possible Apart from this suddenness, its history can be paralleled in scores of undertakings, and in fact this was broadly the experience of almost every undertaking which changed from relatively high flat rates to a two-part tariff with a low running-charge. In a word, the promotional policy in domestic supply, whatever its abstract merits, has been proved to have the ultimate " pragmatic sanction "—it works!

It is difficulty to translate the above into quantitative terms as a measure of elasticity. Unless additional units cost nothing at all, the elasticity must be *something* in excess of unity if price reductions are not to involve a loss. But just how much over unity it must be in order to show a profit, depends upon the magnitude of the incremental cost. Assuming that the cost curve slopes only moderately downwards (and it may, in fact, not slope at all) an examination of the graph suggests that there must have been an elasticity of not less than about 3 in order to give financial results such as those indicated for Oxford.

To set against this " success story " a less favourable example may be cited. The following figures were extracted from a letter in the *Electrical Review* dated May 7th, 1937 :—

	Date of Quarter :		
	March, 1936.	March, 1937.	Change Ratio.
Price of electricity . . .	1·0*d.*	0·48*d*	—52 per cent.
Units per consumer (*a*) . .	332	437	+32 ,, ,,
Consumers (*b*)	2,878	3,082	+ 7 ,, ,,
Units (Millions), (*c*) = (*a*) × (*b*)	0·957	1·348	+41 ,, ,,

It will be seen that after nine months' operation of the new tariff the proportional increase in units was appreciably less than the drop in price. The relative or annual elasticity figure was therefore less than unity, so that a smaller revenue was obtained in March, 1937, than in March, 1936. Since the extra units must have cost something, the immediate results would show an appreciable drop in profits although, of course, the ultimate results might be quite different.

This example is a salutary reminder of the extreme importance of certain factors omitted from the present chapter, namely, time and publicity. Demand has here been isolated and treated as though it functioned automatically *in vacuo*, and as though a drop in price would produce a given increase in sales without the operation of human labour or human ingenuity. Actually it does nothing of the sort. A sales programme concerns everybody in the undertaking, and the instant a price reduction has been decided upon, every department (and most of all the publicity department) must be mobilised if the full results are to be obtained.

Another factor which is related to publicity is that of the facilities for obtaining a supply. Questions of street cabling, assisted wiring, and the support or opposition by landlords of old premises will obviously affect the elasticity of consumers per head, though not affecting the elasticity of units per consumer.

Final Conclusions.—It was said at the commencement that the main purpose of these two chapters is to explain the operations of supply and demand rather than to elucidate the actual shapes of the curves. Nevertheless a few rough generalisations may usefully be essayed, although they are very far indeed from being "final" except so far as this book is concerned. Working both on *a priori* grounds and also inductively from such data as there are, it may be surmised that the lighting load has an elasticity of about 1, the industrial load about 1½ and the non-lighting domestic load from 1½ to 3 or more, depending on the price position. It would certainly appear that the elasticity for purely lighting supplies is seldom much over unity, and very much under if new connections are excluded. It follows that reductions in the lighting rate (or, what amounts to the same thing, in the fixed portion of an "all-in" tariff) are never justified commercially. They are a method of distributing surpluses, not a method of acquiring them.

Even if there are surpluses to distribute, the difference in the two elasticities means that the money will go much further if spent on heating than if spent on lighting reductions, and will therefore give greater benefit to the consumers. To put it in another way, let it be supposed that during a certain year a surplus of 10 per cent. appears in the domestic revenue, and that it is proposed to distribute this by a price reduction the following year. If allocated to the lighting consumers

it will only be possible to grant, say, a 15 per cent. reduction in the tariff without incurring a deficit, and this will give only a 15 per cent. increase in sales; whereas if allocated to the heating consumers it would be possible to grant a 20 or 30 per cent. decrease in charge without deficit, and ultimately to realise thereby something like a 100 per cent. increase in sales.

The outstanding feature of the general survey of domestic demand was the very high elasticity at the lower end of the price range—values of 3 or over for mean prices of about $1\frac{1}{2}d.$ (or follow-on prices of, say, $\frac{1}{2}d.$). Considering that this figure shows the average effect of both lighting and heating, and that the former only contributes a much smaller figure, the elasticity of the latter must be higher still. Under these circumstances a reduction in the heating rate or in the follow-on rate of a two-part tariff will frequently be possible without any ultimate loss in revenue whatsoever. Moreover, if the incremental cost is also known with some accuracy it actually becomes possible to plot the "correct" tariff magnitude, and perhaps to visualise the beginnings of a science of tariff construction.

Lest the above paragraph should lead one into a feeling of optimism it is well to remember that the process of price-fixing, here set out with all the appearance of mathematical precision, in practice can rarely become an exact science. Precise data, especially for the demand, are unobtainable: the curves, though excellent for illustration, usually fail when it comes to calculation, and serve rather to show the process than to indicate the results. Even when the facts are undisputed, policies may differ; and in a changing scene of costs and values the careful plotting of curves is like making an accurate plan of the room before embarking on a game of blind man's buff.

A somewhat considerable omission from these chapters is what is commonly called "human nature," or more correctly, the non-rational and non-economic facets of that nature. In making a case for price reductions in one rate rather than another (*e.g.*, for heating rather than for lighting) no mention has been made of the fact that action which is justified economically may nevertheless not be expedient. In practice a rate must not only be fair, but must *appear* fair, and a one-sided price reduction may have to be abandoned for this reason. It sometimes happens that a lighting price is settled neither by supply costs nor demand values but by sheer popular sentiment.

In conclusion, it cannot be pretended that this study has done more than venture on to the fringe of the subject. It is an enormous and well-nigh virgin field, and the present attempt does no more than suggest a method and make a very slight start on the matter itself. The theory of supply and demand makes an imposing façade to the tariff construction, but until more facts are available it is little better than a coat of scientific paint on a building that is pure empiricism.

CHAPTER III

MARGINAL COST AND THE PRICE STRUCTURE: USE-VALUE CONSIDERATIONS

Economic Cost.—The following is an attempt to explain in a few words the economist's theory of marginal cost in its application to a monopoly such as electricity supply, and where it differs from the treatment usually given by cost accountants. Some of the terms used may be unfamiliar to engineers, and the summary on p. 46 is prefaced by a glossary of terms and definitions.

The basis of the economist's study of productivity is to compare the value of the output with the value of the resources used up. The former is measured roughly by the price (multiplied by volume) at which the output can be sold, and the latter by the expenses incurred; but some discretion is needed on both sides. As regards the latter, not all the expenses rank as costs in an economic sense.

When an expense is incurred, certain of the nation's resources are converted to the particular purpose of the enterprise. If the product has a greater value than the resources used up the enterprise should obviously continue or should expand, whereas if less it should contract or be wound up. But some of the expenditures represent irreversible capital investments or irrecoverable expenses, and to this extent the enterprise cannot economically be contracted. Thus a company which is promoted for purposes of supplying electricity will incur certain non-recurring expenses or it may have to make surveys whose results have no value for any other purpose. Such expenses do not rank as economic "costs."

It will be seen that the method of comparison used by the economist is the same as that so often used by the engineer, namely to postulate certain changes and to examine the results. If output increased how would costs increase? If output were curtailed what resources would be saved or, as it is called, "escaped"? In the limit, if the enterprise were closed down what would be recovered?

In the economic sense the cost of something is the value, to other producers, of the resources which are used to produce it. Cost is measured by computing what expenses would be saved if production were curtailed and resources released for use elsewhere. It will be noted that some expenses such as payments for materials and wages are immediately escapable since they are directly proportional to output. Others such as capital charges (and other "fixed" costs in the engineer's sense) are escapable in the long run but not in the short run.

Thus if a power station were to be shut down at short notice the plant might only fetch scrap prices, but given thirty years' notice it might be possible, by avoiding replacements, to save the greater part of the resources expended. By taking a sufficiently extended period (equal to the longest life of the assets employed) almost all electricity supply costs are escapable and therefore rank as true costs except the minor non-recurring ones such as those incurred in launching the enterprise, or the expenditure on air-raid precautions.

Having isolated the true (*i.e.*, escapable) costs from the total volume of expenses the next step is to divide these costs into two categories, namely, those which are a simple function of the output and those which are not. The former, which the engineer might term " output-related " are called by the economist " divisible " because they can be split up and apportioned to the users according to the amount of their use. The latter, namely unrelated or basic, are called by economist " indivisible " or " common " since they are shared by all consumers in some way that defies simple allocation. This does not mean that the latter are entirely independent of the size of the enterprise : for example, they will include such things as management costs, which are obviously related to the overall size and volume of operations but cannot be rigidly allocated to any specific part of the output.

Marginal Cost.—As regards the divisible cost there are two values which this may have, according to whether the cost is averaged over the whole (mean) or measured by increments (marginal). If the annual output of an undertaking is denoted by x and the annual cost by y, then the mean cost of a unit of output is y/x. The marginal cost is the increase in the annual cost which results from supplying one more unit of output per annum. Mathematically, it is the rate-of-change of cost with output $\delta y/\delta x$.*

* The conception of *marginal* should present no difficulty to the engineer since it results from the familiar process of differentiation. A dependent variable (cost) is related to an independent variable (output), and the marginal cost is the rate-of-change of the former with respect to the latter. The same definition can be applied to the " marginal price " and here it is particularly important to be quite clear what is the variable in respect of which the differentiation is carried out : remembering that items which are constant relative to this variable can be omitted (rate-of-change = zero). The choice of variable must depend on what action is in question. Thus, if a potential two-part tariff consumer is discussing whether or not to install electricity the " marginal price " per unit will be the mean price—*i.e.*, it will include a proportional share of the fixed charge, whereas if an existing consumer is discussing whether to use additional apparatus the " marginal price " will be merely the running charge.

The same principle applies to telephone charges where the average cost per local outgoing call (after allowing for the value of inward calls) might work out at 2*d.* (share of rental) plus 1*d.* (call charge) = 3*d.* total. When debating whether to become a subscriber, the " marginal price " would be this 3*d.* per call since this is what would be saved by refraining. But once the 'phone is installed and the question is whether to make a call or write a letter the relevant " marginal price " is 1*d.*

It might at first appear that the marginal cost is merely the running cost, since the supply of one extra kWh does not in fact necessitate any more plant, staff or buildings. It might even be argued as regards generating costs in an independently operating station that (since the smallest new unit installed is 30,000 kW) once this has been put in, additional kilowatts cost nothing until they total another 30,000. Such an argument would be fallacious : it is like saying that, since a single vote rarely changes the result of an election, therefore voting has no effect. Correct costing can only result if each extra kWh at a given load factor is presumed to bring with it the appropriate fraction of extra kW and consumers* with their attendant quota of costs.

A more precise method is to enlarge the whole conception of output. Instead of regarding output as measured in kWh, and the other factors (kW of demand, etc.) as merely varying attributes of the kWh, it is better to treat them quantitatively and regard them as separate sorts of output in their own right. Just as kWh costs can be isolated and hence allocated according to the measured kWh of the individual consumer, so the kW costs can be isolated and allocated according to the consumer's estimated or measured peak demand. Even the cost of giving a domestic consumer a supply *qua* supply can be regarded as a species of output : the " consumer cost " can be isolated and an individual charge made for it.† The conception of " volume of supply " as used by economists in the definition of marginal cost (see Glossary, p. 300) is altogether inadequate to an electricity service if by " volume " is meant merely the amount of energy transfer.

In support of this quantitative treatment it may be pointed out that these other factors, so far from being static attributes of the kWh which the consumer must accept as best he can, do form a sort of triple output whose proportions the consumer controls, since *he* decides the number of kWh per kW or per consumer (according to his load factor and total load). On this basis the great majority of electricity supply costs are " divisible," and the marginal cost (defined as the rate-of-change of divisible cost with respect to output) equals $\delta y / \delta x$ where x is the output measured by the kWh, kW of demand and number of consumers in the appropriate ratio, and y is all the cost which can be related to the said kWh, kW and consumers. (It will be seen later that whilst, on the production side, a cost may be " divisable " as between system kW, etc., on the sales side, it may not be " divisible " amongst the individual consumers under any practicable tariff.)

* The marginal cost of consumption by existing consumers would, of course exclude this third element.

† The same treatment can obviously be extended to reactive kVA demand, and, a corresponding charge can be made under a power-factor tariff.

Marginal and Mean Cost Differences—The difference between marginal and mean costs has been defined mathematically as the difference between $\delta y/\delta x$ and y/x, and the numerical difference will depend largely upon the composition of y. The marginal or incremental cost δy is the change resulting from a change in output, and hence only the costs which vary with output (*i.e.*, the "divisible" costs) will appear therein. (In mathematical terms, when differentiating y with respect to x the constant terms disappear.) The mean cost on the other hand will include also the unrelated or common costs, in fact all those that are escapable and are therefore costs in the economic sense.

On these grounds therefore one would expect (and under steady price conditions one would find) the mean cost to be higher than the marginal cost. In electricity supply the difference may not be very great because, as already explained, most of the costs can be regarded as divisible on some basis or other. With varying price levels the position is quite different because mean price reflects past conditions as regards plant installation whereas marginal price refers to present additions. The difference will show itself almost entirely in the fixed costs. (The word "fixed" is used here and subsequently in its engineering, not its economic, sense.)

The position at the present time (1950) is that there has been a steady increase in station coal prices for the last fifteen years. (This concerns chiefly the running cost which, being an immediate outlay, affects mean and marginal costs equally.) There has been a sharp rise in plant prices, which are more than twice what they were before the war. This has two effects, an immediate and a long-term one. The immediate effect is that additional kilowatts will cost much more to supply than do existing kilowatts, and in the first years after such a rise the marginal fixed cost may well be double the mean. But as time goes on the old plant will have to be replaced at enhanced prices, and a larger and larger proportion of the installed capacity will have been bought at the higher price level. The mean fixed cost will therefore rise steadily until, after some thirty years of continuous high prices, all the plant in use will have been bought at this level: the mean cost will then approach stability in its old position of somewhat below the marginal cost. (It will be noted that the above is true whether or not the undertaking is expanding, since both replace plant and additional plant is bought at the new prices. But if the undertaking is growing fast the proportion of new plant will rise more quickly.)

The above is of course only a very rough outline of the position. It refers particularly to that major element of fixed cost which is due to the capital charges on the plant. It is not true of the element consisting of management and staffing expenses, in which salary increases will produce an immediate effect on mean and marginal cost alike. Capital charges on non-depreciating assets such as freehold

land are also in a different category. Price increases will cause the marginal fixed cost to go up immediately but the mean cost will not creep up behind it because there is no periodic replacement, and the land originally bought cheaply remains an everlasting low-price asset which will permanently reduce mean cost below marginal. This however is not a large element in electricity-supply capitalisation.

As regards the affect of all this on tariffs, there is usually no question of allowing the difference between mean and marginal cost to be reflected in the actual form of the tariff. It would be impracticable and probably illegal (undue discrimination) to charge new consumers more than present ones or to charge additional kilowatts at a higher rate than existing kilowatts. The industrial consumer paying £5 per annum per kW for a 10-kW load would be more than surprised if he were asked £8 for an eleventh kilowatt. The price charged must in fact be the same for all in similar circumstances.

Nor indeed would marginal cost theory support any such discrimination, and it is unnecessary (even if it were possible) to isolate the additional kW or the new consumer from the remainder.* Every consumer is a marginal consumer in the sense that costs would be greater or less if he decided to consume more or less. Under the marginal cost theory every kW and every consumer should bear marginal cost, based on the cost of an additional kW, etc. The following paragraphs therefore study the incidence of the theory on the *magnitude* of the tariff as distinct from its structure.

Plant Price Changes and Capital Charges.—Consider first the question of depreciation provision. This is discussed at some length in the first volume of this work,† but the following is a summary of the relevant points. The usual view of cost accountants is that, technically, depreciation means expired capital outlay, and that the requirements of strict depreciation provision have been met if at the end of the plant life there is a matured sum sufficient either to amortize the initial loan or (if this is maintained) to purchase fresh plant of equal cost.

If, however, during the plant's lifetime it becomes clear that replace plant of equal magnitude will cost very much more, prudence suggests that this strict depreciation provision should be supplemented by additional provision to meet the increased cost of replacement. Assuming that the price-increases are general, and that the value of the product has gone up correspondingly, it should be possible in this way to accumulate reserves which may avoid the necessity for raising additional capital to finance identical replacements and the necessity for very sharp

* This is not quite so obvious in respect of "consumer costs," and the higher post-war connection costs could in theory at least be attached to the actual new consumers. In fact, this is often done where it is practicable—*i.e.*, in the particular connection charge made to an isolated consumer.

† *Electrical Engineering Economics*, Vol. I: "General Principles and Choice of Plant".

future increases in charges. The plant owner in a rising-price market is like a shopkeeper stocked with large quantities of goods bought at the lower prices. While stocks last he can afford to sell much below replacement cost : alternatively he can put up prices immediately (at risk of the charge of temporary profiteering), and it might even be necessary for him to do so to prevent too rapid a depletion and to provide sums for replenishment.

It might at first appear that only the depreciation provision is concerned in this argument, but further consideration shows that interest charges must also be raised if the marginal cost is to be computed correctly. When plant prices rise the true value of all the plant in use rises also. This value is expressed as the sum of two annual charges for interest and depreciation, both of which are proportional to the capital value and rise with it. If, for example, the turbines were not owned by the supply authority but were loaned to them in the manner of shoe-making machinery, the landlord seeing that a rise in price had occurred would promptly put up his rents even on the old plant. If the authority objected to paying the higher rent their only alternative would be to purchase plant at the new high prices. Conversely, if plant prices dropped suddenly, the landlords would have to lower their rents on the old plant and stomach their losses : for if the undertakings were not offered appropriate rent reductions they would buy the new cheaper plant for themselves.

This difference of attitude to capital charges as between the cost accountant and the economist is not, of course, due to any difference of correctness but merely a difference of aim. As regards the interest charges the accountant has no alternative but to have regard for the payments actually made. As regards depreciation, normal business practice follows certain rules according to which provision for depreciation is made on a given scale, broadly planned so as to keep the enterprise solvent. The annual accounts and balance sheet show whether this provision has in fact been made, and whether the financial state is satisfactory in relation to the plan. These accounts can be used to find the cost of the operations as carried out, and hence the cost of the article or service within the framework of the business practice which is being followed.

The economist is concerned to find the real cost of the operation in terms of the resources absorbed, and his method is hypothetical rather than factual since he is dealing with the future rather than the past. It is therefore to him one must go in order to find what is the economic cost of a unit of electricity, and what is therefore the minimum price below which one must not sell in order to lead to the correct output of the undertaking. On his reckoning, the effect of a doubling of plant prices is that the capital charge element in the marginal cost (comprising most of the " fixed " cost) will be double what it was, and this cost is attached to all the plant employed, new or old.

Practical Considerations.—Apart from any question of correctness, this difference of treatment discloses a serious practical difficulty in an industry like electricity supply, employing large quantities of long-lived plant, because of the time-lag already mentioned. Cost theory shows that, immediately plant prices rise, the true cost of meeting kW demand rises with them ; and that the cost must be measured by reckoning increased capital charges on account of all the plant in use, even though only a small part of this plant has been bought at the higher prices. If electricity is sold at prices below such a cost, this may encourage a development which represents to some extent a living on capital, and will attract resources some of which should go elsewhere. On the other hand, to follow the dictates of economic theory will mean putting prices up before this becomes financially necessary, and will present the undertakings with a large temporary bonus during every year in which they are able to use high-value machinery which they bought cheaply.

The dilemma can be summed up as follows in terms of the kW charge. Since new and old kW cannot be isolated it must be assumed that all kW cost the same, and this cost is measured at the margin by the cost of new plant. Until all pre-war plant has been replaced by new, there is a lengthy time-lag during which the undertaking is sheltered from the full blast of plant-price increases. But to charge according to this marginal cost would unduly swell the revenue, and embarrass the undertaking with the custody of booty which would probably prove irresistible to consumers and employees alike.

The dilemma is likely to be resolved on grounds of policy rather than economics, since it is a question of what is possible rather than what is ideal. A public utility is always a target for popular criticism and would be expected not to put its prices up one minute before it must, through the stern necessity of increased costs actually being met. Moreover, in thus putting off the evil day, there is always the hope that the evil may not fully materialise. Electrical plant lasts thirty years on the average, and in this period there is plenty of time for prices to go down again. The condition under which all the plant in use carries the heavier charges may therefore never arise. It is true that if plant prices rise and later fall again the real cost of a demand kW will in fact have risen and fallen in synchronism. But a utility undertaking would be expected to smooth over both these changes, and not to vary its prices more than absolutely necessary.

The legal aspect should be mentioned. Under the 1947 Act, the British Electricity Authority has the duty of securing that the combined revenues of all the Electricity Boards are not less than sufficient to meet their combined outgoings properly chargeable to revenue account taking one year with another. The sums properly chargeable include provision for depreciation of assets *or for renewal of assets*. [The italics are the author's.] It would therefore appear that there is no legal

disability to depreciation provision on a replacement cost basis. A more serious limitation, of a semi-legal character, is virtually imposed by the Inland Revenue authorities when they refuse to recognise contributions on the larger scale for tax exemption.

The final argument therefore seems to be that at this moment in the nation's history we just cannot afford to do the right thing but must be prepared to live on our capital to some extent, quite apart from the fact that it would be politically almost impossible for tariffs to be raised to cover hypothetical future capital charges much in excess of the charges actually being met. This should not blind us to the fact that in so far as lower prices lead to avoidable consumption* we may thereby be encouraged to consume somewhat more electricity than we can afford. The effect will be a maximum at the moment, becoming small in a few years and zero in thirty if plant prices remain steady.

Marginal Cost and Pricing Theory.—Having discussed the correct magnitude of the marginal cost under conditions of fluctuating plant prices, the following notes round off the marginal cost theory. They show how price may be related to cost, and how the various categories of expenses can be loaded on to the consumers. The quantities concerned are those already defined, and are summarised below under the heading " Schedule of Total Expenses."

Under conditions of perfect competition the price equals the immediately escapable marginal cost (*b*) (i) and fluctuates with it. This is because if the price went below this, managers would curtail output and so save themselves more than they lose in revenue, and *vice versa.* At the other extreme, with a public utility monopoly the aggregate prices will cover all escapable costs (*b*) and (*c*), both divisible (i) and common (ii). Usually they should also cover the inescapable expenses (*a*)—in fact everything except losses due to mistaken foresight.

The cost allocation, and consequent price fixing, is broadly as follows. So far as practicable every section of the load and every separate consumer should pay the marginal cost of his consumption. The divisible cost (i) is therefore allocated to consumers according to their consumptions of the outputs (kWh, kW, etc.) in question. But since the total revenue so obtained is less than the total expenses, it is necessary to spread the remainder in some arbitrary manner over the total consumers. The unrelated cost (ii) and the inescapable expenses (*a*) can therefore be levied at the discretion of the commercial manager, generally on a use-value basis.

In fact there is little choice in the matter but to use price discrimina-

* *E.g.* in the domestic-heating field. Industrial consumption is less responsive to price changes, and a reduction in electricity prices would have entirely beneficial results in lowering production costs.

tion for these expenses, since costs cannot be spread on those parts of the market which will not bear them. It may be assumed that the elastic-demand portion of the load has reached its present state of development because for that particular purpose the electricity is only worth that much, and if the price is increased the sales will collapse. It follows that the unrelated costs and inescapable expenses must be spread where they can be borne without depression—*i.e.*, over the inelastic-demand high-yield consumption. This will achieve optimum expansion under which every consumer, and in respect of each class of load, takes every unit for which he is prepared to pay marginal cost.

The following table sums up this section of the work.

GLOSSARY

Economist's term.	*Engineer's equivalent or explanation.*
Inescapable or Fixed*	Irrecoverable, Non-recurring (*e.g.*, relating to the initiation of the enterprise or to specific equipment which is no longer required.)
Divisible	Output-related.
Indivisible or Common	Unrelated, Basic, Residual.
Marginal	Incremental (*i.e.*, relating to a small addition or subtraction rather than averaged over the whole. By its nature it can apply only to divisible items).

SCHEDULE OF TOTAL EXPENSES

(*a*) Inescapable

Escapable (*b*) immediately, (*c*) ultimately. ("costs")

(i) Divisible (either marginal or mean).

(ii) Common (mean only)

EFFECT ON PRICING

In perfect competition, price $= (b)$ (i).
Under imperfect competition, price may $= \{(b)$ and possible $(c)\}$ $\{$(i) and (ii)$\}$.
In a public utility monopoly, aggregate price should cover $\{(b)$ and $(c)\}$ $\{$(i) and (ii)$\}$ and as much as possible of (a).
Cost (i) will be allocated on the basis of output consumed.
Cost (ii) and expense (a) will be levied on a market-bearing basis.

Danger of Uneconomic Pricing.—Before leaving this part of the subject, some further explanation should be made of the idea of economic cost and the statement that, unless each unit of electricity pays its marginal cost, the resulting development may be uneconomic. The point may be illustrated from the industrial field perhaps more clearly than from the domestic because the former often shows a closer and more measurable competition.

There are factories having a large and steady demand for process steam side-by-side with a corresponding demand for power. When the two demands are closely matched the most economic solution will often be for the factory to generate its own electricity from back-pressure sets whose pass-out supplies the steam for process. On the other hand there are many factories having highly fluctuating power loads, with no heat requirements to match, for which a public supply of electricity is obviously more economic. In between these two extremes there is a water-shed where the solution might flow either way —borderline cases where the economic pros and cons of the two methods are nicely balanced. The decision of where to draw the line between the appropriate fields of the two methods must be made on broad economic lines by expressing every factor (including amenities such as smoke-prevention) on a true cost basis as far as possible. If the industrial electricity load were being subsidised by some other load, or *vice versa*—if in fact the industrial tariff did not accurately reflect marginal costs—there is a danger that the dividing line would be wrongly drawn and natural resources not used to the best advantage.

The same thing is true in a decision between, for example, electric and gas cooking, and in a score of other cases. The final arbiter should be the consumer, but he cannot chose correctly unless he is presented with all the data with their true economic weighting.

Application to Electricity Tariffs.—The dividing-line between costs that are "divisible" and those that are "common" depends ultimately upon the tariff. Common costs are not a visitation from

* This use of the word "fixed" is different from the engineer's as used elsewhere in the book to denote expenses proportional to time rather than kWh output. The two usages will only correspond if "output" is regarded as consisting merely of kWh.

outside like an earthquake: they arise as a result of the normal operations of the undertaking and are in some way related to those operations. If all the services rendered to the consumer could be completely reflected in the tariff, and if the various parts of the tariff could correspond exactly to the operations of the undertaking, there is no reason, in theory, why all the costs cannot be divisible and loaded on to the individual consumers. This would form the "ideal" or perfect costs tariff, though whether it would be ideal in other respects is a different matter. The following paragraphs indicate how far this might be possible in practice in the case of two typical tariffs—the maximum demand industrial tariff and the all-in domestic tariff. In both cases it is assumed that, whilst nominally of a two-part form, the tariff in effect has a third part—a consumer charge—by virtue of a non-proportional element in the standing charge.

In Part II the costs of supply are related to various quantities, in particular to kWh of consumption, kW of maximum demand, and number of consumers. The two-part M.D. tariff for industrial supplies contains just these three elements, and superficially it would appear capable of making the whole of the cost divisible amongst the consumers. In fact there are two reasons which make this impossible. In the first place, the cost-relating into these three components is far from perfect and involves considerable straining. Since it is impracticable to employ a wider range of variables, two of the categories, namely "kW demand" and "consumers", have to serve partly as dumps into which costs are put for which no other home can be found. In the second place, the consumer's metered maximum demand is not an accurate measure of the effective demand made on the supply system, and in absence of some highly elaborate time-varying metering there seems no way of rigidly connecting them. There is, however, a fair degree of correspondence when all the consumers have generally similar characteristics.

In the case of the two-part domestic tariff with a standing charge based on some residential quantity such as house-size or number of rooms, the correlation between this and the system demand is extremely tenuous. The cost estimate on p. 75 suggests that in the supply to the average consumer the three groups of cost are proportioned as follows:—36 per cent. *per kWh*, 54 per cent. *per kW*, and 10 per cent. *per consumer*. Let it be assumed that the first and third are correctly covered in the tariff, and that the second is one-quarter proportional to house-size and three-quarters independent thereof. It could also be assumed that a certain proportion (possibly one-third—see p. 228) of the kW costs could legitimately be levied on the kWh consumption. This would mean that about 77 per cent. of the cost would be divisible and could be allocated to the individual consumer whilst 23 per cent. would be common. The latter should then be spread on a market-bearing basis (for which a house-size fixed charge is eminently suitable).

These figures are highly tentative, and in practice the cost-representation is likely to be less close than they suggest. For example, it is rare for the non-proportional element in the standing charge of either tariff to be as high as the true consumer cost, and there is a tendency for the small consumer (possibly on grounds of expediency) to be charged less than his full costs. Summing up the comparison between industrial two-part and domestic two-part tariffs, it would be broadly true to say that under the former the greater part of the demand-related cost is " divisible " between the consumers, whereas under the latter a large amount is " common " and must be spread on some arbitrary basis.

Use-Value Discrimination.—The procedure which is variously known as " price discrimination," " charging what the market will bear," and " having regard for ' use-value ' or ' consumer-surplus ' " may be defined as making a difference in the price which is not justified by the corresponding cost difference. The alternative would be to charge for each unit of consumption the marginal cost plus a uniform addition.

The 1947 Electricity Act repeated earlier Acts in prohibiting tariffs and agreements showing undue preference to, or discrimination against, any *person or class of person.* It is not always easy to distinguish between " undue discrimination " and price discrimination on a use-value basis, but the words which the author has italicised may give a clue. Broadly, the distinction is that price discrimination is based on some impersonal objective category : it differentiates between different classes of usage (loads) but not between different classes of users (persons). The principle is that whilst a kWh used for lighting may be charged differently from the same kWh used for heating, any consumer can have it for a particular purpose at the same price as his neighbour. It might be thought that to offer a tariff to a domestic consumer that is not open to a shopkeeper would be discrimination against a class of person, but the difficulty here would be to decide what was " undue."

When dealing with a commodity it is easy to distinguish between price discrimination and variations due to cost. If it were decided that, for example, the national coal-price structure was to be based on the principle of a single price for a given grade irrespective of use, this would be a clear indication that no price discrimination would be practised. No such clear-cut indication could be provided in the case of electricity because use affects cost, and a tariff which made price variations for different usages might be merely reflecting cost differences. In order to prove a case for electricity it would be necessary to show that the actual differences made were more, or other, than could be justified by differences in cost.

The principle underlying price discrimination has been stated by Professor Lewis as follows :* " Those who cannot escape must make

* *Overhead Costs,* by W. Arthur Lewis : Allen and Unwin, London.

the largest contribution to the indivisible cost and those to whom the commodity does not matter may escape. The man who has to cross Dupuit's Bridge† to see his dying father is mulcted thoroughly; the man who wishes only to see the scenery on the other side gets off lightly." (The reason being, of course, that *someone* has to pay for the bridge, and the man who crosses for the scenery cannot be charged more or he would not cross.)

Price discrimination in electricity supply makes use of the fact that for some purposes electricity is worth more than it costs to produce, *i.e.*, there is surplus value which may be either pocketed by the consumer or skimmed off by the supplier. The latter process is only possible under monopoly conditions: if electricity distribution were free for all, electricity for any purpose would be supplied at its marginal cost and there would be no "consumer surplus" on which price discrimination could be based. (It is hardly necessary to point out that if electricity distribution were "free for all" the marginal cost would probably be far higher than the present monopoly price, since multiplying the distribution would be a very expensive procedure.) Another reason why discrimination is more possible with electricity than it would be with a material commodity is that electricity supply is an individual service to a consumer and is usually not open to resale.

The phrase "what the market will bear" has an ugly, profiteering ring about it, and the procedure is obviously open to abuse. When a monopoly is in private hands there is a danger that a surplus may be taken in varying amounts off *all* consumers, thus bringing in a revenue much in excess of total expenses, and providing undue profit for the proprietor. Under public ownership it may be presumed that the total revenue is not affected by the procedure, being merely sufficient to balance total outgoings taking one year with another. Price discrimination is then merely an internal distributing device for spreading the indivisible costs where they can best be borne, not an expedient to extract more revenue as a whole. Nor does this constitute improper competition with outside industries provided no supplies are given below marginal cost. It is true that with rising plant prices there is a danger that electricity prices may not cover marginal replacement costs, but the same thing is likely to be true of competing industries.

Use-Value in Electricity Pricing.—So far, this chapter has dealt with marginal-cost pricing theory in general, though with particular reference to monopolistic services. The remainder of the chapter is

† Dupuit was a French civil engineer who studied the problem of charging for a structure such as a bridge or road which costs a certain sum to build but whose cost is not affected thereafter by the number of times it is used (*Annales des Ponts et Chaussées*, 1844). This is the extreme case in which the marginal cost is nothing, and the average cost merely represents the spreading of the common costs over a variable number of users.

devoted to its application to electricity supply, and the arguments for the employment of use-value considerations in electricity tariffs. This raises somewhat controversial issues and it is necessary at this point to turn aside from the strict science of the subject and engage in a little polemic.* The fundamental thesis of this section, that in tariff construction use-value must be considered as well as costs, is frequently attacked, and a few words must be spoken in its defence. It was seen above that the monopolist purveyor of a service is very different from a commodity manufacturer working in a free market. He is by way of being a dictator and can fix the price very much where he chooses. It is sometimes argued that in these circumstances the only safe guide is his own particular costs.†

So prevalent is this idea that it has become almost unconscious and indeed part of the language of the subject. People frequently speak of an " accurate " tariff: meaning, of course, a tariff which accurately represents costs, not one which accurately represents value to the consumer. Actually, as was seen above, cost accuracy extends only to divisible marginal cost, beyond which use-value accuracy should take up the tale. But accuracy is usually spoken of only in relation to costs, and in fact the very use of an exact and absolute word like " accurate " in connection with such a give-and-take matter as price-fixing is evidence of the underlying and unquestioned assumption that the only function of a tariff is to represent costs.

In the aggregate, no doubt, the assumption is a sound one. The undertaking as a whole should pay its way, and on the other hand it should not " exploit " its privileged position. But this is far from saying that it is the business of each separate rate and tariff to recover blindly not only the divisible costs on that particular portion of load but also a uniform proportion of the total expenses, quite oblivious of what service it is rendering to the user. Whatever its peculiarities, electricity supply is a sale like any other sale, and the business of a tariff is to satisfy both parties to the transaction. The buyer will be satisfied if he obtains value for money, *i.e.*, a gratification which he cannot obtain more cheaply by any alternative service. The seller will be satisfied if *on the whole* his expenses are covered with the required margin of profit. The nation will be satisfied if marginal costs are covered in every transaction so that the national economy is not distorted.

* Inevitably, any such attempt is open to the charge of special pleading, and the cynic will say that this section is no more than a smoke-screen under cover of which the tariff designer can get away with anything. It should therefore be stated that, whilst the above arguments are true qualitatively, they make no pretence at a quantitative justification of present tariff practice.

†Even if true, this would not be adequate: the simple formula of selling at cost and making no profit does not provide the answer, because both costs and values vary so widely with the output. One might sell at 8*d.* per unit and make no profit, and one might sell at 1*d.* and have a surplus.

Nor is price discrimination peculiar to electricity supply : telephones and transport, water and gas undertakings, all find it necessary to differentiate between different uses of apparently identical services. The railways do not charge as much for carrying a hundredweight of coal as they do for carrying a hundredweight of gold, although being bulkier they might reasonably charge more for it. The hundredweight of refined gold is like the 50 watts of refined power that works our wireless set or our electric clocks, whilst the coal is like the heating energy of which so much more is required to produce an effective result.

Reasons for Considering Use-Value.—The arguments for employing use-value considerations may, then, be summarised as follows. In the first place, it is the economically correct method of allocating unrelated costs and inescapable expenses. In the second place, it may be partially justified even on cost grounds if a sufficiently long view be taken. A cost assessment should consider not only the actual immediate costs, but also what may be called the consequential costs —remembering that costs are not static but are actually affected by the results of our tariff decisions.

On a narrow view, each kilowatt-hour at any moment costs the same to supply, just as each letter or each passenger costs the same to carry on a railway. The justification of halfpenny postage and excursion trains is that, whilst the charge more than covers the additional running costs, business is obtained which would not otherwise accrue. Hence, even the small contribution this makes towards the overheads lessens the burden borne by the full-rate traffic. In a similar way, low rates for electricity for special purposes can be justified if it can be shown, firstly, that only so can business be obtained, and secondly, that in the long run this will lower rather than raise the price to other users.

One very practical reason for not making tariffs the slave of costs or attempting to make each consumer pay his exact share of the total expenses is the complete impossibility of discovering what this share is. When something like three-quarters of the expenses are overheads, and only one-quarter can be directly related to the metered consumption, costs depend largely upon an unknown quantity, namely peak contribution. Individual costs are to this extent quite indeterminate, and even group costs can only be roughly assigned.

Another reason for having regard to consumer reaction arises in the cost calculation itself, since the most reliable of the many ways of sharing out the costs between the different groups of consumers makes use of consumer response in the cost allocation. This method assesses liability for peak formation at certain times of day by asking the question " if costs are allocated in such and such a way and corresponding charges are made at peak and near-peak times, what is the likelihood of a peak developing at some other time of day ? " The

answer to this question clearly depends upon the consumer's response to price changes.

To neglect consumer reaction would be profoundly unrealistic. The power given by the so-called electricity monopoly is a strictly limited one, because a monopoly to supply electricity is by no means a monopoly to supply heat, light and power: there are always other methods of obtaining the service. The supplier may have an absolute power of determining the price at which supplies are furnished, but he cannot dictate how much will be consumed at that price. If he chooses to disregard the consumer's reaction, the latter will revenge himself, and possibly take up a consumer pattern which will be good neither for the industry nor for the country.

This point can best be shown quantitively. It is suggested on page 214 that purely cost considerations would lead to a very much closer approximation of, say, lighting and heating prices. Consider then, what would be the effect of carrying this to its logical conclusion of a single flat rate of, say, $1\frac{1}{2}d.$ per unit for all domestic purposes. Such a figure would be above what the electricity was " worth " for heating, and much below what it was " worth " for lighting (" worth " being measured by the cost of other methods of satisfying the same need). The result in a very few years would be a complete change in the pattern of consumption, and a consequent change in the very costs by which the price change had been justified. Much of the capital equipment, particularly of the distribution system, would soon be badly under-employed, and specific costs would rise in all directions.

It will be noted in this connection that a knowledge of consumer reaction, even if it is not to influence tariff construction, is essential to the supply engineer. His plans must depend on estimates of future load growth, and if the future is to witness tariff changes he will need to have some idea as to their probable effect upon the consumption.

Finally, whatever the arguments, if one surveys the actual tariffs there is plenty of evidence that they have been profoundly influenced by considerations of consumer response. It is shown in later chapters that both the forms of electricity tariffs (industrial as well as domestic) and the magnitudes of the ratios between their different parts are based as much on use-values as upon costs. The differences in character between industrial, commercial and domestic tariffs, or the differences in size between lighting and heating flat-rates cannot possibly be explained purely on cost grounds, and all attempts to do so are the sheerest casuistry. But whereas costs are openly consulted, the influence of the market has been largely unconscious, or at least shamefaced. One object of the present study is to bring these suppressed motives up to the surface of consciousness, and to rationalise them into a scientific working hypothesis.

Summary.—The rules governing electricity pricing are the same in principle as for any other service or commodity. Costs which can be numerically related to a consumption variable should, as far as practicable, be covered by a corresponding variable in the tariff. Other expenses can be loaded as and where they can best be met, which in practice will usually mean spreading them according to what the market will bear. Electricity supply differs from other industries only in respect of the *proportions* in which these two principles operate.

The outstanding features of electricity-supply economics from the pricing aspect are as follows :—

(*a*) The very high ratio of overheads to running costs.

(*b*) The fact that many of the former are indivisible, *i.e.*, they cannot be rigidly related to any practical output variable (kWh, kW or consumers).

(*c*) The difficulty that even of the divisible portion, those which are demand-related cannot be precisely allocated to consumers through ignorance of their effective demands on the supply system (which is a very different matter from their individual maximum demands).

(*d*) The very high use-value of certain portions of the consumption which encourages " market bearing " methods of allocation.

The hard core of cost-relating has therefore a soft and woolly exterior ; it casts an extensive penumbra, and over this dark and dubious area the tariff-framers have ample scope for their operations and a correspondingly large margin of error.

PART II

COSTS

Chapter IV deals with the division of costs into two according to their proportionality to power demand and energy consumption. Chapter V pursues the same division but modifies the proportions in accordance with the operations of diversity. The next two chapters re-examine the cost division in greater detail and relate it to other variables besides power and energy. The power-related portion is then allocated to the groups of consumer deemed to be responsible. A final chapter in this section deals with load studies.

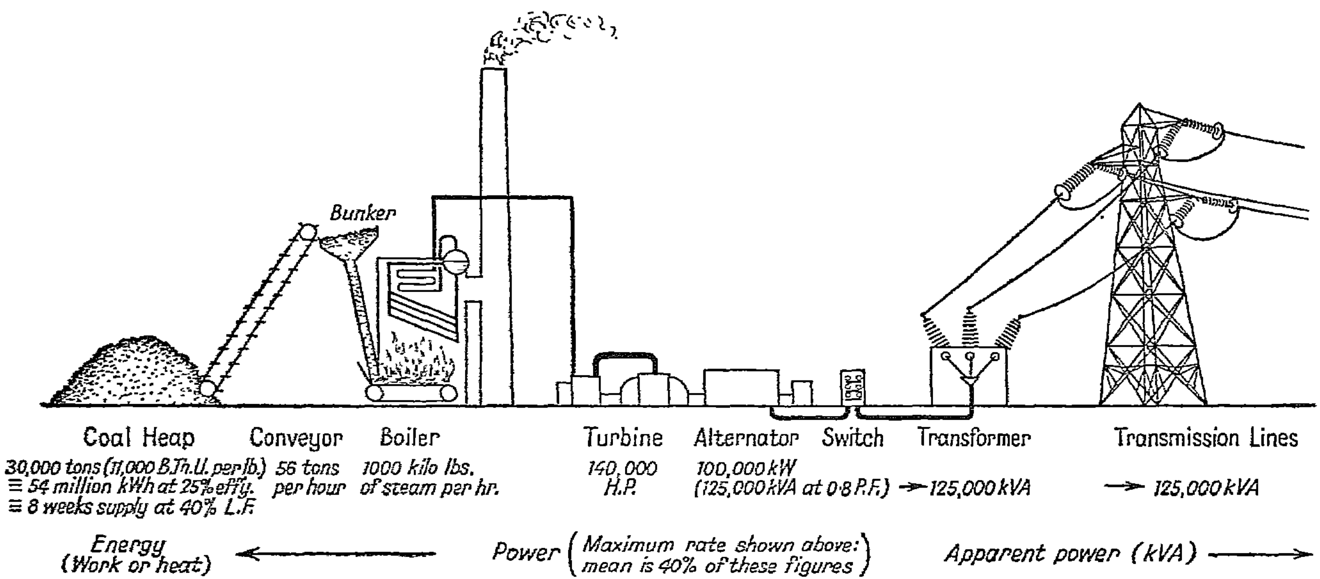


FIG. 8.—Energy, Power, Load Factor and Power Factor.

CHAPTER IV

COSTS ON TWO-PART BASIS

Work and Power.—Two quantities are of supreme importance in electricity-supply costs, namely the *work* or *energy* taken by the consumer, and the *power* or *rate* at which that energy is taken. Roughly speaking, the amount of coal consumed is dependent on the former, whilst the amount of plant utilised is dependent on the latter.

The work done denotes the actual amount of accomplishment, as in lifting a given weight a given distance. The unit of work or energy (the two words mean the same thing) is the foot-pound, the horse-power hour, the joule or the kilowatt-hour. Heat units such as the calorie or the British Thermal Unit are of the same kind, since heat is a form of work. Power denotes the speed of accomplishment, or rate of doing work or transferring energy, as in lifting a given weight at a given speed. The unit of power is the foot-pound per second, the horse-power, the watt or the kilowatt.

The relation between power and energy is the same as the relation between speed and distance. It is a time relationship, namely, power equals energy divided by time, or energy equals power multiplied by time. Thus one horse-power or 746 watts maintained for one minute equals 33,000 ft. lbs. or $42\frac{1}{2}$ B.Th.U. A kilowatt ($1\frac{1}{3}$ horse-power) maintained for an hour is a kilowatt-hour.

The relationship can be very well seen in the case of combustion. One pound of good coal contains some 14,000 B.Th.U.'s of potential (chemical) energy, and if this is converted at 25 per cent. efficiency it gives about 2,500,000 ft. lb., or 1 kWh. If this pound of coal is burnt very rapidly it will represent a large power, but if slowly the same work will be spread over a longer period and so represent a smaller power. The size of grate, boiler, etc., required will therefore depend on the rate at which the coal is to be burnt, *i.e.*, on the power. On pound of gunpowder contains only about one-tenth as much latent energy as a pound of coal, but since it is usually burnt more rapidly, the power it develops is greater and lasts a shorter time. A *high-power* explosive such as dynamite burns even faster than gunpowder, and is therefore correspondingly more powerful and destructive while it lasts.

Fig. 8 has been drawn to illustrate the relationship between energy, power, load factor and power factor. It shows in diagrammatic form the generation of electricity from coal and its overhead transmission. A coal stack of 30,000 tons equivalent to 54,000,000 kWh is used at

a maximum rate of 56 tons an hour to give an output of 100,000 kW. The mean rate throughout the day is only 40 per cent. of this, and hence the daily output of the station is $40{,}000 \times 24 = 96{,}000$ kWh. At this rate the stock of coal will last 8 weeks.

Dual Costs.—It will be seen from the foregoing that the total work received by the consumer is measured by the energy, whilst the rate at which he takes this work is measured by the power. Furthermore, the energy will require a certain amount of *coal* consumption in order to furnish it, whilst the power will require a certain size of *equipment* (boilers, alternators, cables, etc.). But there is this difference between the two : the energy can be added up throughout the day and year, and the total sum will represent the fuel burnt for this consumer, whereas the power fluctuates, and it is the *largest* power at any one time that represents the size of equipment required by this consumer (modified by other consumers' joint use of the same plant). In order to assess the expenses of supply to a given consumer it is therefore necessary to know his total or integrated energy consumption over the period and his maximum power requirements at any time in the period. This latter is known as his *maximum demand* (M.D.).*

The total costs of supply then comprise two main elements corresponding to the two quantities described above. In a similar way the costs of a motorist might be said to be dependent on two main factors, the mileage or distance he wishes to cover, and the maximum speed at which he wishes to travel. The petrol consumption will be largely a function of distance, but the engine-size required (for a given accommodation) will chiefly depend on what maximum performance he requires. Thus there will be a running cost dependent on the distance travelled and an overhead cost dependent on the maximum rate demanded.

In the case of most commodities and some services it is only the quantity which passes that is important, whilst the rate or regularity

* The word " demand " is often loosely used as an abbreviation for " maximum demand." This is convenient where there is no ambiguity, but the better plan is to use the word " demand " as synonymous with " load " except that, instead of being an instantaneous quantity, it is averaged over successive integration periods such as half-an-hour. Thus one can speak of the demand in the neighbourhood of any given time (such as lunch-time). More precisely, the demand at, say, 6.15 p.m. would mean the average kW or kVA load between 6 p.m. and 6.30 p.m. (*i.e.*, twice the kWh or kVAh taken during this half-hour). " Maximum demand " signifies the largest of these successive demands in a stated period, the assessment period. Usually the period is a year, unless it is qualified in some way as when speaking of the " September maximum demand " or " the average of the monthly maximum demands ".

The word " peak " is used to describe the largest demand in relation to demands in its immediate vicinity. A very short assessment period may then be visualised, as when speaking of a morning peak and an afternoon peak in the same day. This implies nothing as to the relative magnitude of the two peaks but merely that each stands out from a valley on either side. When a number of such peaks are involved over a longer period such as a year, the largest is described as the " absolute peak ".

with which it passes is comparatively immaterial. In such cases a simple quantity charge or flat rate is all that is required ; and in the past even electricity tariffs have been legalised on such a basis, the kilowatt-hour being the original Board of Trade " unit " for charging purposes. (The word " unit " is a convenient though somewhat loose synonym for kWh, and when no ambiguity exists it will frequently be so used in the present book.)

More accurate costing has shown that, on the average, only one-quarter of the total costs of electricity supply are represented by coal or items proportional to energy, whilst three-quarters are represented by fixed costs or items proportional to power, etc. If therefore only one rate is to be levied it would appear more logical to charge only for power and neglect the energy,* were it not for certain practical difficulties of which the following are two. In the first place the effective power demand on the system made by any particular consumer is extremely difficult to estimate, and is very different from the individual maximum demand metered at the consumer's terminals. Secondly, a purely power tariff would probably lead to a waste of energy to a greater extent than a purely energy tariff leads to a waste of power.

Load Factor: Definition.—The load factor can be defined as the actual energy consumption divided by the consumption which would have occurred had the maximum power been taken all the time. It may refer to any particular load or part of a load, and it may be taken over any specified period such as a day or a year. When the apparatus concerned takes its full power all the time it is connected, then the load factor is merely the time of connection divided by the total time—thus plant which takes, say, 100 kW whenever it is connected, and which is in circuit eight hours a day, will have a load factor of one-third, or $33\frac{1}{3}$ per cent., whilst if it is in circuit six hours a day the load factor will be 25 per cent. Unfortunately, most apparatus not only is not connected continuously, but does not take its full current all the time it is connected, and this results in a still further reduction of load factor. The load factor can also be defined in terms of the maximum demand, as follows :—

$$\frac{\text{actual energy consumption over a period}}{\text{maximum power demand} \times \text{length of period}},$$

the numerator and denominator being expressed in the same units, namely energy.

A graph which plots power to a base of time (Fig. 9) is known as a load curve, since it exhibits the fluctuations of load throughout the period concerned. Since the area of such a graph represents energy,

* With hydro-generation, especially from run-of-river plants where the power-component cost ratio is even higher, such a plan is not unknown.

the load factor is measured by the shaded area divided by the total area of the surrounding rectangle, or the mean height of the diagram divided by its maximum height. Such a curve is frequently plotted, with one day as its base, to show the load on a station or cable. Some undertakings cut out their load curves on cards and stack them side by side to form a three-dimension "load model," and a year's stack of this kind is shown in the frontispiece.*

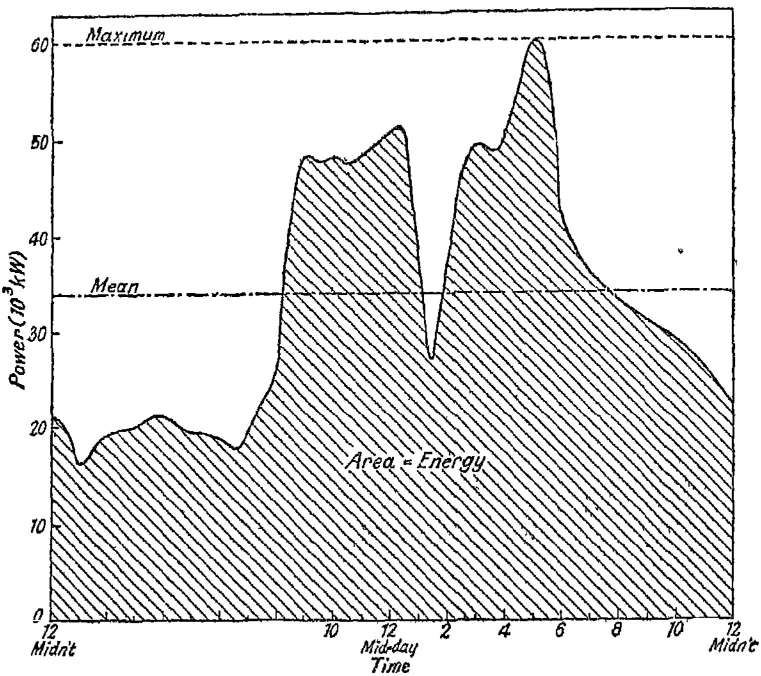


FIG. 9.—Daily Load Curve.

The particular curve in Fig. 9 represents a day in late October 1937 for an undertaking (Coventry) having a well-developed industrial and off-peak load. The day's output was 818,000 kWh, giving a mean height of 818,000/24=34,000 kW (or 34 megawatts, MW). As the maximum height was 60 MW, the day's load factor was 57 per cent. But since the year's maximum was higher than this, and since the mean daily output (averaged over the year) was lower, it follows that the *yearly* load factor (which is the determining factor in the matter of

* These and other special methods of load representation are described in "Means of Load Representation" by P. Schiller : *British Electrical and Allied Industries Research Association* (generally referred to as the "Electrical Research Association" or "E.R.A.") *Technical Report* K/T 107.

costs) had a much smaller value. The actual figure for the year in question was 36 per cent.

There is something resembling load factor in almost all forms of production. Machinery, ships, lodgings, taxicabs, in fact almost all capital goods are necessarily idle during periods of their existence; and during these periods they cost money in interest, depreciation and standby charges, which has to be paid for during the working period. The longer (per annum) this idle time is in the case of any given plant, the greater will be the capital charges then incurred, and the less the earning time available. Hence the improvement of load factor is at all times a thing to be aimed at, although in many branches of production the economic limit is soon reached beyond which further improvement is not worth its cost.

Thus in a factory employing machine plant and working eight hours a day, the actual capital charges per item of product will necessarily be more than they would be if an overtime or night shift be worked: but unless the machinery is very expensive the saving resulting from overtime would be more than balanced by the higher cost and lower efficiency of labour during these extra hours, quite apart from the human inconvenience involved. In most factories, therefore, overtime is rather a temporary expedient than a permanent economic advantage, except as regards particular and highly expensive machines. With electricity supply, as will be seen below, the balance between machinery and labour is altogether different, and an improvement of load factor will almost always produce a very great cheapening of supply.

Magnitude and Effect.—It is easy to see that the load factor of an individual consumer will generally be low. Thus a power consumer working a nine-hour shift, which, allowing for short time, holidays, repairs, etc., may be estimated to extend over 300 days per annum, even if his mean rate of consumption during the shift were 75 per cent. of his maximum consumption, will have a load factor of only $\frac{9 \times 300 \times 0{\cdot}75}{24 \times 365} = 0{\cdot}231$, *i.e.*, 23·1 per cent. A domestic consumer using electricity for lighting is likely to be even worse. Thus, suppose that he has twenty equal lighting points, of which at night time an average of four are employed continuously and a maximum of seven at any one time.* The mean daily hours of burning might be estimated at the time of the equinox, say, from 6.30 to 11 p.m., *i.e.*, four and a half hours, but the effect of the Daylight Saving Act has been to reduce this somewhat. Taking a mean of four hours per evening, his

* The maximum, if metered, will probably be read from an instrument taking twenty to thirty minutes to reach its final position. Thus accidental overloads or temporary switching on of extra lights will not be registered.

load factor would be only $\frac{4 \times 4}{24 \times 7} = 0{\cdot}095$, *i.e.*, 9·5 per cent. Fortunately these low individual load factors are redeemed to a considerable extent by the diversity factor discussed below.

The effect on the supply costs of increasing the load factor n times is that the fixed cost will be spread over n times as many units and will therefore be $1/n$th as much per unit whilst the running charge will be the same as before. The gain will therefore depend on the initial proportions of the two costs.

Assume that in the first instance three-quarters of the total cost is due to items dependent on size of equipment and one-quarter to items proportional to energy. The initial cost per unit can then be represented by $\frac{3}{4}$ (fixed) + $\frac{1}{4}$ (running) = 1. If now the system load factor be doubled, the fixed costs per unit will be halved, and the total cost per unit will be $\frac{3}{8} + \frac{1}{4} = \frac{5}{8}$, *i.e.*, little more than half what it was previously. It would not be true, however, to say that if an individual consumer doubled his own load factor the costs of supplying him would be almost halved, because his improved individual load factor will be largely offset by a reduced diversity. This fact is recognised in many two-part tariffs in which the ratio of the two parts is much less extreme than would be suggested by the costs ratio mentioned above.

It will always be found that in electricity supply, whether from coal or water power, the fixed costs form much the greater part, and the effect of load factor is therefore paramount. Whatever changes we may make in our organisation or in our technique, an improvement in load factor will still remain the shortest cut to price reductions. This is because (and the fact must never be lost sight of) the chief expense in the production of electricity lies not in mining or otherwise obtaining the potential energy, but in converting it into a useful form.

Many imaginative writers, in planning their Utopias of the future, have suggested that when we can tap some cheap inexhaustible source of energy such as that of the atom, all our troubles, at least as regards cheap power and labour saving, will be at an end. But if the plant required to convert this energy into a utilisable form were only 30 to 40 per cent. more expensive than our present plant for converting coal energy into electricity the total cost would be just as much as it is now. What would be altered would be the *proportions*, and supply economics in the atomic age may prove to be something like that now applying to run-of-river hydro-electric stations.

Diversity Factor.—If there were a hundred consumers connected to a station all with identical loads in every respect, the energy consumption would be 100-times the individual one, and the maximum demand would also be 100-times, so the station load factor would be the same as each consumer's load factor. But actually the consumers vary, so that whilst the station consumption is always the sum of all

the individual consumptions, the station demand is less than the sum of the individual demands. For even if two consumers had the same loads and load factors they would probably not take their maximum currents at the same instant, and, in fact, their consumptions might not even overlap, so that the joint demand would be far less than double the individual one. Owing to this fact of different consumers taking their maxima at different times of the day and year, the maximum demand on the station is always less than the sum of the consumers' maxima, and hence the station load factor is always better than the average consumer's load factor. In an extreme case it would even be possible for two consumers, each with a load factor of 50 per cent., to combine to give a station load factor of 100 per cent.

The sum of the consumers' maxima divided by the actual maximum coming on the station is called the diversity factor. Since the maximum demand forms the denominator of the load-factor fraction (see p. 59),

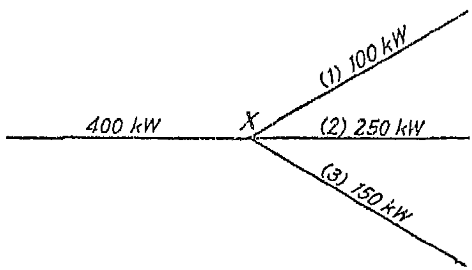


Fig. 10.—Diversity of Maximum Demands.

and since the numerator (actual consumption) is necessarily the same for both consumers and station (apart from losses) it follows that the station load factor will be the mean consumer's load factor multiplied by the diversity factor. (The mean, of course, refers to a mean of consumption, not of consumers.) Diversity factor can be expressed not only as between consumer and station, but also can refer to the conditions before and after any point at which a number of lines or cables meet. It will be seen that diversity factor cannot be less than unity, whilst load factor, like power factor, cannot be greater than unity.

Consider any point X in a distribution system (Fig. 10). A number of distributors, 1, 2 and 3, on the right connect to a single feeder on the left, *i.e.*, power is flowing from left to right. In a given period, say one month, the largest power recorded is 100 kW in distributor (1), 250 kW in (2), and 150 kW in (3). Owing to the fact that these do not all occur simultaneously, the maximum power in the feeder in the same period is 400 kW. The diversity factor existing at the point X is then $(100 + 250 + 150)$ divided by $400 = 1{\cdot}25$.

The importance of diversity factor in lowering the maximum demand on a station and so improving the load factor is very great. A lighting consumer taking energy for a few hours and at a particular time each day has both a bad load factor and comparatively little diversity factor with reference to other lighting consumers; but even here the item is considerable, since some users start earlier, others finish later, and very few synchronise exactly their biggest demand. Power and heating users will generally have better individual load factors, and they also vary considerably amongst themselves—bakeries, restaurants and domestic-heating consumers taking their biggest loads when factories are not running—whilst the diversity factor with reference to the lighting load will, of course, be excellent. Hence the importance to the supply authority of getting consumers not only with good individual load factors, but also using electricity for as many different purposes and at as many different times of day as possible.

Degree of Capitalisation.—The outstanding economic feature of electricity supply may be said to be the high ratio of standing charges to working expenses. This feature is to some extent common to all types of service (*e.g.*, gas, water, telephone and other supplies, transport, etc.) where the overheads are almost always bigger than in the manufacturing or distributive industries. It is largely due to the high price and rapid obsolescence of the plant employed, coupled with the relatively small labour costs.

The position is intensified in the case of electricity supply by the virtual impossibility of storage. In this respect electricity is well-nigh unique, even among the most immediate and personal types of service. With commodities, one expects that some kinds will have to be ordered in advance, whilst others, more standard in character, frequently go temporarily out of stock. Even with an individual service, as when one calls in a doctor or orders a taxi, there are busy times when one has to wait one's turn. At rush hours, again, transport is held up, telephone lines become engaged, and crowded buses refuse one admission. But with electricity, at the moment of pressing the switch, the alternator has to be working that self-same instant. The service is immediate or it is nothing at all. Moreover, the kilowatt we are demanding is ours alone for as long as we need it, and no one else can share it with us. Probably the nearest parallel is the water supply, since (on the distribution side at least) there is here no effective storage; and it is noteworthy, in the figures below, that the water undertaking offers an even more extreme example of high overheads.

In order to see the position which electricity occupies in this connection it will be well to give figures showing the amount of capitalisation, first in a manufacturing industry and then in certain services. Single undertakings may give misleading results, because expansions may have been made when plant was expensive, and policies may

differ as to charges and loan redemption. But an average result will smooth out individual variations, and in order to eliminate differences of management the supply figures refer only to municipal undertakings.

In contrasting the generation and distribution of electricity with the manufacture and delivery of other commodities, a useful basis of comparison will be with the engineering industry, itself a highly capitalised one. According to the 1935 Census of Production, the total engineering firms of the country produced a yearly output of about 100 million pounds, of which approximately 50 per cent. went in payment of material and 30 per cent. in wages. Hence the whole of the overhead and capital charges absorbed only 20 per cent. of the total, and if this sum represented an average of 10 per cent. on the invested capital, the total investment would be just double the total annual receipts. In electricity supply the invested capital is about seven times the total annual receipts, and as a consequence a large proportion of these receipts are absorbed in capital charges.

In comparing electricity with other sorts of supply, probably the simplest guide to the degree of capitalisation is to find the amount of capital expenditure required to produce a certain annual revenue or turnover. These figures are given in the table below, which is reprinted from Volume I.

Capital Expenditure Per Cent. of Turnover

Gas Supply, 1936–7	396
Electricity Supply, 1936–7 . . .	694
Water Supply, 1936–7	1,380
British Railways, 1937	597
Mechanical Engineering, 1935 . .	282
Electrical ,, 1935 . .	329

The natural result of finding electricity supply (in its economic aspect) occupying a mid position between those of gas and water, is to look for a method of charging for electricity intermediate between those which have been found suitable for gas and water supplies. And since gas is very generally sold at a fixed price on the metered quantity and domestic water on a fixed rating independent of quantity, it seems reasonable to combine both methods of payment in the charge made for electricity. The two-part tariff is therefore the logical outcome of an economic survey of the supply industry.

Two-Part Costing.—Having shown from the actual figures of supply undertakings the inadequacy of a purely quantity rate and the necessity for dual costing, the next step is to set about this division into two. The analyses which follow are primarily aimed at allocating the various

costs into their power and energy components. A two-part formula can then be constructed representing the cost of supply at the point in question. This formula will take the same form as that of the maximum-demand tariff commonly employed for large industrial and bulk supplies, namely, so many pounds per annum per kilowatt of maximum demand plus so many pence per kilowatt-hour.

In the event of such a tariff being adopted it may even be possible to identify cost formula and tariff, and to regard the power component of the cost as paid for by the standing charge of the tariff and the energy component by the running charge. It would be premature, however, to make any such general assumption, and the tariff to be submitted to the consumer is likely to be a very much modified and possibly unrecognisable version of the cost formula. It may not have a kilowatt basis and may not even be of a two-part form, although in all cases the two-part analysis will be found to be an essential preliminary to its construction. Retail tariff construction is dealt with later, and the present section concerns itself purely with the cost formula.

In addition to the division into power and energy components (or "fixed" and "running" costs*), these analyses also separate generation from transmission and distribution. The first case is a simplified generalisation of the whole of public electricity supply in England and South Scotland in the year 1948–9. The second case is treated in more detail, and is a particular example representing one typical system. It is a synthesis as well as an analysis, since in this case a hypothetical "costs tariff" is built up to represent the cost of supply at a number of different points. A practical example is given in the next chapter showing how an actual consumers' tariff can be developed from the cost figures.

The basic principle used in costing the supplies at any given point is to add together all the expenses of the kind in question (fixed or running) up to the point, and then to divide the total by the kW or kWh supplied at that point. The quotient then gives the annual charge per kW, or the charge per kWh, as the case may be. As distribution proceeds, additional costs will swell the numerator of the quotient, whilst diversity and losses will affect the denominator in a manner which can be better seen in the examples themselves.

Some explanation should be made as to the method of allocation between fixed and running. When plant is installed to meet consumers' power demand it has to be paid for each year thereafter in the form of interest, rent, maintenance, etc. But when coal is

* Whilst very broadly the two parts correspond to power and energy respectively there is considerable fringing on both sides, and in particular the "power" component includes a number of overheads not strictly related to size of plant. In future the words "fixed" and "running" will therefore be used in preference to "power" and "energy".

burnt to supply energy it is paid for directly on a quantity basis, into which time does not enter. Hence the first group of costs must include every item which is a function of time (and frequently of plant size) and which goes on from year to year independent of output. It must therefore include all capital charges on the plant and buildings (interest, depreciation, insurance, etc.), maintenance (except in so far as affected by output), rents and rates. It must also include the majority of the management and operating expenses, wages and salaries, since these are but little affected by the energy output.

The second group of costs should include only those items which are proportional to the actual units generated—*i.e.*, chiefly the fuel and some other generation working expenses. The fixing of the dividing line between the two groups must be to some extent arbitrary, especially as regards the generation working expenses. As a first approximation, the whole of the fuel can be put in the energy section and all the other expenses included in the other group. This involves some inaccuracy in both directions—a certain amount of fuel, which may be anything from 5 to 20 per cent. of the whole, is required to keep the plant " banked " ready for action, and on the other hand there are other items such as wages and repairs which are partly dependent on output. Moreover there are certain charges and appropriations commonly regarded as " overheads," such as income tax, the profits of a company or the rates contribution of a local authority undertaking, which should perhaps be spread over both fixed and running groups. A more scientific allocation of the generation expenses is described in the section below. There is also a special group of distribution fixed costs which are not dependent on any electrical quantity, and which are best expressed as a cost per consumer.

As regards the division of costs down the system, the usual dividing line (on technical grounds) is between generation on the one hand and transmission and distribution on the other. A better plan here is to group main transmission and interconnection with generation, and secondary transmission with distribution. Capital invested in main lines between power stations takes the place of expenditure on fresh generating sets, since it serves instead of stand-by plant. It is therefore best regarded (economically) as part of the generation system. The only difficulty which has in the past arisen in this connection is with the management expenses, rates, etc., when these were given as a single total for an undertaking carrying out both generation and distribution. The bulk of these must then be regarded as incurred in distribution.

Sources and Dates.—The chief source of information on public electricity supply in Great Britain has been the *Return of Engineering and Financial Statistics* published annually by the Electricity Commissioners. There was also a useful and more up-to-date summary of

generation published by the same authority. The Returns related to the calendar year in the case of the company undertakings, to the 12 months ending March 31st in the case of the English local authorities, and May 15th in Scotland.

The last Return in which full statistical details were given was that published shortly before the war and related to the year 1937/8. An abridged Return covering the five war years 1938–9 to 1942–3 was published at the end of 1946. Annual returns, but not in the full pre-war detail, have since been published for each of the five later years, covering the period up to Vesting Date. The last two were published by the Ministry of Fuel and Power.

On April 1st, 1948, all authorised electricity undertakings outside the North of Scotland vested in the British Electricity Authority and the fourteen Area Boards under the 1947 Electricity Act. Four months later the Electricity Commissioners were dissolved. The Boards' first Annual Reports were published early in 1950 and covered the 12 months ending March 31st, 1949. Unless otherwise stated, the data made use of in this book is taken from this Report and refers only to the area of the British Electricity Authority.

Both generation and distribution in the area north-west of a line running roughly from Stirling to Dundee is in the hands of the North of Scotland Hydro-Electric Board. Annual reports are issued, but so far these have been of a somewhat different character, and serve rather more as a guide to the Highlands than to the economics of its electricity supply.* However, in terms of kWh sold the area represents (electrically) only about 1½ per cent. of the country, and the omission of this from the figures analysed here will not materially distort the general picture.

Another omission is that of the non-statutory undertakings. Most of these are quite small and are likely to merge into the appropriate Area Board before long. A few, somewhat larger, are associated with collieries and may continue their operations under the National Coal Board for the time being. The most serious gap in the statistics is due to the absence of figures for private generation, chiefly by large industrialists. The figures obtained in the last Census of Production (1935) are now out of date, but another Census is being taken and should yield valuable information.

Generation Expenses Allocation.—It might at first appear that the running charge at the generating station is merely the cost of the coal, oil, water, and suchlike materials divided by the number of units sent out. This, however, would give too high a figure because even if there were temporarily no output, merely a liability for one, some coal would be consumed in preparedness. (It is true, of course, that if there were permanently no kWh there would be no coal needed, but there would

* Later reports have taken the same form as that of the other Electricity Boards.

be no plant needed either. Such a *reductio ad absurdum* argument leads nowhere. What matters is not the limiting condition but rather the slope of the curve over the range of variation actually involved. The curve does not point to the origin, as will be seen below.)

Generation works costs are in three main groups: fuel (including ashes and handling); operation (salaries and wages); and repairs and maintenance (including oil, water and stores). The method of

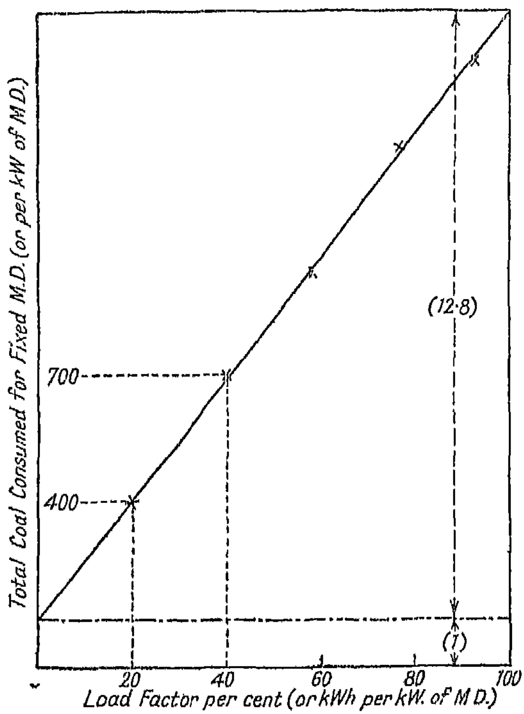


FIG. 11.—Fuel Costs Allocation.

allocating these between fixed and running components can be instanced in the case of fuel, and can best be illustrated by a hypothetical example.

Consider any generating station, and let the number of kWh and the maximum demand be measured over a given period such as a day or a week. Depending on the demand, certain boilers and sets have to be in use and in readiness at various times throughout the day; but since the load factor is not unity they are not fully loaded all the time. Consider any two such days on which the M.D. is the same, say, 100,000 kW. On the first day, the kWh generated are half a

million (giving about 20 per cent. load factor), and the coal consumed is 400 tons, thus giving one of the points marked with a cross on the graph, Fig. 11. On the second day the kWh are twice as many, but the coal consumption is less than twice—say 700 tons for an output of a million units. This gives a second point, as indicated by another cross on the graph.

If tests could be made on a number of such days, each with the same M.D., the coal consumptions would be represented by a number of points such as those shown on the graph. They would be found to lie approximately on a straight line not passing through the origin, showing that there would be some coal consumed even with zero output. A very similar line was first drawn by Willans to show the steam consumption of an engine, and is frequently known as the "Willans line." It is characteristic of a large number of engineering processes in which there are two groups of losses—constant and proportional.

If the above test were made on another day in which the M.D. was twice as great and the kWh twice as great (giving the same load factor), probably the coal consumption then would be just double. This is because the proportion of standby and partially-used plant would be the same in the two cases. (The exact figure would depend on the particular loads and sizes of sets, but on the average this statement should hold true.) If, therefore, the results of a number of different days or even of a number of different stations were logged by plotting the coal per kW of M.D. against the load factor they would all tend to lie on a line such as that shown in the figure. (*N.B.* If the coal *per unit* were plotted, for a given load factor, the result would be a hyperbola standing on a rectangle as in Fig. 35 on p. 193.

Results from a number of stations have been compared by the Electricity Commissioners, and used in allocating the cost of production at selected generating stations.* The Commissioners have determined the running portion of the fuel cost to be 12·8-times the fixed portion when the load factor is unity. (*See* Fig. 11 : the straight line represents this result, but the crosses are purely imaginary.) Hence, if the load factor is L per cent. the proportion of fuel to be allocated as fixed costs is $\frac{100}{100 + 12{\cdot}8\ L}$. In the case of salaries and wages, the constant is 0·38 instead of 12·8. In the case of maintenance, etc., it is 0·876 for a station that is continuously in commission. The rest will be running costs in each case.

It should be explained that the above formula is based on figures obtained many years ago on independently operating stations and is

* Statutory Rules and Orders, 1929, No. 1015. *Electricity (Allocation of Cost of Production) Regulations.* H.M. Stationery Office. *See also* the reference on p. 129.

not necessarily representative of a modern interconnected system. In particular, there is reason to believe that the proportion of fuel cost allocated to fixed costs is too high. Unfortunately, although out-of-date, it is not yet superseded by any other accepted standard formula and, for want of a better, it is employed elsewhere in this book and for purposes for which it was never designed.

Supply in Great Britain, 1948/9.—The following is a rough first approximation to an analysis of public electricity supply in the year ended March 31st, 1949, in the area covered by the British Electricity Authority (*i.e.*, excluding the North of Scotland). The aim of the analysis is to group all the essential outgoings of the Boards under four heads, namely, fixed and running components of generation, and fixed and running components of distribution. In order to simplify the picture, only the strictly electricity-supply activities are covered, and certain items (such as work done for consumers) are omitted from both sides of the account. For the same reason some purely financial items are omitted from both sides. The following notes refer to the numbered items in the table below.

		CENTRAL AUTHORITY.				*AREA BOARDS.*	
				Allocation: Generat'n.	*Main Transm'n.*		
	Running Costs (£ million):						
1	Fuel (allocated per S.R. & O. 1015).	63·85					
2	Operation (Sals. and Wages) (allocated per S.R. & O. 1015).	0·85					
3	Repairs and Maintenance (allocated per S.R. & O. 1015).	3·89					
4	Total Generat'n running cost	68·59	36·0%				
	Fixed Costs (£ million):						
5	Generation: Fuel, Operation, Repairs, etc., "fixed" allocation	21·32	11·2%	21·32			
6	Generation: Total Works Cost (fixed and running)	89·91	47·2%				
7	Main Transmission and Distribution Operation	1·11	0·6%		1·11	18·67	9·8%
8	Rents, Rates, Insurance, etc.	5·37	2·8%	(no	split)	8·34	4·4%
9	Administration and General	3·40	1·8%	,,	,,	17·41	9·1%
10	Interest, *less* Receipts	7·50	3·9%	6·59	0·91	7·72	4·0%
11	Depreciation	13·24	6·9%	10·48	2·76	18·07	9·5%
12	Total Fixed Costs (lines 5 and 7 to 11)	51·94	27·2%			70·21	36·8%
13	Total net Costs (lines 6 to 11)	120·53	63·2%			70·21	36·8%

Total Net Costs (line 13)	190·74
Surplus	4·39
Total Turn-over	195·13
Income from Sale of Electricity	191·36
Income from Meter Rents, Contracting, etc., etc., *less* Energy Purchased from outside .	3·77
	195·13

The generation working costs are in three groups—fuel, operation (salaries and wages), and repairs and maintenance (including oil, water and stores). These are allocated between fixed and running in the manner described in the previous section, and the running portions are shown in lines 1 to 4. The three fixed portions are grouped in line 5 and the total works cost of generation is shown in line 6.

All the other costs and charges are allocated entirely to the fixed group. Such an allocation is without question in the case of the capital charges (interest and depreciation) since these are directly proportional to the capital expenditure and therefore to the maximum demand in kW or kVA. As regards the operation costs of main transmission and distribution, and the administration and general expenses of the Electricity Boards, whilst these are mainly dependent on the size of the equipment and the organisation, it is arguable that they are slightly affected by the actual number of units carried from day to day. (The analogy with generation operating costs is no guide because there is no operation comparable to the flow of fuel.) The effect is likely to be small and no allowance has been made in the present analysis.

Where possible, a split has been made between generation and main transmission, but the association is necessarily a close one both technically and administratively. On technical and economic grounds, main transmission should be grouped with generation rather than with distribution ; since in this country it takes the form of interconnection between generating units, which are operating almost as one big station, rather than long-distance transfer of large blocks of power. In fact, in a tightly-coupled system such as we have in Great Britain, the high-voltage grid is more of a bus-bar than a transmitter. Finally, since main transmission comes under the Central Authority, it is not possible to make a split of the rents, rates and most of the administration and general expenses, although these could be arbitrarily divided in the ratio of the capital expenditures.

Line 7 shows the works cost (operation, repairs and maintenance) of main transmission and of distribution. The next two lines show the rents, rates, insurance, etc., and the administration and general expenses incurred by the Central Authority and by the Area Boards. Finally, the interest and depreciation charges are shown separately

for generation, main transmission and distribution. Included with the interest are certain financial expenses, less receipts, and included with depreciation are the amounts written off intangible assets.

The addition of the above items gives the total net cost, and this, together with a small surplus, can be equated to the total receipts less the cost of energy purchased by the Central Authority. (*See* end of table.) It will be noted that in this year the Electricity Boards as a whole made a slight profit or surplus, which was put to reserve. Regarding this item as a small fluctuating quantity, plus or minus, which is necessary to give the necessary flywheel effect, it can be omitted from the cost analysis as has been done here. If, however, it became a permanent (positive) feature, it would have to be regarded as a small overcharge to be spread over both fixed and running portions. The same treatment would have to be applied to income tax, which was a negligible item in this year's accounts.

Mean Costs Formula.—The final stage in the analysis is to express each of these costs as *rates* by dividing by the appropriate electrical quantity existing at the point in question. The resulting quotient will then give a costs formula which, if desired, could be used as the basis of a tariff applicable to a purchaser at that point. Starting with the running cost for generation, line 4, and dividing this by the kWh sent out (43,582 million), the running charge at this point is found to be 0·378*d.* per kWh. Distribution does not increase the running cost, but the losses (approximately 11 per cent.) decrease the denominator giving a larger quotient, namely, 0·427*d.* per kWh sold. (In dividing the costs by the units sold, it is necessary to make an adjustment for the electricity purchased from outside, since its cost has been excluded.)

Taking next the fixed costs, the total for generation is given in the first column of line 12. The maximum demand met by the whole system on the peak day of the year (February 4th, 1949) was 10,016 MW. The sum of the maximum demands met at all the grid supply-points will be greater than this simultaneous M.D. in the ratio of the diversity, which for the present estimate is assumed to be 1·15. The fixed charge at the grid points then becomes

$$\frac{51{\cdot}94}{10{,}016 \times 1{\cdot}15} = \pounds 4\ 10s.\ 0d. \text{ per annum per kilowatt.}$$

In finding the total fixed cost of the supply as delivered to the consumer, it is necessary to add the distribution costs as shown in the columns for the Area Boards. This total must then be divided by the aggregate of the consumers' maximum demands in order to express it as a rate per kW. (The distribution fixed costs can more accurately be expressed per kVA of demand, but for the present analysis it will

be assumed that the power factor is constant so that generation and distribution costs can be added on a uniform kW basis.) This aggregate of the individual consumers' demands is a somewhat loose conception when taken over the whole field of consumption, domestic as well as industrial, but in order to get a rough overall figure an additional diversity of 1·39 will be assumed. This gives a total diversity of $1{\cdot}15 \times 1{\cdot}39 = 1{\cdot}6$ between station and consumer, and the capacity to meet demand will therefore be increased in this ratio. On the other hand there will be a fall in kW capacity due to losses in the distribution system. The proportion will be somewhat less than the corresponding loss in kWh, and a figure of 9 per cent. has been assumed.*

It will be evident that, as distribution proceeds, a number of the fixed costs are not proportional to the kW of demand but to the number of consumers connected. Much the biggest capital item is for mains and services, and the cost of a service connection is in part independent of either kW or kWh. Many of the administration expenses and selling costs (*e.g.*, showroom expenses, metering and billing) are similarly a function of the consumers rather than of the electricity. This matter is discussed in greater detail later, and for the present calculation it is assumed that these consumer-related costs amount to the following annual sums :—£1 10*s*. per domestic consumer, £2 per commercial consumer, and £3 per industrial consumer. (These figures are *average* costs, affected largely by pre-war installation prices. Marginal costs —per *new* consumer—would be higher.)

The total number of consumers is given in the Report, and for this analysis the proportions are assumed to be the same as in 1937, namely, 87 per cent. domestic and farm, 11½ per cent. commercial, and 1½ per cent. industrial. Multiplying these out gives a total of £19¼ million consumer-cost to be subtracted from the total distribution fixed cost, the remainder being proportional to kW. No attempt has been made to separate out these consumer costs specifically from the individual items making up the total, but in calculating the rate per kW an overall *pro rata* adjustment has been made. The final figure is then

$$\frac{51{\cdot}94 + 70{\cdot}21 - 19\tfrac{1}{4}}{10{,}016 \times 0{\cdot}91 \times 1{\cdot}6} = 7{\cdot}06 \text{ (£ per kW).}$$

* This figure is not given in the Report, but the loss in units is shown as 11 per cent., and the loss in kW is likely to be less than this for the following reason. The ratio $\frac{\text{units delivered}}{\text{units sent out}}$ (given as 89 per cent.) is a measure of the all-day efficiency of the transmission and distribution system. The corresponding ratio in kilowatts (taken as 91 per cent.) is a measure of the efficiency at the time of maximum demand. Distribution plant such as transformers and rotaries having considerable iron and shunt losses are likely to result in a higher efficiency at peak periods than their all-day efficiency, assuming they are continuously electrified. This clearly does not apply to series copper losses nor to the case when additional plant is connected at peak periods.

It will be noted that on this estimate the consumer cost is 10% of the total. The kW cost is 54% and the kWh cost 36%.

The resulting cost formulæ can be summarised as follows :—

At the bulk-supply point—£4 10*s*. per annum per kW plus 0.38*d*. per kWh.

At the consumers' terminals—£7 1*s*. per annum per kW plus 0.43*d*. per kWh plus a consumer charge (£1 10*s*. to £3 per annum, depending on type of consumer).

(*N.B.* These cost formulæ must in no wise be confused with the tariffs, though they may be made the basis of a tariff construction. Even where the form was appropriate, the values would require adjustment, *e.g.*, to allow for what is described later as "differential diversity".)

Ratio of Fixed to Running Expenses.—Whilst a high ratio of fixed to running costs is characteristic of electricity supply, the actual value of this ratio fluctuates with the various price levels. A general price increase, such as has occurred in the last fifteen years, produces a temporary fall in the ratio because many of the fixed expenses are capital charges on plant bought many years ago. These therefore lag behind the running expenses in their response to price changes. Another factor, possibly more permanent, is that power-station coal has risen in price quite out of proportion to the general price level.

Taking the thirteen-year period from 1935 to 1948, the mean price per ton of coal into power stations rose from 15*s* to 52*s*. 11*d*., *i.e.*, 3½ times, but most of the plant in use in 1948 was purchased at pre-war prices and the capital charges thereon were still very much at their old level. For the time being, therefore, running costs have risen steeply and fixed costs only slightly. Much of this change in ratio is only temporary. New plant now being installed is costing some 2½ times as much as pre-war, and the proportion of new plant must rise steadily until it reaches 100 per cent. in 30 years or so. Wages, salaries and most other expenses have also risen. Moreover, if and when prices fall again, the time-lag will operate in the other direction.

The consequence of these various changes can be seen in the figures listed above. An analysis made on these lines for the year 1935 showed the running cost to be exactly one-eighth of the total, whilst the fixed costs were seven-eighths. The corresponding ratio for 1948 is 36 per cent. running and 64 per cent. fixed (lines 4 and 12 of the table). The running-cost proportion has probably now reached its peak and unless

there are further substantial increases in coal prices the ratio is likely to decline steadily due to the increasing installation of post-war plant. The ratio is hardly likely to revert to its pre-war value of 1 to 7, but it would not be surprising if (at present price levels) it ultimately settled down to stability at 1 to 4 or 5.*

The effect of interest rates must also be considered. Nationalisation has reduced certain of the capital disbursements in respect of the company undertakings, and all future capitalisation will be on the basis of gilt-edged rates. Thus it was stated at the time that shareholders in the aggregate would lose some £4½ million annually as a result of the conversion of ordinary shares into Government stock, and this should mean a corresponding reduction in the capital charges of the new authority. On the other hand, gilt-edged interest rates may rise, and it is by no means certain that future capital can be raised at the rates ruling at Vesting Date. There is also the continuing tendency (characteristic of all modern industry) to install ever more elaborate and expensive equipment in order to reduce running costs. Moreover the restrictions on capital development following the war, and the load-shedding which became necessary, resulted in an artificially high load factor. In effect, fixed costs were being pegged down whilst running costs were unrestricted.

Two-Part Diagram.—This has been devised by the author to represent the two-part cost analysis and the maximum demand tariff in a semi-pictorial fashion so that they can be at once visualised, and yet in such a manner that quantitative exactness is not sacrificed. The diagram can be used as in Fig. 12 to illustrate the foregoing cost analysis, or it can be elaborated as in Fig. 13 to build up a two-part cost formula for a number of different points of the system. When the formula is used as a tariff basis, the diagram serves not only to link up and explain the price charged in terms of the various cost items but also to show what the price might be if the energy were taken at a different load factor or in another form or place.

The cost formula is in the form already described, namely, a simple two-part expression of so much per annum per kW or per kVA of maximum demand plus so much per kWh consumed. These two components are different in kind and must be plotted quite separately. They cannot be added except on the basis of some known load factor, yet they need to be plotted side by side since they are affected similarly (though not equally) by such items as losses in the plant and cables. In the type of diagram here shown, these two components are plotted vertically below and above a horizontal "zero" line, so that they

* A figure of 1 to 3 (*i.e.* 25% running, 75% fixed) has been used elsewhere in the book as a representative figure for a few years ahead.

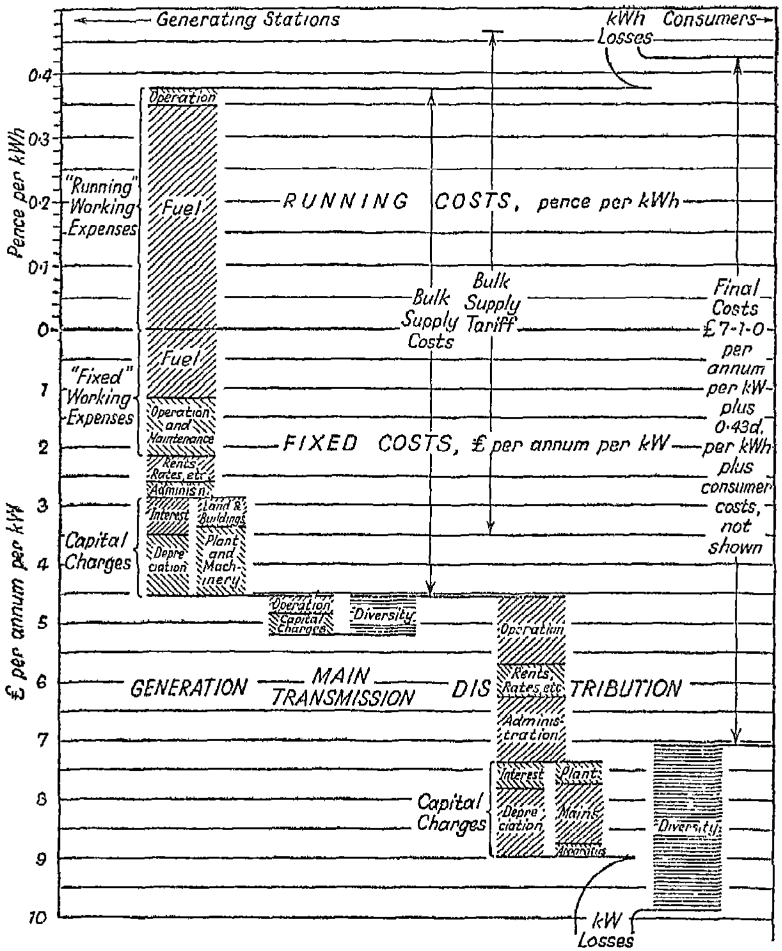


FIG. 12.—Two-part Diagram for 1948/9.

progress side by side and can at any point be added if the load factor is known.

A glance at the next two diagrams will show that they run parallel to this horizontal line, and below this line the ordinates (measured downwards) represent annual costs per kW of demand and are scaled in pounds sterling. The ordinates above the line represent the cost per kWh and are scaled in pence. (In what follows, this is referred to as the " energy " scale.) The left-hand side of the diagram may be said to represent the generating station and the right-hand side the consumers' premises.

The two-part diagram shown in Fig. 12 has been drawn to illustrate the allocation carried out above. It will be seen that the two sets of costs form a belt or river gradually building up as it flows from left to right and is swelled by various tributaries. The scale of the lower portion is in £ per annum per kW of demand, and on this basis are plotted the various costs shown in the table. Generation fixed costs are shown occurring at the extreme left, followed by main transmission, with distribution costs on the right. Central Authority costs which cannot be split, *e.g.*, administration, are shown under generation, The effect of the distribution losses in decreasing the kW available to meet demand is shown as a tributary increasing the price per kW. Diversity produces the opposite effect, and is here shown by horizontal shading leading to a reduced ordinate.

The top portion of the diagram is scaled in pence per kWh, and it illustrates graphically the first part of the table. It will be seen that the various components combine to produce two sets of two-part costs, one for the bulk-supply point and the other for the consumer. By way of comparison, the mean bulk-supply tariff is shown in the figure, namely, £3 10*s.* per annum per kW plus 0·46*d.* per kWh. The running charge has been adjusted according to the mean price and calorific value of the coal consumed.

The generation costs here calculated give a higher fixed charge and lower running charge than the bulk-supply tariff, the overall effect (distance between arrows) being approximately the same. This indicates a certain difference in the basis used for proportioning the fixed and running allocations, and possibly some adjustment for differential diversity. (The basis used here is that of S.R. & O. 1015 described on p. 70.)

The stream of costs shown in the figure can evidently be regarded as a whole, not merely as two independent portions. That is to say, for any given load factor the two can be added or resolved into a single overall price per kWh. Now the effective system load factor (based on the potential demand) was 47 per cent., but the mean consumer's load factor will be lower than this in the ratio of the overall diversity

factor (1·6). It is also necessary to allow for the difference between the loss in kW and the loss in kWh, giving a final load factor for the average consumer of nearly 30 per cent.

As a check on this, it will be found that at this load factor the two-part charge of £7 1*s.* plus 0·43*d.*, together with the consumer costs, becomes equivalent to 1·18*d.*, the actual mean overall figure. Furthermore, in this particular diagram the two scales have been so chosen that at 30 per cent. load factor all vertical distances represent pence per kWh on the scale shown in the upper part. The total widths of belt therefore give a visual impression of the importance of the various items to the average consumer.

It must be clearly understood that the above, although representing the total costs of supply for the country on a two-part basis, by no means represents the price to be charged. Even when the form is is suitable for an actual tariff (*e.g.*, for industrial power supplies) the magnitudes would probably not be. The diagram represents a simplified allocation of costs and receipts rather than a possible tariff; moreover, it is a generalisation covering a wide range of variations, high and low-tension supplies, domestic and power consumers. Of the three main variations concerned, load factor, diversity factor, and distribution costs, only the first named is differentiated in the diagram, the other two being averaged over the whole country.

As regards the differentiation between transmission and distribution, high and low tensions, etc., and the framing of possible tariffs at the various points, a single case is taken below and treated in detail. This follows the same lines as the previous one, but generation, main transmission, and distribution are here taken separately, and the diagram as it develops from left to right represents a definite sequence of stations and gear.

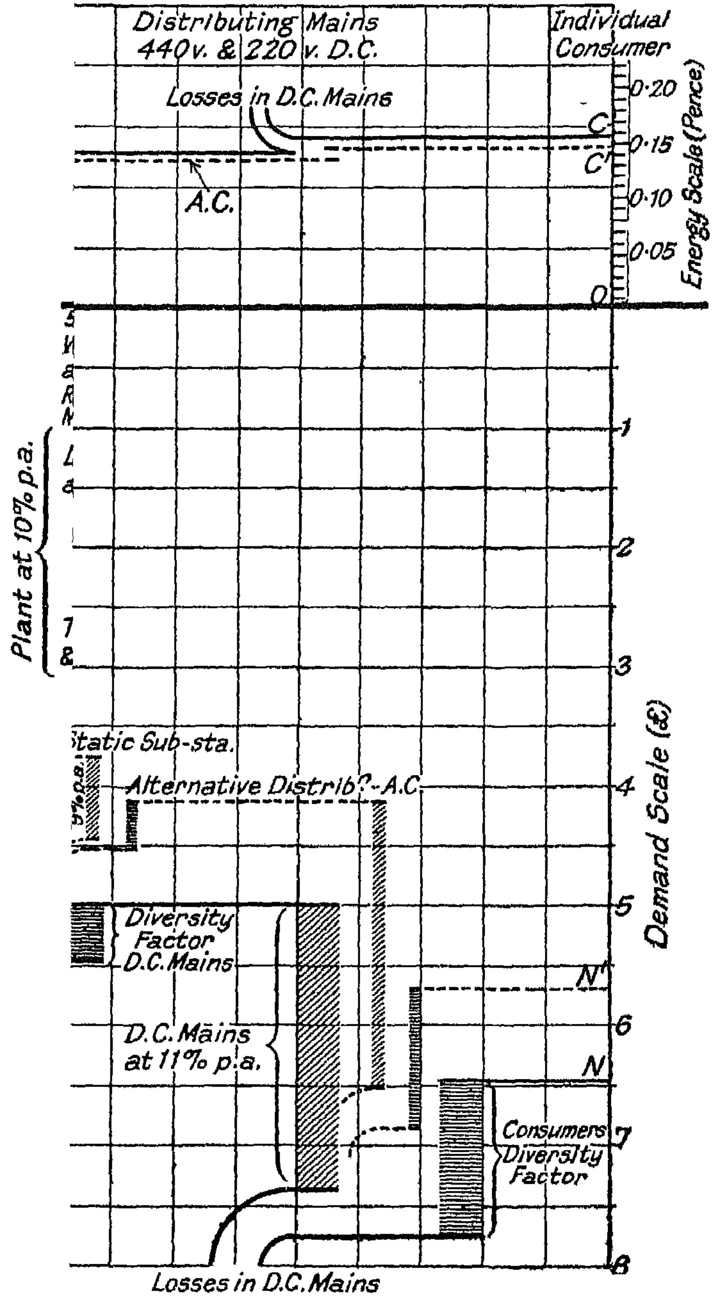
Single Supply System.—The data here employed is taken from a paper by the late S. J. Watson, presented to the Manchester Association of the *Institution of Civil Engineers*, on January 27th, 1926. A single station of 100,000 kW capacity is postulated, and a complete system of generation, main transmission, and distribution is worked out. Using the then cost figures a two-part tariff is indicated for each point in the system, capable of covering all the costs up to that point.

The figures are taken exactly as they stand in Mr. Watson's paper, with the object of showing how a costs formula can be built up step by step and then plotted in a diagram of this kind. They were selected as a good example of a complete, self-contained, independently operating supply system.

MAXIMUM STATION LOAD, 70,000KW. STATION LOAD FACTOR, 40 PER CENT. (THE TOTAL INSTALLED IS 100,000 KW.)

		Fixed		Running	
	Annual Cost (÷ / 1,000).	kW available (— 1,000)	Annual Cost per kW of Demand	Annual Units delivered (— 10^6)	Cost per Units (Pence)
Fuel, etc.—95 per cent. of £134,000 .		—	—	245	0·125
5 per cent. of £134,000 .	7				
Wages, rates, etc. . .	64				
Station first cost (at 10 per cent. per annum) .	150				
	221	70	£3 3 0		
Diversity factor main cables of 1·10 . .		77	2 17 3		Point A.
Cost of main cables and sub-stations (at 9 per cent. per annum) .	40·5				
	261·5				
Losses in main cables and sub-stations (kW reduced 2½ per cent., units 3½ per cent.)* .		75	—	236·4	0·129
Diversity factor 6,600V. cables of 1·05 . .		79	£3 6 2		Point B.
Cost of cables, etc. (at 8 per cent. per annum) .	32				
	293·5				
Losses (1 per cent. reduction in kW and units).		78	£3 15 0	234	0·1307 Point C.
Cost of rotary subs. (at 10 per cent. per annum) .	80				
Running expenses of ditto	24				
	397·5				
Losses (kW reduced 7 per cent., units 11 per cent.)		72·6	—	208	0·147
Diversity factor D.C. mains of 1·10 . .		79·8	£4 19 5		Point D.
Cost of D.C. mains £20 × 79.8 (at 11 per cent per annum) . .	175·6				
	573.1				
Losses (7½ per cent. reduction in kW and units)		73·8	—	192·4	0·159
Diversity factor individual consumers of 1·20 .		88·6	£6 9 2		Point E.

* *See* footnote on p. 74.



[*facing page* 80

As regards generation (which includes transformation up to 33,000 volts), 95 per cent. of the cost of the fuel, oil, etc., rank as running expenses proportional to the energy generated, and are paid for per unit. The remainder of the fuel and all the other generating costs are reckoned as fixed costs to be paid for by an annual charge per kW of demand. Main transmission takes place at the above-mentioned pressure to sub-stations where the supply is transformed down to 6,600 volts for secondary transmission. At this point the supply is considered to be split up into several portions, only one of which is here shown,* namely, that to rotary sub-stations for conversion to three-wire, D.C., 440 volts between outers.

The main figures are summarised in the foregoing table. These are necessarily greatly condensed, being inserted merely as a check on the diagram (Fig. 13), and any details and explanations required should be sought in the paper from which they were taken. In the double columns are shown respectively the kW and units available at each point and the corresponding prices per annual kW and per unit. The lettering of points A to E corresponds with that on Fig. 14.

Commencing at the left-hand side of the diagram (Fig. 13), the following points should be noticed. In the first place the cost of generation and transformation to 33,000 volts is found by adding the total expenses at the power station, these being expressed on an annual basis. For this purpose the whole of the plant and equipment is reckoned at a uniform figure of 10 per cent. per annum on the capital cost, and it will be seen that roughly one-third of the first cost went in land, buildings, etc., one-third in the steam plant, and one-third in the electrical gear. As a figure of 8 per cent. would cover interest at 5 per cent. and repayment in twenty years,† and since at least the first of the three groups of expenditure would not require renewing so soon as this, the allowance of 10 per cent. should provide a reasonable margin for insurance and reserves against any unexpected change in the fortunes of the supply authority or the requirements of the public.

Assuming that the sum of the individual maximum demands on the main cables reaching the central station is 10 per cent. more than the aggregate demand which they combine to make on the station, the generating cost becomes £2 17*s.* 3*d.* per annum per kW of demand plus 0·125*d.* per unit. This is therefore the cost price which could be charged to a consumer buying at the power station terminals, assuming that his cable showed the above diversity factor to the main body of the demand. In the diagram (Fig. 13) the two scales have been

* From here onwards the figures of the original paper are doubled as though the whole distribution were D.C.

† A full discussion of interest and depreciation rates, and the influence thereon of lives and salvage values, is contained in Volume I.

so chosen that, for a load factor of approximately 25 per cent., ordinates on the under side of the diagram represent pence per unit to the "energy" scale. Hence for this load factor the dotted bracket represents to scale the generating cost expressed as an overall price per unit, and this equals 0·44*d.* per kWh.

From here the diagram works to the right and shows the effect in turn of E.H.T. cables, main sub-stations, 6,600-volt cables, rotary sub-stations, and D.C. distributing mains. For each of these items there is a loss both in energy output and in kW capacity to meet demand, so that the price per unit and the annual price per kW are each correspondingly increased. There is also the capital cost of the various distribution gear, reckoned at percentages from 8 to 11 per annum, and this serves to swell the lower part of the diagram.

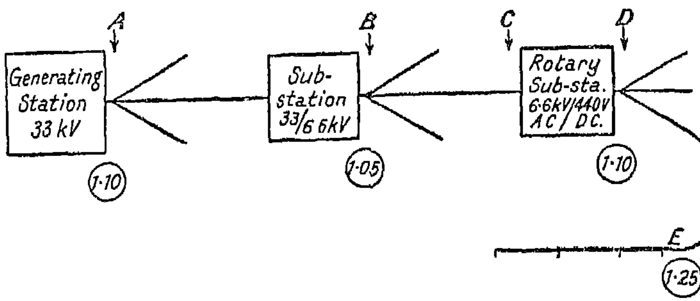


FIG. 14.—*Diversity Factors in Supply System.*

It will be noticed that since the cost per unit or per kW is in the form of a quotient—Total Cost/No. of Units or kW—this may be increased either by an increase of the numerator (greater cost) or a decrease of the denominator (*i.e.*, by losses). The former is shown on the diagram by shaded additions to the cross-sectional area of flow, the latter by curved tributaries flowing into the main stream. (A more exact interpretation would be to regard the total volume of money starting at the right and flowing leftwards, being spent or dissipated in the various ways shown.)

The effect of a diversity factor is to increase the number of kW available to meet demand, owing to that demand not all taking place simultaneously. This increases the denominator of the above quotient, and so reduces the cost per kW. On the diagram this is indicated by horizontal shading, and it takes place wherever a number of cables meet at one point, *i.e.*, at each station or sub-station, and also when a number of individual consumers' lines enter the street mains and distribution pillars.

Fig. 14 elaborates this point and shows diagrammatically the system lay-out, and the diversity which occurs at each meeting point. The

diversity factors are shown in rings, and the points are lettered A to E to correspond with points in the table. The overall diversity from consumer to power station is the product of all the separate factors, and is the same as that assumed in the previous example, namely, 1·6.

The composite price at each stage of the process is shown on Fig. 13 to the two scales marked, and the final price to the D.C. consumer is £6 9*s*. 2*d*. per annum per kW plus 0·159*d*. per kWh. In the original paper the power station is taken as working at a load factor of 40 per cent., but owing to the various diversity factors and to the fact that the constant losses reduce the kWh more than the kW capacity, the average load factor of the individual consumer is reduced to 24·8 per cent. The two scales on the diagram have therefore been chosen so as to correspond at a load factor of approximately 25 per cent., and for a consumer having this load factor the overall price per unit is represented by the total distance *CON* to the " energy " scale and equals 0·87*d*. per unit. For any other load factor the lower portion of the diagram must be multiplied by the ratio 0·25/new L.F. before adding it graphically to the upper portion, in order to find the total overall price per unit.

Conclusions from Diagram.—Some of the points which the diagram particularly illustrates may be briefly mentioned. The cardinal feature is, of course, the high proportion of the fixed charges, and the influence thereon of the load factor. This is particularly true of the second case, based on pre-war coal prices, in which the enormous preponderance of the lower portion of the diagram is very striking. Another point to notice, in the same diagram particularly, is the way in which the transmission and the distribution costs mount up in a steadily increasing ratio as the supply proceeds. The earlier, higher voltage stages not only show much smaller energy losses (as would be expected), but even as regards capital charges they appear far less serious than the later items, and it is abundantly evident that low tension distribution is an expensive business, and was so still more when D.C. was employed. The economy resulting from A.C. distribution lies particularly in the sub-stations, and in the paper from which these figures were obtained an alternative is worked out for static stations delivering to consumers at 400 V and 230 V A.C. This is lightly sketched on the right-hand portion of the diagram (with dotted lines), and it results in a final price of £5 13*s*. 11*d*. per annum per kW of demand plus 0·150*d*. per kWh (*C′ON′*).

A further use for the diagram is in showing the effect of changes in the price of the different " ingredients." The price changes which are likely to produce the biggest effects are changes in the cost of fuel, in the cost of electrical plant, and in the cost of capital. Rates and management are also considerable and are growing larger, but wages are only a small and lessening item.

As regards fuel, a change in its price will produce an exactly proportional change in the whole of the upper portion of the diagram. Changes in the cost of electrical plant and machinery could be shown by a proportional change in the capital items of the lower part of the diagram. The cost of capital, as evidenced by the rate of interest which has to be paid, will also affect the capital items. In this case, however, the change would not be a proportional one, since interest and depreciation would be affected in opposite directions.

It will be noted in conclusion that no mention has been made of power factor, a complication which is avoided in the D.C. distribution illustrated in the second diagram. The economic effects of a bad power factor are in many respects similar to those of a bad load factor, and are dealt with in detail in a later chapter.

"Profits" Method.—The system of costing employed above may be regarded as the normal straightforward method, and the one most suitable for obtaining a general view of the whole field of supply costs. There is, however, an alternative method which must be mentioned because it is based on a somewhat different view-point.

In the above analysis, capital charges are regarded as costs, and the undertaking is visualised as hiring all the necessary capital at a fixed interest rate, and amortizing or repaying the loan over a given period (usually corresponding to the life or the depreciation rate of the plant employed). This is the natural view-point of a non-profit-making body such as a local authority or public board whose only object is to supply electricity to its constituents at the lowest possible cost. The position is somewhat different when the undertaking is a company whose object (or one of its prime objects) is to earn as large a rate as possible on its ordinary capital. In such a case, the decision of what to quote a special consumer may be based on the rate of return to be anticipated from the special expenditure involved.

The method of costing which follows from what may be called the "company" view-point is as follows. (This treatment concerns only the fixed costs: the running costs are dealt with exactly as before.) The outgoings which occur necessarily in an annual form, such as rents, rates and management, are classed as overheads and expressed as an annual charge (per kW of demand) in the ordinary way. But the expenditures on plant purchase are not "annualised" in terms of their interest and depreciation rates, but are expressed as capital sums (per kW of demand).

The fixed costs for a consumer then consist of two portions—an annual overhead charge and a capital expenditure. This does not therefore immediately give the appropriate fixed charge or tariff for the consumer, and such could only be derived by assuming interest rates. Alternatively, if a tariff value is assumed, it is possible to construct an equation consisting of income minus overheads (excluding

capital charges) divided by capital expenditure, thus finding the annual return on this expenditure.*

Although there may be advantages in this method when contemplating special expenditures, for routine costing it is undoubtedly preferable to regard capital requirements as a fixed cost rather than as a basis for fluctuating profits. The annual cost of owning a generator or cable is then no different in kind from the rent of a building (which would not be altered in character even if the building were to be bought outright). The usual plan is also much simpler, since all fixed costs are then of the same kind and can be added together.

Bulk Supply Tariff.—Under the 1947 Electricity Act the Central Authority is authorised to generate or acquire electricity and to provide bulk supplies for distribution by the Area Boards. The charge for these bulk supplies is in accordance with the tariff fixed by the Central Authority from time to time and need not be the same for each Area. These tariffs must be framed to show the methods by which, and the principles on which, the charges are made as well as the prices to be charged, and they must be published. The following are the details of the interim tariff for supplies to the Area Boards.

Standing Charge.—£3 10*s.* 0*d.* per annum per kW of maximum demand (increased to £3 15*s.* 0*d.* on April 1, 1950). In respect of each year of account the figure taken is the average of the largest half-hour demands in that year and the previous year. Normally, the maximum demand will occur between 7 a.m. and 7 p.m. on working-days (or 7 a.m. to 12 noon on Saturdays), but should there be a night-time demand in excess of this the charge is based on the day-time maximum plus one-third of this excess. The demand is separately metered at each point of supply and the charge is made on the aggregate of these M.D.s, but in future it is proposed to base the charge on the simultaneous M.D. of the Area in the year of account.

Running Charge.—0·335*d.* per kWh ± 0·0007*d.* per kWh for each 1*d.* by which the mean price of the coal consumed at all stations in the area exceeded or fell below 38*s.* per ton. Where the coal has a calorific value above or below 11,000 B.Th.U. per lb. a *pro rata* adjustment is made.

The above tariff applies to all areas but the actual running charge payable varies considerably owing to the wide differences in mean coal

* A good example of such costing is contained in a paper by Messrs. Woodward and Carne, *Journal I.E.E.*, 1932, 71, p. 872.

prices in the different areas. In the period 1948/49 there was a difference in the running charge of about 0·2*d*. per kWh as between the cheapest and the dearest coal areas. Moreover, whilst the above interim tariff framework was intended to operate unchanged for several years its actual incidence during the first two years from Vesting Date was modified in each area so as to taper off the differences between the pre-vesting and the final figures. It will be noted that the tariff does not vary with power factor nor with changes in local rates.

CHAPTER V

DIVERSITY

Load Factor and Diversity Factor.—The figures of the previous chapter all go to show the extreme seriousness of load factor in electricity supply. No other element appears to have so much effect on the cost of electricity ; and if coal were as free as air or could be mined and transported for absolutely nothing, this would have less effect on the price than would be produced by a comparatively small increase in the load factor. Yet as regards the individual consumer it is questionable whether very much can be done, or much improvement can be hoped for. Change circuits, storage heaters and other such devices are helpful so far as they go, but in the main the consumer wants his electricity *when* he wants it, and cannot be bothered to take it at such times as suit the supply undertaking.

Fortunately there is another element, namely diversity, which operates in exactly the opposite direction. Diversity factor is the silver lining to the dark cloud of load factor. It is the magic by which the consumer can have a low load factor and the supply undertaking a high load factor at one and the same time. This it is which enables a tradesman with a small stock to serve a large and fluctuating body of clients ; and, by the same means, the supply engineer is able to give each consumer what he wants without ruining himself.

It is perhaps well to realise that electricity undertakers are not the only people with big overheads and a service which cannot be stored. The cost of many things, such as hotels and taxi-cabs, lies far more in the fixed charge than in the running expenses, and only a very small part of the taxi fare goes to pay for the petrol. But even in these cases the position is far less bad than it would be if there were no diversity. The average middle-class person probably only takes a taxi half a dozen times a year, and his individual load factor is of the order 0·05 per cent. Yet the load factor on the taxi itself (although still low enough to account for much of the cost) is many times higher than this owing to the diversity with other users. The taxi service is unique in just the same way as the electricity service or the telephone service —no two people can use the same taxi or the identical kilowatt at the same time. But this does not prevent the same apparatus serving a number of different people consecutively.

Of the two ways of improving system load factor, namely, by raising the individual load factor and by increasing the diversity, there can be little doubt that the second holds out far greater possibilities.

In the case of the power load there is usually little chance of storage or load levelling, and most factory managers have no off-time uses to fall back upon. The same thing is true of the traction load and of the street-lighting load, each of which may have an annual load factor by itself of not more than 30 to 40 per cent. But put the three together and it will be seen how the traction load makes up for the power load in the early mornings, evenings and lunch times, whilst the street lighting carries on the tale throughout the night. Were it possible to mix these three in appropriate magnitudes a station load factor of 60 per cent. might be achieved.

In the case of the domestic load, the same principle operates. For some purposes, heat storage apparatus can be used to bring up the individual's load factor. But for many operations, *e.g.*, lighting, grilling, cleaning, etc., this is impossible, and for the most part one must rely on the diversity between consumers to bring up the load factor on the station and the mains. It is better also from the psychological point of view, since it does not leave the consumer with a suspicion that he is being restricted in some way, or that his consumption is being modified to please the supply undertaking.

The advantage of a cooker, a cleaner or a water heater over a lamp (regarded in each case merely as a piece of consuming apparatus) is not that the former will necessarily have better load factors than the latter, but that they have better diversity characteristics. A single vacuum cleaner will have no better load factor than a single lamp —probably a good deal worse. But 1,000 vacuum cleaners will give a very useful load factor at the sub-station whereas 1,000 lamps may be very little better than one.

The case just cited is an extreme one, so that the explanation is obvious. Lighting is associated with particular hours of the day whereas vacuum cleaning is a purely spasmodic load and may occur at any moment during sixteen hours out of the twenty-four. If there are 1,000 cleaners in a given neighbourhood, each used for an average of one hour a day, there will only be a probability of, say, 200 being used at any one time, whilst the chances of them all being in circuit simultaneously are exceedingly remote. But the same thing is true, to a greater or less extent, of any irregular consumption of electricity, the hours of which depend on the diverse personal habits of the user.

Types of Diversity.—In the last chapter a definition of diversity was given, and its general effect on supply costs was considered. In the present chapter it is proposed to analyse this in some detail, *i.e.*, to split diversity up into its constituent parts. This will show how the supply costs already worked out on a two-part basis must be modified before they can be put before the consumer as a correctly representative tariff.

The total diversity that exists between any final consumer and the

load on the power station can be regarded as the composite result or product of a large number of separate diversities. Each time that two or more connections are taken from one point a ratio of demands can occur, and a fresh diversity can thus arise. But in order not to complicate matters unduly it will be sufficient to distinguish between two main sorts of diversity.

On the one hand there is the obvious diversity which exists between different *kinds* of load. A case has already been cited to show how at certain times of year a power, a traction and a street-lighting load may contrive to complement one another and build up a comparatively smooth curve, and a very much better load factor than any of them had individually. This inter-group diversity is the type occurring at points *Y* and *Z* in Fig. 17. It is dealt with more fully in the example below.

On the other hand, there is the diversity between individual consumers of the same kind—or, to put the matter less personally, between installations of identical apparatus (1,000 cookers, 1,000 vacuum cleaners, and the like). This latter diversity clearly depends on the spasmodic nature or otherwise of the particular consumption, *i.e.*, it depends on the extent to which the consumption is associated with particular hours, and it also has a reciprocal relationship with the individual's load factor. This, which may be called mass or group diversity, is the type occurring at *X* in Fig. 17, assuming for simplicity that a_1, a_2, etc., represent single pieces of apparatus such as cookers or water heaters. It will be referred to in detail later in the chapter. (*N.B.* Most installations comprise several pieces of apparatus, and both sorts of diversity would then be found at *X*.)

Diversity between Groups.—As regards the diversity between different kinds of loads, Fig. 15 has been prepared in order to clarify the matter. This shows the load curves for a hypothetical station serving a well-diversified urban area. They refer to the twenty-four hours during which the year's maximum demand was recorded, this occurring between 5.30 and 6 p.m. on a day towards the end of December. The sum total load curve *S* is analysed into five components, namely, Domestic, *D* ; Industrial (factory power), *I* ; Commercial (office and shop lighting and heating), *C* ; Traction, *T* ; and Street Lighting, *L*. The only serious omission in this analysis is that the domestic load covers two sorts of consumption (lighting and heating) with quite different characteristics.

It should be stated at the outset that this figure is in the nature of a composite portrait. Each curve is an actual load diagram, but not all from the same undertaking : moreover, whilst preserving the shapes, the magnitudes have been adjusted so as to represent reasonable proportions for an authorised undertaking in this country having a good traction and public lighting load.

The salient points, in so far as they concern diversity, have been summarised in the table below. Column (*a*) gives the maximum height of each curve, *i.e.*, the biggest load occurring on the day of station peak. Column (*b*) gives the height at the time of day when the peak occurs. The sum of these figures in column (*b*) gives the peak on the station, and this is less than the sum of column (*a*). Column (*c*) gives the maximum height of each type of curve or group of consumption throughout the year. Data were not available for this, and

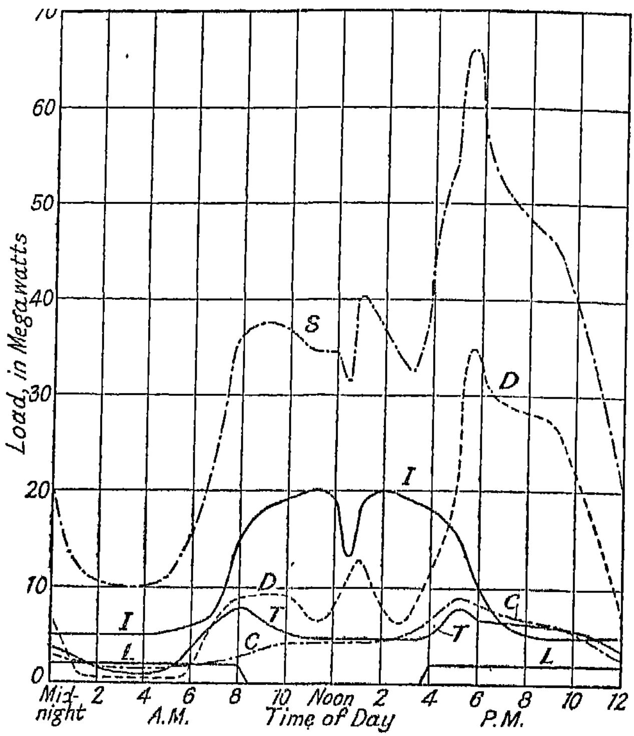


FIG. 15.—Diversity between Groups.

it might be thought that it should be no higher than column (*a*). But even though all the loads tend to be high at this time of year, there is no reason to suppose they all reach their absolute maximum on the same day. Nor would this necessarily be the peak day, since the latter arises owing to the fortuitous overlapping of the high portions of several curves, and is governed by their times rather than their maxima. Column (*c*) has therefore been taken as 20 per cent. greater than column (*a*).; this increase will cover the daily fluctuations which cause the maximum to be higher than the mean (even of the cold and dark days).

Group.	(a) Maximum Load on Day of Station Peak MW	(b) Load at Time of Station Peak. MW	(c) Maximum Load throughout Year [(a) + 20 per cent.]. MW	(d) Diversity Factor [(c)/(b)].	(e) Load Factor per cent. [Annual mean load ÷ (c)].
Domestic .	34·5	34·5	41·4	1·20	19·4
Industrial .	20	13	24	1·84	41·4
Commercial .	9·2	8·9	11·0	1·24	29·9
Traction .	7·7	7·3	9·3	1·26	45·2
Street lighting	2	2	2·4	1·20	39·6
Sum .	73·4	65·7	88·1	1·34	40·1

The diversity factor of each group, column (*d*), is found from the quotient of (*c*) divided by (*b*) since it is the group's peak divided by the group's contribution to the station peak. The yearly load factors, column (*e*), are easily found because the annual consumption of each group is known, and hence the mean load throughout the year. This, divided by the maximum load from column (*c*), gives the load factor of the group.† The station load factor is similarly calculated from its total units, the divisor in this case being the station peak (*i.e.*, sum of column (*b*) not (*c*)). It will be noted that the load factors of the group are, in general, lower than the load factor on the station owing to the operation of the diversity factors just calculated. In fact, if a correct average of these separate load factors were made it would come to 40·1/1·34 = 30 per cent. Thus the station acquires a load factor of 40 per cent. although the constituent loads average only 30 per cent. If each group were separately metered and charged for on a two-part basis, the average price per kW of group demand could therefore be made only three-quarters of the price per kW of station demand.

Later Examples.—The example given above was a build-up of a hypothetical supply system using component load curves suitably proportioned. The curves were selected somewhat at random, and their shapes have no great significance in relation to present-day loads. The whole purpose of the exercise was to illustrate the meaning of diversity. The studies which are referred to below represent much more serious attempts to arrive at the quantitative results. They should therefore be referred to for guidance as to the actual shapes likely to be followed by the component load curves, and the values obtained are more likely to be representative of present conditions.

A build-up on the lines of the previous example, but for post-war conditions, was made by P. Schiller and F. H. Dennis.* The purpose

* *Electrical Research Association Technical Report*, K/T 126.

† By dividing by (*b*) instead of (*c*) the "*effective* load factor" of the group on the station is obtained.

was to establish the load factors rather than the diversity factors of the four components making up the total of the hypothetical undertaking, and these are given in the table below. The overall diversity factor, namely, the sum of the individual M.D.s of the component loads divided by the undertaking M.D. was also estimated, its value being 1·1. In general, the shape of the component curves was nearer to that of the combined curve (and hence the diversities were lower) than in the previous example.

	M.D.	Load Factor.
Domestic . .	11·4 MW	30 per cent.
Commercial .	3·3 ,,	25 ,, ,,
Industrial . .	10·1 ,,	35 ,, ,,
Traction . .	1·6 ,,	30 ,, ,,
Aggregate . .	26.4 ,,	
Overall . .	24 ,,	35 ,, ,,

The third example represents not a build-up into a hypothetical whole but a break-down of an actual whole into its estimated components. In the Report of the Committee to Study the Electricity Peak-Load Problem in Relation to Non-Industrial Consumers† (The 1948 "Clow" Committee) the load on the grid during peak conditions in December, 1946, is split into five consumer groups, and these are split again into types of consumption in each group.

Naturally, any such break-down can only be an estimate, and the figures given below (which are scaled off the curves) are quoted in order to illustrate the diversity.

LOAD ESTIMATES
(Analysis of Peak Day Demands).

Consumer Groups.	Load in MW at 8 37 a.m. (time of system M.D.):				Group M.D.:		Diversity Factor.
	Lighting.	Heating and Cooking.	Power.	Total.	Time.	MW	
Domestic	550	2,260	—	2,810	17.20	4,030	1·43
Commercial	1,390	780	340	2,510	10.00	3,430	1·36
Industrial	920	600	2,890	4,410	8.28	4,520	1·02
Traction	—	—	360	360	8.37	360	1·00
Public Lighting	80	—		80	15.00 to 23.00	120	1·50
Total	2,940	3,640	3,590	10,170	8.34	12,460	1·22

† Cmd. 7464: H.M. Stationery Office.

In comparing these various estimates and analyses certain differences of date must be allowed for. In the earlier set (p. 91) the domestic curve refers to what would now be regarded as an under-developed area, in which lighting heavily predominates. Not only has the domestic load a late-afternoon rather than an early-morning peak, but *this is so pronounced as to dominate the whole system and impose* an afternoon peak thereon. Result : Low domestic-group diversity and high diversities for the groups with morning peaks. In the third set of figures the domestic load still has its peak in the late afternoon, but owing to greater heating-load development this is not nearly so pronounced and is insufficient to prevent the system peak from occurring in the early morning. In the second set of figures the domestic load is even more fully developed, with a morning peak very close to the system peak and hence a much lower diversity.

These differences in peak-time make any numerical comparison of diversity factors between the three sets very difficult, and incidentally they show how erratic such figures become when the system peak-time *is unstable. The present use of these figures is merely to illustrate the* meaning of inter-group diversity and how it may be estimated. For example, the load-curve for domestic and farm consumers reached its maximum of 4,030 MW at 17.20 (*i.e.*, 5.20 p.m.) whilst its height at system peak-time (8.37 a.m.) was only 2,260 MW. The diversity factor is therefore given as $4{,}030 \div 2{,}260 = 1{\cdot}43$. Had the domestic load been supplied from a separate generation and transmission system the plant required to meet the domestic demand would have been 1·43 times as great as was actually monopolised. (One reason why the third set gives higher diversity figures is that it was obtained by scaling off values from smooth curves and is therefore based on instantaneous-demand readings instead of the usual half-hour integrations.)

Group Diversity.—So far, the analysis has been into groups : the next step is to divide up a single group into its constituent elements. If one of the group curves of Fig. 15, say the industrial *I*, is split up into the separate curves of all the different power consumers the result would be something like that shown in a simplified form in Fig. 16. That is to say, the individual curves would not be miniature duplicates of the group curve but would be unique irregular lines giving a smooth curve only by addition. Particularly if the consumers were small ones, the graphs would show great irregularity (and therefore poor load factors), whilst the various maxima would occur at all different times of the working day. The separate M.D.'s of such consumers would add up to *very considerably more than the M.D. of the group*—*i.e.*, there will be a further diversity allowance to be made.

The diversity which exists between a number of individual consumers of the same kind has been called the mass or group diversity. In the case shown there are eight consumers and their separate maxima

total 30 megawatts whilst the group curve has a maximum of 20 MW. In an actual case there might be some hundreds or thousands of consumers, and the diversity ratio would be even greater. This diversity of a single consumer within the group bears a very close relationship to the load factor of the consumer. What this relationship is will appear from the discussion which follows. But before embarking on this, it will be well to attempt a more precise definition of the diversity of a single line.

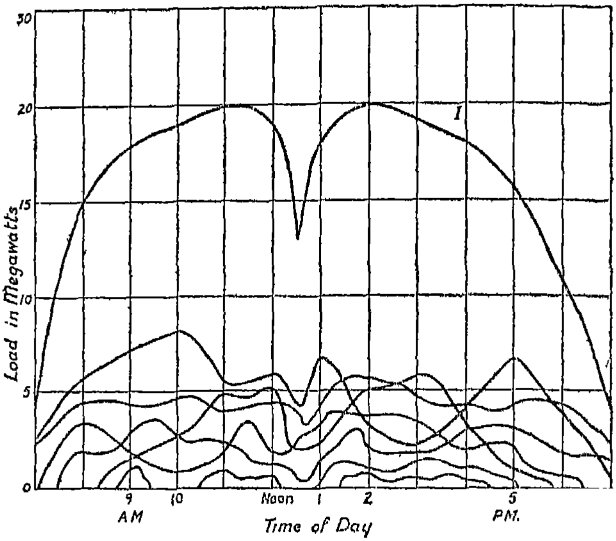


FIG. 16.—Group Diversity (simplified).

Diversity of Particular Line: Numerical Statement.—It is now possible to attach a more precise meaning to the concept of diversity, and even to assign a numerical value to the individual consumer or line. In the above paragraphs, the diversity factor has been treated as an overall figure referring to all the lines in general. Thus in the case of the inter-group diversity at the point Y (Fig. 17) where three lines, b_1, b_2, b_3, join to make one line c, if the aggregate M.D. on the right amounts to one-and-a-quarter times the M.D. on the left, the diversity at this point will be 1·25. But this does not tell us the particular diversity of any one of the b lines. Such a thing only becomes possible when there are a number of similar connections, or when time-of-day is brought into consideration.

The same thing is true of group diversity, as at the point X where a number of lines or individual consumers, a_1, a_2, etc., join to form a common line b_1. By definition, there is a generalised diversity of the

a lines relative to b_1 equal to the sum of the maximum demands at a divided by the maximum demand at b. In a similar way there is a diversity of a relative to any point c further back in the system, which can be defined in the same way. But this does not tell us the individual diversity of any one of the a lines. If, however, all the a lines have the same load factor (*i.e.*, the same ratio of units to M.D.), it will be fair to say that each a line has a diversity relative to b of the value defined above. For in this case there will be a certain fixed ratio between the load factor of any a line and that of b, and the diversity factor of a can be defined as the reciprocal of this ratio.

To simplify matters, suppose that the a lines consist of a number of water heaters each with a load factor of 15 per cent. and that the load factor of b is 30 per cent. Since the units in b equal the sum of the units in a this must mean that the M.D. in b is only half the sum

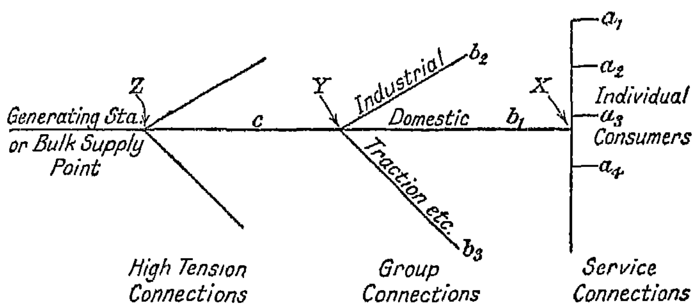


FIG. 17.—Skeleton Supply System.

of the M.D.'s in a. Hence there is a diversity factor between a and b of 2, and this can be credited to each of the a lines. But if a_1, a_2, etc., differ in load factor, it is impossible on the above definition to know with what diversity to credit any one of them.

The individual diversity of a single connection can best be defined in the following way. The diversity of any one line a_1 relative to b is the reciprocal of the probability of a_1 being in circuit at the time of M.D. in b. For this purpose, it is necessary to suppose that a_1 itself represents a number of precisely similar (but not identical) loads, *i.e.*, loads having the same magnitude of units and of M.D.'s but the latter not necessarily synchronising. Three different statements will then define and illustrate the diversity of a relative to b. It is the number of 1 kW loads of the type a necessary to raise the M.D. of b by 1 kW; it is the sum of the M.D.'s in a divided by that portion of the M.D. in b which is due to a: it is the sum of the kW demands in a divided by the kW in a *at the time of the* M.D. *in* b.

The above statements are worded as though the M.D. were measured at a particular instant instead of being spread over fifteen-,

twenty- or thirty-minute periods, but precisely the same form of definition will serve for the usual methods of metering. Taking half-hour periods, the last definition can be written as follows : *The diversity of an* a *line relative to* b *is the sum of the maximum half-hour recordings (kWh) in* a *divided by the number of kWh taken by* a *during the half-hour when* b *is a maximum.* This definition has the very great advantage that it does not require any measurements in *b*, merely a knowledge of the time of day that *b* records its maximum. It can be used to define the diversity of any single line relative to any other point in the system. It represents actual costs of supply, since it tells precisely how much a particular connection raises the maximum load on a feeder or the fixed cost of the bulk supply. But when applied to an individual consumer rather than a composite line it needs a certain amount of imagination to create in the mind a group of consumers of the type considered, and then to visualise their load characteristics.

After-Diversity Maximum Demand.—When there are a number of connections of the same kind it is convenient to know the average effective demand on the system due to each. This is called the after-diversity maximum demand (A.D.M.D.), and it equals the group M.D. divided by the number of connections making up the group. Thus, if a group of 100 cookers produce a group M.D. of 150 kW the A.D.M.D. is 150/100 = 1·5 kW *per cooker*. (Note that this is not strictly the effective demand of any one particular cooker but the average of them all The term is meaningless except in relation to a large group of similar connections to one system.)*

The convenience of the A.D.M.D. conception is that the figure can be attached to a single apparatus or installation (when there are a number) and used as though this were its actual demand. The effect of diversity has then been allowed for, and (with some reservations) can thereafter be neglected. For example, in the table below, the A.D.M.D. of the heaters in group A is 50·95/66 = 0·77 kW and of those in group B it is 68·88/60 = 1·15 kW. These figures, however, would mean little unless the groups were uniform.

Another advantage of the A.D.M.D. conception is that it will often substitute one guess for two, and a safer guess at that. Thus, to estimate the effective load due to 100 water heaters one might proceed to estimate their individual demands and then to estimate their diversity, whereas a single estimate of A.D.M.D. will give the answer directly and can be made with greater certainty.

The reason for this greater reliability is that, with a given energy consumption, a high individual demand (*i.e.*, a poor individual load factor) is likely to be associated with a high diversity. These two

* In the same way, the average demand per connection (of a group of connections) at *any* time is called the after-diversity demand, A.D.D. When this time is the time of M.D. of the group, the A.D.D. becomes the A.D.M.D.

individual variations tend to cancel out, giving a relatively uniform figure for the resultant A.D.M.D. Thus a batch of water heaters might be fitted with 2-kW or, alternatively, with 3-kW elements. For a given hot-water service, the latter will result in little or no greater effective load because the 3-kW heaters would be in circuit a shorter time, with less chance of overlapping each other (*e.g.*, more diversity). Unless the diversity estimate is made with a strict regard for the loading (*vis-à-vis* the usage), more reliable results will be obtained by estimating the A.D.M.D. (This inverse characteristic is elaborated in subsequent sections.)

Diversity and Numbers.—Enough has been said to show that diversity factor (at least when it refers to an individual consumer) is a sort of average, and must therefore imply a number of connections —and, moreover, similar connections. It is meaningless unless there are several of a kind. When one says that a load *a* has a diversity of 2 relative to *b*, this means that a *number* of loads of the kind *a* would only produce a demand on *b* of half their aggregate demands.

This fact stands out most clearly if the instantaneous conception of demand is employed, and especially in connection with apparatus such as heaters, which are on full load all the time they are connected. A single piece of apparatus of such kind cannot really be said to have any diversity at all. It cuts in at certain times and cuts out at other times. Either it is in circuit at the moment of maximum demand, or it is not. In the one case its diversity factor is merely unity, in the other case it is infinity: it cannot have any value in between.

If there were actually only one such load, a diversity factor could therefore only be said to arise as a result of ignorance. It merely expresses the *chances* of the load occurring or not occurring at the time of M.D., and once the fact is know the diversity disappears (neglecting, that is, the possibility of a partial overlap). It follows that in speaking of the diversity of a single piece of apparatus it is essential to imagine others of its kind. A diversity factor of 2 means that there is exactly a half-and-half chance of its being in circuit at the time of M.D. —in other words, that if there are 100 of them, 50 will be in circuit at this time. A single apparatus on a circuit is like a single voter in an election. To realise his importance he must be regarded as typical.

The theory of diversity is therefore essentially a theory of probability, and requires numbers for its operation. To find the effect of *a* upon *b*, we must suppose a considerable number of *a*'s. If we toss a penny once, it must come down either head first or tail first—it cannot be half-way between. The correct proportion of heads and tails can only arise when a number of tosses are made.

This dependence of diversity upon numbers can best be seen in the case of group diversity. Unfortunately diversity is a quantity which is very rarely measured in practice, and owing to the number of

variables a great many readings are necessary in order to establish satisfactory results. Thus, for group diversity not less than 50 to 100 similar appliances should be connected to a single line, and their individual and collective maximum demands measured over a considerable period. One of the first substantial tests of this kind to be

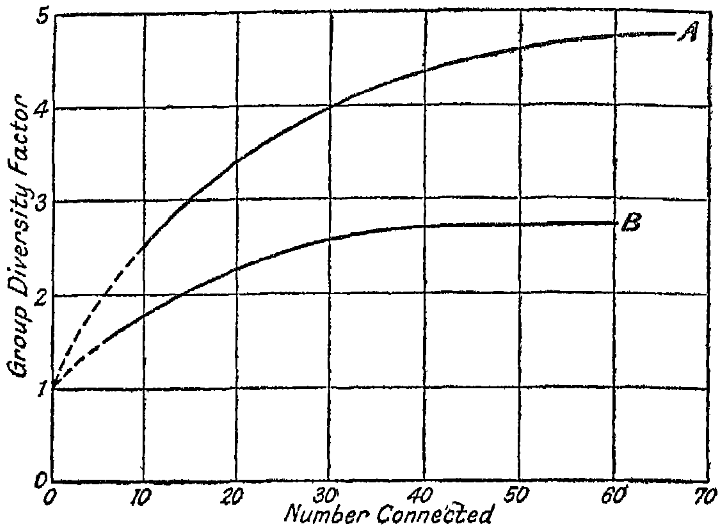


FIG. 18.—Group Diversity

Group.	Number of Heaters.	Mean Individual Maximum Demand.	Group Maximum Demand.	Diversity Factor.	After-Diversity Maximum Demand	Mean Weekly Consumption.	Load Factor (per cent.).	
							Group.	Indiv.
A	66	3·69 kW	50·95 kW	$\frac{3·69 \times 66}{50·95} = 4·78$	0·77 kW	34·6 kWh	26·7	5·68
B	60	3·02 kW	68·88 kW	$\frac{3·02 \times 60}{68·88} = 2·72$	1·15 kW	50·7 kWh	27·2	10·0

recorded was the survey of water heaters carried out by the American National Electric Light Association.* Groups of 50 or more similar heaters were tested over a normal domestic cycle with the following results.

The group diversity factor rose with the number connected, being of course unity with one heater, and reached a steady maximum value

* *N.E.L.A. Proceedings*, Vol. 89. New York, 1932.

by the time some fifty to sixty were connected. Greater numbers than this were not found to produce any appreciable further change. The diversity factor (with fifty or more connected) reached a figure of between 2 and 5, depending on the load factor, and two typical curves are reproduced in Fig. 18 (data below the curves).

It will be noted that, although the diversity rises rapidly with the first few connections, a considerable number is needed to produce the maximum effect. Even when ten are connected the curve is little more than half-way up to its asymptote. Another point of note is that a reciprocal connection is clearly evidenced between individual load factor and diversity factor. The individuals in group *B* have nearly twice the load factor of those in group *A* (10 in place of 5·6) and their diversity factor is somewhat over half (2·7 instead of 4·8). (It will be realised that in the case of group diversity one is dealing with the probability of a number of similar loads being on together, not with the probability of a certain load being on during a particular half-hour—the metering half-hour of the system.)

Similar tests were carried out by the Electrical Research Association in connection with their "Supply Technology" investigations described in a later chapter. The tests in this case were made on dwellings, not on single pieces of apparatus, but the dwellings were uniformly equipped and the phenomenon tested was the same, namely, the drop in the demand per connection (or the increase in diversity) as the number of connections increased. The results were not published, but the curve below is reproduced by the courtesy of the Association.

Three sets of tests were carried out, similar results being obtained in each. The curve (Fig. 19) refers to tests on an estate of 87 prefabricated bungalows each equipped with electric cooker, wash-boiler and immersion heater. The total installed load per dwelling was about 14 kW. Thanks to the foresight of the local management, the sub-station supplying the estate contained facilities for measuring the load of various numerical combinations of groups of consumers, and by temporarily installing additional instruments it was possible to obtain measurements on one group of 87 dwellings, three groups of 29, and varying numbers of groups of 10, 9, 8, 7, 6 and 5 dwellings. Ten instruments were also installed in individual dwellings selected at random.

The results were analysed for one week with adjoining Sundays, and the full curves in Fig. 19 show the largest values of the A.D.M.D. during the working week, with a separate curve for Sundays. (These curves plot the reciprocal of the quantity plotted in the previous figure.) With single consumers there is of course no inter-consumer diversity, and the maximum demand is the same before diversity and after diversity, namely, between 7 and 8 kW. The A.D.M.D. drops rapidly with increase of numbers of consumers up to about ten, and less rapidly after that, becoming almost level after fifty. From the shape of the

curves it would appear that with an infinite number of consumers the A.D.M.D. per consumer would be about 1 kW on weekdays and 2 kW on Sundays.

The measured points, illustrated by crosses on the graph, represent not merely the maximum recorded (during the week or the Sundays) for a group but the largest of the maxima recorded for any of the groups. For comparison purposes the *average* (over the groups and over the week) of the daily maxima is shown, for the working days, in a dotted curve. (The number of values averaged is shown against each point. For example, there were ten individual consumers

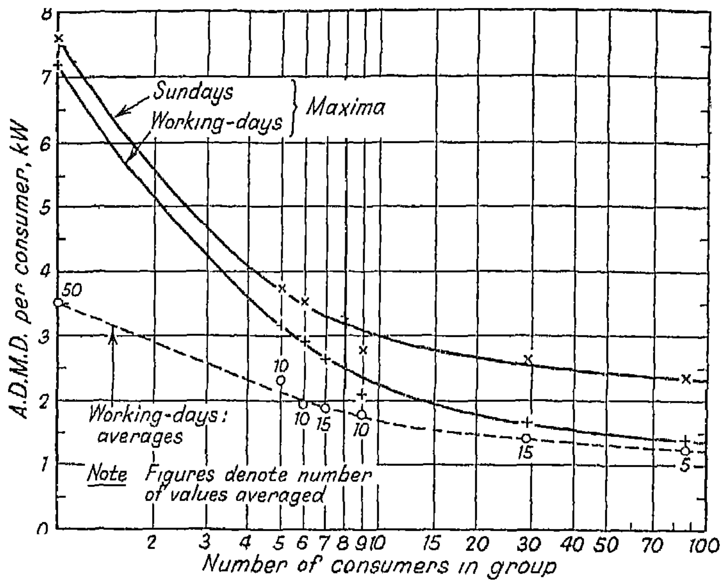


FIG. 19.—After-Diversity Demand and Number of Consumers.

metered, so that over the five working days of the week there were 50 maxima to be averaged in order to give the first point on the dotted curve.) Naturally, the demand value for the single consumer is much lower than in the full curves, since the averaging implied in the A.D.M.D. conception is already partly in operation. With larger numbers the dotted and full curves tend to coalesce.

From the averaged results the inter-group diversity factor, defined as the sum of the individual M.D.'s divided by the collective M.D. of the group, can be found for any size of group. It equals the height of the dotted curve at the left-hand end, divided by its height on the right. Thus, for the full group of 87 the working-day diversity factor $= 3{\cdot}5 \div 1{\cdot}2 = 2{\cdot}9$.

Time Differentiation.—In order to study the operation of diversity with a single consumer or apparatus it is best to start with a purely theoretical "ideal" case. The probable divergence of actual cases from the ideal can then be estimated.

The first step in the argument is to postulate a consumption (spread over a period) which is entirely without any specific time characteristics during the period. This means that it is just as likely to be on at any one minute as at any other. For example, a motor in a shop employing individual-drive tools might be subject to use at any time during the factory working hours—say, 8 to 12 and 1 to 5. During these hours, it shows no partiality whatsoever for any particular time. It may be objected that such characteristics never occur in practice, since if they did, the effect of several consumptions of this kind would be to cause the factory demand to be absolutely uniform—a very rare occurrence. But this objection overlooks the need for numbers, dealt with above. In order to obtain any approach to uniformity of load, there would have to be at least twenty, and preferably fifty or a hundred of the same kind.

Other illustrations could occur in domestic practice. It might be found, for example, that an iron or a vacuum cleaner was liable to be used at any time from 8 a.m. to 8 p.m., and that, taking some hundreds of households, there was no tendency for it to be used at any one time more than at any other. A thermostatically-controlled space-heating load might also, if climatical fluctuations were ignored, be regarded as another possible example ; and if there were a sufficient number of them, the precise time of cut-in and cut-out of any one of them might be regarded as a matter of pure chance. Even a water-heating load, although inevitably biased towards certain times of day by the habits of the household, may approximate to the required characteristics when sufficient households are included.

The above may be illustrated by means of a hypothetical case. For simplicity it will be best to take some appliance whose loading does not vary during use—it may be some domestic apparatus or a works motor giving a constant torque. It will be supposed that, as between one house and another or one factory and another, the appliance is liable to be used at any time from 8 a.m. to 8 p.m. The true load factor (*i.e.*, over the twenty-four hours) is 20 per cent., so that the load factor during the use period is 40 per cent. It is further supposed that the use of the appliance shows absolutely no time differentiation, *i.e.*, that it is used indiscriminately at all times during the period.

In the first place let it be supposed that there are five such appliances of 1-kW loading, each used for 24 minutes and then disused for 36 minutes every hour. These uses are perfectly regular and perfectly spaced as between the different appliances. This case is illustrated in Fig. 20, where the first appliance starts at 8.00, 9.00, 10.00, etc.,

the second at 8.12, 9.12, 10.12, etc., the third at 8.24, 9.24, and so on. The result is of course that at all times two and only two appliances will be on at once. The maximum demand on the system (represented by the overlap on the shaded belt) will be 2 kW, since this is the constant aggregate load throughout the period. Another way of expressing it would be to say that the after-diversity maximum demand per heater is 0·4 kW.

If, instead of being used 24 minutes every hour, they are used 24 seconds each minute the result will be exactly the same, provided the spacing is again uniform and symmetrical. Once again, it only requires five appliances in order to provide a constant 2-kW load on the system. The diversity factor, as before, is $5/2 = 2\frac{1}{2}$.

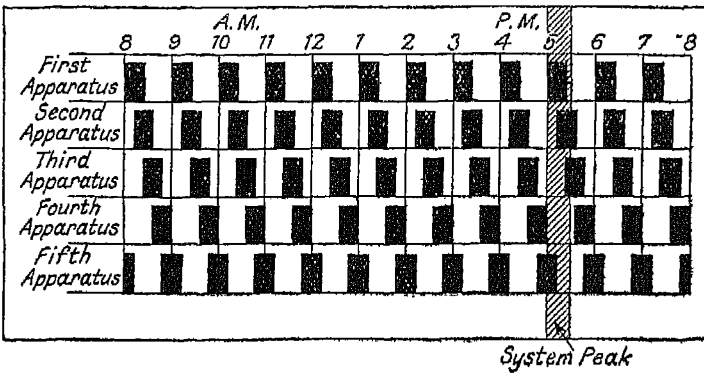


FIG. 20.—Ideal Case.

It will be said that such an exact symmetrical spacing is quite unthinkable and could never happen in practice. The answer to such an objection is that this fact only affects the number of appliances required, not the general principle. Provided there are sufficient of them, and provided they have undifferentiated time characteristics, the results will be approximately as described, whatever the spacing. For the meaning of this characteristic is that there is no more likelihood of the appliance being in use at one moment than at another. In the present case the load factor during the use period is 40 per cent., so that at any instant there is exactly a four-tenths chance of any one of them being on. If there are 100 appliances there will be approximately forty in use and sixty out of use at any one moment.*

* A sidelight on this supposition is thrown by tests on installations of ripple-controlled water heaters (p. 154). By arbitrarily switching them off and on as rapidly as possible, and noting the change in the feeder current, the water-heater load was picked out. The figures when plotted lay on a smooth curve, showing that, with sufficient number of appliances, individual fortuity cancels out.

The (proportional) closeness of the facts to this mathematical statement will then depend on the number of similar appliances connected. If there are only a dozen or so, the non-symmetrical spacing will affect the results, and the actual maximum demand in the metered period may be either more or less than that calculated above. But if there were five million of them it could be stated with confidence that, however erratic the spacing, the M.D. would be 2,000,000 kW to a close degree of approximation.

If the loading fluctuates during use, instead of being of the "on or off" character, the same thing is true. Provided there are sufficient appliances of random time characteristics, the M.D. will be the sum of the mean loads during the use period, and the diversity factor will be the reciprocal of the load factor during this period.

Deduction and Divergence.—The surprising fact emerges that with the above hypotheses the load factor during the use-period has no effect on the true maximum demand on the system although it directly affects the metered M.D. of the consumer. If it is a factory load that is under consideration and if all the factories work eight hours a day (and the same eight hours) the collective load factor will be $8/24 = 33\frac{1}{3}$ per cent., and such consumers can be given the benefit of such an assumption. To make the case more specific, let the maximum power be 100 kW and the annual energy consumption be 146,000 kWh. The true load factor is $\frac{146,000}{100 \times 8,760} = \frac{1}{6}$, but this is due to the combined effect of a use-period or "hours" load factor of $\frac{1}{3}$ and a load factor during the period of $\frac{1}{2}$. From the point of view of supply costs, the load factor during the use-period is immaterial and can be regarded as unity. The effective load factor (for supply costs) therefore is $\frac{1}{3}$ and the effective M.D. is only 50 kW.

In a similar way, if it is a domestic load that is under consideration, and if this may occur at any time indiscriminately between 6 a.m. and 10 p.m. the collective load factor will be 16/24, or 67 per cent. But in order to achieve these results two things are necessary, neither of which will ever be fully realised. There must be a sufficient number of similar loads for the law of averages to operate fairly well, and they must have purely undifferentiated time characteristics. As regards the former requirement, probably at the power station, the bulk-supply point, or even at a main sub-station the numbers will be sufficient. But in assessing distribution costs the numbers at any one distribution point may be too few to obtain even an approximate agreement with the theory. The second requirement (as to time characteristics) will be harder to realise.

Any attempt at assessing time characteristics is necessarily very tentative. Probably a factory power load will show the least deviation

from the above requirements. If the consumption for lighting and heating is separated off, the remainder may well show a very fair approximation to undifferentiated time characteristics. The effective load factor will then be only slightly lower than the hours load factor *i.e.*, working hours per diem ÷ 24. But the working hours must be taken as those common to all the factories in the area concerned. Thus, if the normal working day is eight hours, unless all factories start and stop together, the use-period may range over 10 hours of the day of which only six are common to them all. The probability of demand during the fringing hours will be less than during the common period, so that it would not be correct to estimate the load factor from 10/24.

Taking next the domestic non-lighting load, this will be fairly well undifferentiated in the case of irons and cleaners, less so with water heaters and still less so with cookers unless of the thermal storage type. The full use-period may be from 8 a.m. to 8 p.m., with a "fringing" period of several hours on the outside of each time. During these fringes the probability will be less but not zero. By combining these loads a nearer approach may be obtained to undifferentiated characteristics. Cookers and cleaners, etc., may to some extent dovetail, even within a single household; but for the most part one must rely on large numbers of adjacent consumers to provide the diversity. In the case of space-heating appliances, similar remarks apply except that the regular use-period might be, say, from 8 a.m. to 10 p.m. during four or five months of the year only. The "hours" load factor will be correspondingly reduced, but the use within these hours may be fairly undifferentiated apart from climatic fluctuations.

In the case of the lighting load, this cannot possibly be regarded as even approximately undifferentiated, since it is largely a function of darkness, and this occurs at about the same time throughout the country. The load therefore gravitates to particular hours of the day and year, so that the random character necessary for probability principles to operate is not present.

In all the above cases it is assumed that the use-period fully overlaps the particular half-hour during which the M.D. of the system occurs. If it does not overlap, the effective M.D. on the system will be zero, and if it only partially overlaps, the M.D. will be reduced in a corresponding ratio.

Summary.—The results may be summed up as follows for the individual consumer: bad load factors during the use-period are to a greater or less extent compensated for by diversity factor. When the use has no specific time characteristics within the period, this compensation will be complete within the ordinary laws of probability.

In order to express the results mathematically, it is necessary to

split the load factor up into its component parts. Let L be the fraction—daily use-period in hours divided by 24—and let F be the load factor during this period. Then the diversity due to irregular use during the period may be empirically expressed as $\left(\frac{1}{F}\right)^n$ where n is a constant less than unity. The collective load factor will then be LF^{1-n} The value of the index n would approach unity with a large number of appliances connected, having entirely undifferentiated time characteristics. It might be of the order 0·6 to 0·8 for a power load, and 0·4 to 0·7 for a non-lighting domestic load. To be on the safe side, a figure of 0·5 could be taken for any load without pronounced time preferences. The diversity factor thus becomes the reciprocal of the square root of the load factor during the use-period.

The reciprocal of the diversity factor is called the *coincidence factor* (often expressed as a percentage) and measures the degree of coincidence with other loads. On the above assumption, the coincidence factor equals F^n, and when $n = \frac{1}{2}$ the coincidence factor equals the square root of the load factor during the use-period.*

It will be noted that, using the above symbols, the true or overall load factor is the product of two terms (LF) but only one of these terms, namely, F, is related to the diversity factor. No credit for increased diversity can be given for load factors whose lowness is merely due to their short hours of service (assuming that these hours overlap the peak period). It will further be noted that only the group diversity of a single consumer or apparatus within a group of similar ones is governed by this relationship. There is another factor, namely, the inter-group diversity between composite lines, and this has no connection with their load factors but only with their times of day. The total or overall diversity of a consumer will be the product of these two separate diversities as explained below.

No mention has been made of loads having a predilection *away* from the peak period. Such a case is less likely, since the peak period is by definition the time when most people are using electricity and therefore when it is generally most useful. But if this time occurs in the afternoon there will be some loads having a preference for the morning (*e.g.*, vacuum cleaners), and if it occurs in the weekday there may be some which excel on Sundays (*e.g.*, cooking). The result

* The work of some American engineers has confirmed the existence of a relationship of this kind, though less simple than is implied in the empirical formula suggested above. (Constantine Bary, H. E. Eisenmenger and R. F. Hamilton. See especially *American Institute of Electrical Engineers: Transactions*, 1945, **64**, pp. 623–629.) In these results, curves plotting coincidence factors against *overall* load factors were only roughly of the type $(LF)^n$ where n is less than unity. The chief difference was that the coincidence factor after at first rising rapidly with load factor, became almost steady at about 84 per cent. between load factors of 35 to 70 per cent. At higher load factors, it approached the theoretical lower limit of the relationship (coincidence factor = load factor). The difference in results is probably due to the difference between use-period load factor and overall load factor.

would be a system diversity greater than $\frac{1}{F}$, *i.e.*, n greater than unity. It would be unsafe, however, to presume on this, particularly as conditions may change and valleys may become peaks. There is also the danger of localised peaks developing even when the system as a whole is well diversified. It is therefore better to leave any possibility of favourable discrimination to serve as an occasional bit of "jam," and to legislate on the assumption that the best condition to be met with is complete indiscrimination.

Verification.—Anything so tentative as the theory here outlined can hardly be said to be susceptible of "verification," and would in any case require far more data than are at present available. The utmost that can be done is to show that it is generally consistent with observed facts and current practice in tariff fixing. In the case of the American water-heater tests described above, fourteen groups of heaters were tested, having eight to sixty-six heaters in each group. In each case the diversity factor for the whole group was obtained and also the mean weekly load factor of the individual heater. By plotting the logarithms of these values it was found that they could be represented very closely by an index of $n = 0{\cdot}5$ and a value of $L = 0{\cdot}54$, *i.e.*, a use period of $0{\cdot}54 \times 24 = 13$ hours.

These results are in accordance with common sense as well as with the present theory. They suggest that the full use-period is thirteen hours a day, *e.g.*, from 8 a.m. to 9 p.m. That, within this period, the use is only partially undifferentiated, the diversity factor being the square root of the reciprocal of the individual load factor during the period.

Much the best proof of the correctness of the above theory, and the best guide to the value of n, is the flatness (or otherwise) of the group load curve. In the industrial load analysed in Fig. 16 the curve shows a pronounced hump on either side of the lunch interval. Fig. 9, on p. 60, which is a mixed-load curve for a predominantly industrial area (Coventry) on a day in late October, shows a very rectangular morning load, and the afternoon is similar but somewhat lower until the lighting peak begins. In general it may be said that the typical industrial curve (apart from lighting) is very nearly a rectangle during the working hours of, say, 8 a.m. to 5 p.m., with a narrow lunch-time trough, and with the sides tailing steeply off to a low value throughout the rest of the day.

Such a result can only arise through the general operation of the above principles. For the individual power consumers' curves are certainly not of this shape, but often have comparatively severe peaks at all sorts of different times during the working hours. If then the aggregate curve has no very marked peak, this can only be because the individual loads have no special predilection for any particular time of day.

The same fact can be inferred from the magnitudes of the curves as well as from their shapes. The load factors of small industrial consumers are often extremely low (figures of 5 to 10 per cent. are quite common according to many authorities) and for these to combine into a power curve with 25 to 30 per cent. load factor there must be fairly complete staggering of demands over the working period.

Estimate of Total Diversity.—In order to crystallise the above results let us estimate the diversity factor of a single consumer—say a small factory owner forming part of the industrial group I in Figs 15 to 17. Suppose that his yearly load factor is 10 per cent. and that his factory working hours are forty-eight a week and are the normal hours of other factories in the group. Then $L = \dfrac{48}{7 \times 24} = 0{\cdot}286$, and since $LF = 0{\cdot}10$, F must equal $0{\cdot}10/0{\cdot}286 = 0{\cdot}35$. In words, the "hours" load factor is 28·6 per cent, and the load factor during those hours is 35 per cent.

Assuming a value of 0·7 for the index n, the group diversity factor of this consumer relative to his fellows would be $\left(\dfrac{1}{0{\cdot}35}\right)^{0{\cdot}7} = 2{\cdot}09$. This would be the figure to take for the diversity relative to the cable b_2 supplying the group, and would determine his share of the fixed costs of this cable. But the group as a whole has a diversity relative to the supply station of 1·84 (p. 91). Hence the total diversity of the consumer relative to line c would be $2{\cdot}09 \times 1{\cdot}84 = 3{\cdot}8$, and this would be the figure required in allocating the fixed costs of the station or bulk supply. (*N.B.* This particular inter-group figure is rather high. Usually the industrial group peak, especially if there is appreciable factory lighting, will almost coincide with the system peak when that occurs in the afternoon, giving an inter-group diversity of little more than unity. A likely value would be about 1·2, and the total diversity of this particular consumer would then be 2·5.)

Effect on Supply Costs: Two-part Basis.—It will be seen from the foregoing that the effective load factor and the demand made on the supply system are determined far more by the length of the use-period and the degree of its coincidence with that of other users than by the individual maximum demand as metered at the consumer's terminals. In the reduction of supply costs it is more important for a consumer to spread his demands over a long period than to keep his demands uniform during the period. Whatever the meter readings say, a fourteen-hour factory load is likely to be nearly twice as satisfactory as a seven-hour one, assuming that in either case the peak period is overlapped. The aim should always be the improvement

of the *system* load factor, and the only justification for an elaborate tariff is that it shall contribute directly to this end.

In the preceding chapter, costs were worked out on the dual basis of a fixed charge per annum per kW of demand (with an allowance for the average diversity factor) and a running charge per kWh of consumption. If these costs are passed on to the consumer as they stand, in the form of a two-part M.D. tariff, the fixed charge will be levied on the consumer's individual M.D. instead of on his effective demand on the system. The consequence will be that the low-load-factor consumers will be over-charged (since they are given insufficient credit for their greater diversity) whilst the high-load-factor consumers are under-charged.

The weakness of such a tariff when applied to the small individual consumer is that it treats load factor as a variable and diversity factor as a constant. Each separate consumer is levied with a fixed charge based directly on his metered load factor, but he is given a "lump sum" fixed allowance for diversity the same as other consumers in the group. But, in practice, diversity factors vary from consumer to consumer almost as much as load factors, and moreover, in the opposite direction.

The result can well be seen by comparing two imaginary consumers (*a*) and (*b*) having the same working hours. Consumer (*a*) has a very steady loading—say his use averages six hours a day at full load (maximum demand reading) throughout the year. His load factor will then be 25 per cent. Consumer (*b*) only operates his plant for the equivalent of three hours' full-load use per diem, so that his load factor will be $12\frac{1}{2}$ per cent. If their energy consumptions are identical, consumer (*b*) will have to pay double the standing charge of consumer (*a*). But a group of (*b*) consumers will not cause double the load on the system that an equal group of (*a*) consumers cause because they will have more diversity amongst themselves. Consumer (*b*), merely because his use is more spasmodic, has a lower probability of coming on to the peak period than has consumer (*a*).

Another way of putting the same thing would be to say that whilst the costs estimates on pp. 74 to 83 allowed for an average diversity for all l.v. connections of 1·6, this is too high for some consumers and not high enough for others. This can be seen in the above example, and still better by taking extreme cases. If I have a motor and use it for only one hour in the year, what are the odds that it is in use during the hour at which the M.D. on the system is being registered? Not as low as 1 in 8,760, but certainly not as high as 1 in 1·6. If my neighbour has a motor driving a ventilating fan which works at a constant load all day and every day, the load factor of this will be 100 per cent. Yet he also is credited with a diversity of 1·6 although actually he can have no diversity whatever.

Hard cases make bad law but they make good illustrations, and that

is why these extremes have been cited. A man who only uses electricity once a year should not have to pay £5 or £10 a kilowatt-hour any more than the man who only takes a taxi once a year has to pay for all the time it has stood in the rank. In both cases the principle applies that the lower the load factor the higher the diversity, and it is this reciprocal relationship which makes it necessary to temper the (apparent) justice of the two-part tariff with a little mercy towards the occasional user.

The same point can be put from the distribution aspect. One sub-station supplies a group of textile factories with steady loads throughout the working-day. Their individual load factors may all be in the 30 to 40 per cent. range. Another sub-station supplies a number of miscellaneous factories, repair shops, rubber and other works using heavy power-presses, etc. Their individual load factor may be only of the order of 10 to 20 per cent., yet the shape of the load curve on this sub-station may be almost identical with that on the other. This indicates that the costs of supply are similar up to this point, and that only the costs of the last stages of distribution are affected by the individual load factors.

The author has coined the term "differential diversity" in order to call attention to this relationship. Differential diversity may be defined as the probability that a low individual load factor will be associated with a high diversity, or that there will be a reciprocal relationship between load factor and diversity factor. The word individual is inserted as a reminder that such a probability arises as a consequence of a large number of connections. Bulk supplies cannot be expected to have much diversity with one another.

There is a very simple way of compensating for differential diversity within the compass of the standard two-part tariff, and that is to reduce the fixed charge and increase the running charge so as to bring in the same revenue as before. There is abundant evidence (some of which is given below) that this has, in fact, been done with many present-day tariffs, though it is difficult to say whether this has been from conscious economic motives or merely from a desire to meet the user's wishes. Possibly some higher "engineering sense" has guided the decision. It is a common experience in engineering that correct design has often been achieved more by an instinctive sense of proportion than by conscious reasoning. The founders of our craft built better than they knew, and something of the same sort appears to have happened in connection with tariffs. The correct decision seems often to have been made without, or even in spite of, the apparent reasons and necessary data. It is difficult otherwise to account for the complete absence of the maximum-demand tariff in domestic supplies, the large number of flat-rate tariffs still surviving, and the fact that even when a two-part M.D. tariff is employed (*e.g.*, for industrial or commercial supplies) the proportions of

the two parts are not at all those indicated by cost considerations.

M.D. Tariff: Ratio of Parts.—A further confirmation of the thesis advanced in this chapter can be derived from tariff practice and proportions. So far, only the costs have been considered, and the details of actual consumer tariffs are given in later chapters. It is, however, necessary at this point to anticipate by saying that the majority of industrial (and many domestic and commercial) tariffs still consist of flat rates per unit of energy, with or without sliding scale reductions for quantity. Since only one-quarter of the total cost is proportional to energy, this would appear to be a hopelessly uneconomic type of tariff. But in the light of the present work, such a tariff (if fixed according to the likely use-period of the consumer) is seen to be not so far out after all.

In other cases the tariff consists of a fixed annual charge per kW of demand plus a running charge per kWh consumed. This has already been referred to as the "Hopkinson," or "M.D." tariff, and the numerical ratio between the annual fixed charge in pounds and the running charge in pence will be referred to as the "ratio." It is in connection with this ratio that some interesting facts emerge. It will be recalled that this dual charge was used as the basis of the cost reckoning in the last chapter. It might therefore be thought that when the actual tariff was to follow precisely the same form, all that would be necessary would be to work out the costs and then increase each part sufficiently to cover profits and contingencies, leaving the ratio between the parts the same as before. An examination of existing tariffs of this type serves to show that this is not what has happened.

Takings the costs first, the value obtained for the above-mentioned "ratio" will largely depend on the assumption made as to diversity factor. If no such factor were assumed, the ratio obtained by the cost analysis would be 14 to 25 (depending on the coal/plant price-ratio) at the generating station and would rise to 30 to 60 at the low-tension consumer's terminals. This is because the expenses of transmission (excepting the small element due to line losses) consist almost entirely of fixed costs and so go to swell the standing charge. But this tendency is opposed by a contrary factor, namely, the diversity, which mounts steadily up as distribution proceeds. Considering only the final consumer, the total diversity assumed in both the analyses on pp. 74 and 83 was 1·6. Since this has to be an average for all consumers (including those with steady loads overlapping the peak, whose diversity is almost zero), it is doubtful whether a larger figure could safely be assumed. On this basis, the "ratio" between the two parts in the cost estimates was nearly 17 in the post-war example and would have been two or three times as great with pre-war coal prices. The ratio in the second example (old date) was 41.

Another pre-war example of a tariff built up on costs was that of J. A. Sumner in his paper on "Private Plants and Public Supply Tariffs."* In this case, only the loan charges were scheduled as fixed costs, all other expenses being put onto the running charge in order to obtain a more acceptable tariff. Even on this basis, namely, adding bare capital charges to the initial "grid" tariff, the standing charge to the low-tension consumer in the above instance came to over £14 per annum per kW before diversity had been allowed for. Had the usual allocation been employed the figure would not have been less than £16 per kW of system demand, or (assuming 1·6 diversity) £10 per kW of consumer's demand. In a similar way, the running charge including all proportional costs would have been about 0·25*d.* per kWh. The ratio between the parts would then have been 40.

Turning now to actual tariffs, the British Electricity Authority's tariff for bulk supplies has a ratio of about 8 with 1948 coal prices. Consumers' tariffs for power supplies vary enormously, the ratio in the case of published two-part tariffs of authorised undertakings varying from about 2 to 30, with an average of approximately 10. (It is significant that the ratio in the high-tension and bulk-supply tariffs is about the same as in the low-tension ones.) Thus the value of the ratio in the costs estimate, even on the most moderate assumptions, is about four times its value in current tariffs.

A final illustration of the divergence between costs and tariffs can be seen by examining the running charge only. The energy distributed in this country is purchased by the Area Boards at a bulk-supply tariff with a running charge of ⅓*d.* to ½*d.* The only considerable item in the distribution costs which can directly affect the running charge is the I^2R loss, which averages about 11 per cent. It follows that the running charge of a distribution tariff which was based strictly on costs could not be higher than about 0·4*d.* to 0·6*d.* per kWh—but where will one find such a tariff? The standing charge, on the other hand, if worked out on similar cost principles is likely to be considerably higher than the figures current in present-day tariffs.

It will be appreciated that the above discussion concerns only the comparison between costs and tariffs, as though this were all that is involved. It must not be forgotten, however, that the construction of a suitable tariff is a matter of policy as much as of strict economics, since the electricity has to be sold as well as made. The standing charge is not popular at the best of times, and a figure of £10 to £15 a kilowatt, even though coupled with a low running component, is likely to make it still more frightening. It is therefore quite impossible to say how far the low ratio to be found in existing tariffs has been due to commercial rather than to cost considerations. At the same time it could hardly have persisted, and have functioned as well as it has, were it not for its fundamental economic soundness. The same

* *Journal I.E.E.*, 1935, **77**, p. 310.

thing, though to a lesser extent, might be said regarding the continued persistence of the flat-rate tariff.

Summing up the above sections, it may be said that there is the following evidence in existing tariffs for the phenomenon termed "differential diversity."

(*a*) The continued existence of flat-rate tariffs although only one-quarter of the costs are proportional to kWh.

(*b*) When two-part M.D. tariffs are employed the ratio between the two parts is far less than can be justified on simple cost grounds even allowing the largest possible figure for mean diversity. In fact, the average value of this numerical ratio is about 10 in the tariffs and 40 in the cost estimates. (Pre-war coal prices in both cases.)

Economic Tariff Construction: Adjustment of Parts.*—It has been stated already that the method of allowing for differential diversity within the framework of the standard M.D. tariff is to adjust the proportions of the two parts. The standing charge is lowered and the running charge raised in such a way as to maintain the required total revenue. This will have the effect of reducing the overall price per unit to the low-load-factor consumers and raising it to the high-load-factor ones, thus giving something of the compensation desired.

If the theory advanced in this chapter is mathematically correct, and there is a constant value for the index n, this readjustment of parts will not produce exactly the effect required, but it will do so very nearly. Moreover the theory was put in this form merely to give it coherence and tangibility, and on grounds which were frankly empirical. The essential fact is that there *is* an inverse connection between load factor and diversity factor, whatever its exact form; and this being so, an adjustment of the tariff parts in the direction indicated will necessarily improve it, even though not making it exactly and economically correct.

The process of "biasing" a two-part tariff can best be followed from a numerical example. The cost values employed, whilst somewhat on the high side, are correctly proportioned (pre-war values) and have been chosen so as to give convenient round numbers in the result. On a system with fairly high distribution expenses, the cost might be expected to reach the figures just mentioned, namely, £16 per annum per kW of system demand plus 0·25*d.* per kWh. This would be built up by adding all distribution costs to the bulk supply tariff, on the lines of the last chapter. It is further supposed that the load factor of the industrial load at the bulk supply point is 24 per cent.

The problem is to devise a suitable Hopkinson tariff for a number

* This construction and subsequent criticism of the m.d.tariff refers more particularly to its use for factory and office supplies. With domestic loads, it is still further from representing either costs or use-values, but as it is so rarely employed in this field there is no need to waste time here on either its construction or its criticism.

of small consumers, and the first step is to make an allowance for their overall diversity. If all the consumers served are on such a tariff, this figure could actually be measured from the ratio of their aggregate M.D.'s to the total M.D. coming on the system at this point. Failing this, it could be estimated from the relationship given at the beginning of the chapter. Assuming a use-period of eight hours a day (*i.e.*, $L = \frac{1}{3}$) and a value of $n = 0{\cdot}6$, the average consumer's load factor would be 15 per cent. and the average diversity $24/15 = 1{\cdot}6$. The tariff then becomes £$16/1{\cdot}6 =$ £10 per annum per kW of consumers' demand plus 0·25*d.* per kWh.

The above tariff, besides being unpractical, is economically unsound, and the next step is to "bias" it. The object of this biasing is to load the tariff against the high- (and in favour of the low-) load-factor consumer whilst keeping it the same for the average man. Now the "average" consumer has a load factor of 15 per cent. and his standing charge therefore amounts to

$$\text{£}10 \times \frac{240}{365 \times 24 \times 0{\cdot}15} = 10 \times 0{\cdot}183 = 1{\cdot}83d. \text{ per kWh}$$

when spread over his unit consumption. He therefore pays an overall price of $1{\cdot}83 + 0{\cdot}25 = 2{\cdot}08d.$ per kWh, and this is therefore the mean revenue per unit obtained by the undertaking. The following alternative tariffs bring in the same mean revenue :—

£8	per annum	per	kW	plus	0·62*d.*
£6	,,	,,	,,	,,	0·98*d.*
£4	,,	,,	,,	,,	1·35*d.*

It will be noted that in each case the standing charge in £ when multiplied by 0·183 and added to the unit charge gives the same overall figure of 2·08*d.* per kWh. Any one of these will therefore bring in the correct revenue at the average load factor, and they all compensate to a greater or less extent for differential diversity. But it is not clear how far the tendency should be pushed, *i.e.*, which of them compensates to the right degree, or whether some other figures might not be better still.

The complete range of possibilities is shown graphically in Fig. 21. The falling and rising lines represent the standing and running portions of a two-part tariff as shown by the ordinate scales on left- and right-hand sides respectively. They are so drawn that any vertical line intersects a pair of prices which will yield 2·08*d.* per kWh overall at 15 per cent. load factor. The base line shows the numerical ratio existing between the two parts of the tariff. This is not uniformly divided but is so scaled that the two tariff graphs are straight lines.

The base of the figure also represents the extent to which diversity is regarded as being affected by load factor. If the diversity is taken as a constant figure of 1·6 regardless of the individual load factor, then the correct tariff is that given by the left-hand boundary, namely,

£10 plus 0·25*d*. If on the other hand the diversity is considered to be fully proportional to the reciprocal of the load factor (so that individual load-factor variations are entirely neutralised by diversity), then a flat rate would be correct, and this is represented by the extreme right-hand ordinate (2·08*d*.). Assuming the facts to lie somewhere between these two assumptions, it becomes necessary to select some intermediate position for the ordinate.

Probably the choice in practice will be made on commercial grounds depending on which value of ratio is likely to be most acceptable to

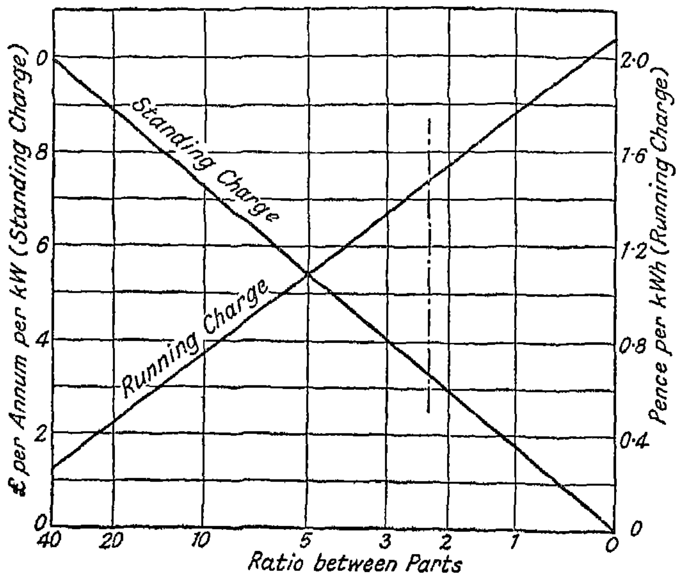


FIG. 21.—Two-part Tariff Proportions.

the consumers concerned. Sometimes they are allowed to choose between two alternatives, as in the case of a London company which offered the choice of £8 per kVA plus 0·4*d*. and £4 per kVA plus 0·75*d*. This seems to suggest that the company were not able to "bias" correctly on a cost basis for a range of load factors, since it is obvious that with such a choice the low-load-factor consumers would merely take advantage of the low fixed charge, and *vice versa*.

Precise Method.—A less haphazard method of solving the problem is to apply the principle mentioned earlier in the chapter. If the diversity of any individual consumer is related to his load factor in the manner described, it is possible to estimate precisely what effect

his consumption will have on the system demand. A "true costs" formula can then be constructed; this will be too complicated to serve as a tariff, but a two-part tariff can be devised to give substantially the same overall effect over the range of probable load factors.

Using the same symbols as before, namely, L for the ratio of use-period to total period, and F for the load factor during the period, it is suggested that the diversity is $\frac{1}{F^n}$ and the effective load factor on the system is therefore not LF but LF^{1-n}. It is further suggested that for the normal industrial user L may be taken as $\frac{1}{3}$ (averaging eight hours a day use) and n as 0·6. Now, a standing charge can always be expressed as a price per unit when the load factor is known. Thus £1 per annum per kW of standing charge means.

$$\frac{1 \times 240}{365 \times 24 \times \text{load factor}} = \frac{1}{36{\cdot}5 \times \text{load factor}} \text{ pence per kWh.}$$

In the present case the true cost of supply is £16 per annum per kW of system demand plus 0·25*d*. per kWh, and the effective load factor is LF^{1-n}. Hence the correct overall price per unit is $\frac{16}{36{\cdot}5\ LF^{1-n}}$ $+ 0{\cdot}25d$. But if a normal two-part tariff is levied of £q per annum per kW plus p pence per kWh, the overall price will work out at $\frac{q}{36{\cdot}5LF}$ $+ p$ pence. Putting in values of $L = \frac{1}{3}$ and $n = 0{\cdot}6$, the two formulæ become: true costs $= \frac{1{\cdot}32}{F^{0{\cdot}4}} + 0{\cdot}25$; two-part tariff $= \frac{q}{12{\cdot}2F} + p$ (both in pence per kWh). The question to be settled is, what values of q and p will bring the tariff nearest to representing the true costs equation over the range of likely load factors?

One of the best ways of illustrating the overall price of a tariff is to plot it to a base of load factors having an inverse or reciprocal scale. (A fuller description of this type of graph is given in Chapter X.) In Fig. 22 the load factors shown at the base points are the reciprocals of the distances of these points from the right-hand end (marked ∞). The merit of this scale is that the effect of any given standing charge q (expressed as a price per unit) can be represented by a falling straight line reaching zero height at the infinity mark. The height of the line at, say, 10 per cent. load factor will be $\frac{q}{3{\cdot}65}$ pence per kWh. Since the vertical scale is a uniform one, a running charge of p pence per kWh is represented by a constant vertical height p. Hence, the two-part tariff (£q per annum per kW plus p pence per kWh) is represented by a straight line parallel to the q line and situated p units above it.

In the case of the "true costs" formula, the overall price per unit

is represented by the line shown on the graph, having a slight downward concavity. In order to find the nearest equivalent two-part tariff it is only necessary to draw the closest straight line to this curve, which in the present case is the one shown chain-dotted, and representing £3 3*s.* per annum per kW plus 1·44*d.* per kWh. In order to show the components of this tariff, a parallel line is drawn (dotted) passing through the ∞ mark. This line is 1·44 units below the other,

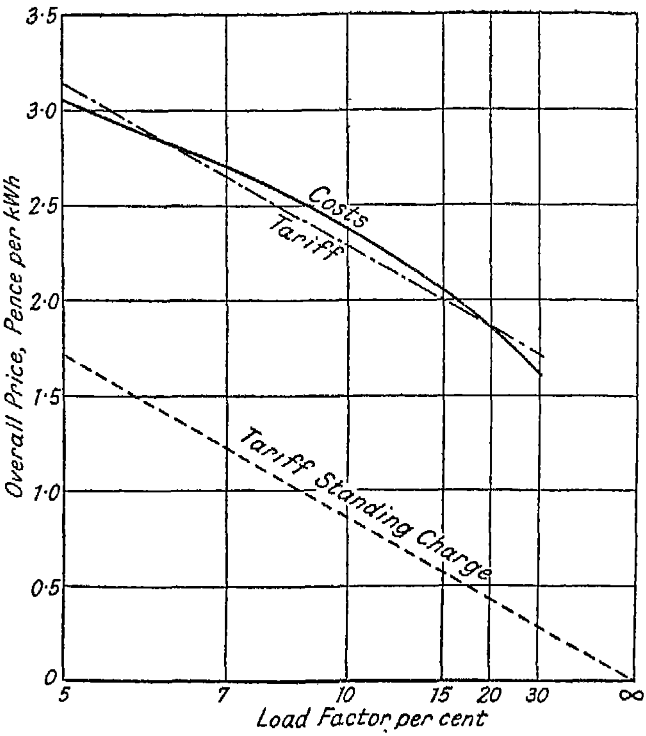


FIG. 22.—True Costs and Two-part Tariff.

and at a load factor of 10 per cent. its height is $\frac{3 \cdot 15}{3 \cdot 65} = 0 \cdot 86d.$ per kWh.

It will be seen from the curves that a plain two-part tariff can be constructed to represent very closely the true costs on the data assumed. Actually, the divergence is never greater than 0·1*d.* over the range of load factors illustrated. The ratio between the two parts (2·2) is extremely low, and results in a much nearer approach to a simple flat rate. (The chain-dotted vertical in Fig. 21 shows the exact position of this tariff.) Had a lower value been taken for *n*, the ratio would have come out higher.

It may be remarked that the curves only refer to a limited range of load factors, and that on either side of this range the agreement will be far less. This is true, and it should be emphasised that such a tariff is only intended to represent costs for a group of fairly uniform consumers—in this case, factories having similar working hours. The individual load factors are not likely to be much below 5 per cent., and they cannot be more than 33⅓ per cent., since this would mean unity load factor during the use-period (eight hours a day). Hence a range of 5 to 30 per cent. covers all the likely cases.

Use-Value Aspects.—It will be seen that when the cost figures are adjusted in the manner described above, they result in a very low standing charge indeed, and a correspondingly high running charge. In fact, the correct ratio between the two parts, instead of being higher than that usually employed, turns out to be lower. Initially, the costs ratio was 40, but after biasing it became only about 2½, as compared with a normal figure in actual tariffs of about 10. Before going to this other extreme, however, there is a commercial aspect that must be considered.

It has been mentioned already that the M.D. tariff, although designed purely to represent costs, has a certain correspondence also with use-values. Owing to internal diversity the large consumer, *ceteris paribus*, will always tend to have a better load factor than a a small one. In fact, any large well-diversified industrial concern working an eight-hour day, and with no special time characteristics, can be expected to approach a load factor of some 30 per cent. (apart from lighting). Thus the M.D. tariff will tend to favour the large concern, and this is in accordance with value requirements, since the large industrialist is in a better competitive position and should be offered a lower price. But if the tariff is modified by whittling down the fixed charge, this tendency will be reduced : in fact, the correct economic construction results in so small a ratio of fixed charge that the advantage of the large user almost disappears.

The best practical solution (if a plain M.D. tariff is to be employed) would therefore appear to be a compromise value of about 10 for the ratio between the two parts, at pre-war coal prices, and this in fact is what was employed in the average tariff of the country. It should be realised, however, that this compromise is the result of trying to make one thing serve two different purposes, namely, costs and use-value. A better design would be to employ two different elements in the tariff structure for these two purposes. True cost considerations can be satisfied by a very low ratio between the two parts of the tariff, whilst use-value requirements are best met by " stepping " or " blocking " one or both parts.

Future Policy.—It is difficult not to press the matter further and

to ask how far a maximum-demand tariff is necessary for the small individual consumer. Is its extra complication justified by its closer representation of supply costs? The answer to such a question depends entirely on the degree of time specialisation possessed by the consumers in question. In the case of the power load, the aggregate curve on the day of system peak is frequently very nearly a flat-topped figure from about 8 a.m. to 5 p.m. with a drop at lunch time and with a much lower figure over the remaining fifteen hours of the day. This means that over this nine hours' period the individual loads have shown no predilection for any one time more than another. The index *n* in the above formula would then be unity. Even when the aggregate curve is far from rectilinear, as in the case already illustrated, the peaks in the curve will not necessarily coincide with the peaks on the station, or even with those on the distribution mains unless the power load is thoroughly localised.

The conclusions seem to be that in all cases where the use occurs largely over certain well-defined hours common to all consumers in the group, and where the group curve is tolerably flat-topped, time characteristics within the period can be largely neglected. Every consumer, whatever his individual load factor, can then be credited with the load factor of the whole group, and a simple flat-rate tariff will serve for them all. Referring back to Fig. 21, it will be evident that the higher the value assigned to the index *n* the lower becomes the corrected " ratio " and the more nearly do the pair of costs merge into a single charge.

Such conclusions are highly unconventional, but it is believed that once supply engineers have realised the implication of the comparative smoothness characteristic of many group curves there will be a considerable revision of opinion on the subject. It will then be recognised that within any flat-topped portion of a load curve the undertaking gains nothing by individual good load factors and loses nothing by individual bad ones. Twenty bad-load-factor consumers each taking supplies for short periods during the working day, or taking very " peaky " supplies may yet in the aggregate produce the same effect as twenty good-load-factor consumers taking supplies uniformly over the same working day. For the latter cannot in any case produce a better curve than a flat-topped one, whilst the former may (and by hypothesis do) produce just such a curve. (These arguments, whilst applying strictly to generation costs, apply to distribution costs only in so far as the twenty consumers are close together.)

The same thing may be seen in the case of the irregular curve *I* shown in the figures at the beginning of the chapter. Provided the consumers are sufficiently close together, it does not matter whether the top curve in Fig. 16 is produced by a thousand consumers each with a " good " curve of exactly the same shape as *I* but smaller in height, or whether it is produced by a thousand irregular and

spasmodic curves of the type sketched in the lower part of the figure.

On the above hypothesis, there would appear to be very little economic merit* in a two-part tariff for power consumers whose demands are limited to the normal working hours. The chief advantage of such a tariff is not to induce consumers to " level up " their demands during factory hours (diversity does this for us) but to persuade them to spread their demands over the rest of the twenty-four hours. If this possibility exists, a Hopkinson tariff will encourage this extension, and will make a fair allocation of the supply costs entailed. But when there is little hope of extending the use-period, such a tariff is of doubtful utility and appears somewhat of an impertinence.

There is something distinctly pathetic in the sight of a factory manager employing a " tell-tale " maximum-demand indicator which rings a bell or shows a light whenever a certain value is exceeded. Whereupon the foreman dashes wildly about and shuts down a buffing motor or a grindstone that has been left running unnecessarily! In another factory down the road a precisely similar comedy is being enacted except that it happens at a different time of day and on a different day of the week. Is it not time the supply authorities realised that this sort of spasmodic local peak hardly matters at all, although they charge the unfortunate factory owner £5 a kilowatt for it?

The fact is, we have often thought far too much of the individual load factor, as though the group curve were merely the individual curve writ large. It is time we did something to correlate load and diversity factors, and to realise that there are different *kinds* as well as different *magnitudes* of load factor. A purely quantitative statement of a consumer's load factor is therefore no guide whatever to the costs of supplying him. It leads to the conclusion that the best possible consumer is one having a load factor of 100 per cent. Whereas of course the best consumer (whether his load factor is 8 or 80) is one who avoids the peak. Whatever be the condition and the load factor of the system he will necessarily improve it.

Summing up the position, if the working hours are the same it matters little as regards generation costs whether the supply is to an engineering or rubber works with a highly fluctuating load or to a textile mill with a very steady load. The same is true of much of the distribution cost provided there are a number of such consumers in proximity. The two-part M.D. tariff is then defective as a cost

* The commercial merit, that of favouring the large consumer, can be better supplied in a more direct manner, namely by the block tariff. There is, however, another commercial merit in the M.D. tariff which must not be overlooked, namely, that of reflecting alternative costs of supply governed by load factor. Differential diversity levels up the undertaking's costs but it does not help the small private power producer. His costs are a strict function of his individual load factor, and the M.D. tariff is to this extent competitive (*cf.* p. 191).

vehicle though it well reflects use-value and "what the market will bear."

Among the remedies that may be suggested are to employ a two-part tariff which is heavily biased in the manner described, to have the maximum demand recorded only during such times as the system peak is liable to occur, to take more frequent M.D. readings, or (when the use-periods are uniform) to employ a flat-rate tariff. The useful future of the M.D. tariff would seem to lie in the costing of bulk supplies where diversity is small, and for large industrial supplies where there is a narrow margin between Area Board costs and consumer values. On a smaller scale, it is useful when the hours of use are very variable (*e.g.*, shop and office supplies).

It need hardly be added that the remarks of this chapter are in nowise a criticism of the maximum-demand principle, but merely of its application to certain retail tariffs. The principle remains as firmly established as ever, and the two-part analysis is still the only possible basis for any scientific cost allocation. It may even be safely applied as a tariff whenever internal diversity has already operated, *e.g.*, for large consumers and bulk supplies. In these cases the scope for differential diversity is only slight, and metered M.D. becomes a fairly accurate guide to peak responsibility.

CHAPTER VI

COST DETAILS: CLASSIFICATION AND RELATION

Introduction: Stages of Work.—In the two previous chapters the subject of costing was treated broadly, and chiefly in relation to two variables—power and energy. In this chapter the same ground is covered, but in greater detail, and a possible cost formula is constructed. The following chapter elaborates a single element of the cost structure, namely, the allocation of demand-related cost.

The complete process of attaching the costs of electricity supply to those who benefit from the service involves several stages. The costs must first be listed in order and sequence under appropriate headings. They must then be related to the magnitude of the function performed or the service rendered. Finally they must be allocated to the consumers responsible for incurring them.

There is no general agreement as to the names and definitions of these several stages. The following was adopted by an Electrical Research Association Committee studying the subject.* (The first two stages shown are dealt with in present chapter, and the third stage in the following chapter.)

I.—*Cost Classification*

The identification, arrangement and grouping of cost components by methods of accountancy and on a basis of function or process.

II.—*Cost Relating*

Grouping or subdivision of cost components on the basis of relationship to energy, time, power demand, number of consumers or other reference not directly amenable to methods of accountancy.

III.—*Cost Allocation*

Sharing a cost among parties on behalf of whom it is deemed to be incurred.

* A Sectional Committee of the E.R.A. was set up, largely at the author's instigation, to do what is nowadays called "operational research", *i.e.*, to study problems (amenable to scientific and objective treatment) occurring in the actual business of electricity supply as such, and unconnected with specific materials or apparatus. Such problems, frequently statistical in character, and involving field work outside the scope of the ordinary university or manufacturer's laboratory, are particularly appropriate to co-operative research; and these two chapters are based largely upon the work of this E.R.A. Committee. (Now transferred to the British Electricity Authority.)

The following table shows the work in a diagrammatic form. (The K/T references are to E.R.A. Technical Reports ; the S.R. & O. reference is to the publication described on page 70.)

I.—Classification (K/T 112)	II.—Relating (S.R. & O. 1015).	III.—Allocation (K/T 106 & 109).
Capital Expenditure under similar heads to Revenue a/c and added thereto in the form of annual charges.		
Revenue A/c Expenditure: Generation: Fuel, Wages etc.	Fixed costs related to demand	P.a. per kW or kVA of demand during peak or potential peak period.
	Running costs related to consumption	Per kWh.
Transmission, Distribution, Installations, Common Services, etc.: Wages, Materials etc.	Consumer costs related to consumers.	P.a. per consumer.
	Unrelated costs referred to one of the above?	Other variables?

Classification : Main Divisions.—The main division will be on a functional basis, *e.g.* :

I. Generation
II. Primary Transmission and Interconnection
III. Secondary Transmission and h.v. Distribution
IV. L.v. Distribution
V. Installations and Apparatus
VI. Common Services (Administration and Specialised sections)

It will be noted that Sections I to V trace the processes of supply in sequence from power station to consumer, and the last of these (Section V) concerns matters beyond the consumer's terminals. The functional basis is less appropriate to distribution, and this may have its main divisions on a departmental basis with cross-headings of a functional nature.

Section VI may be used for all expenses incurred in centralised departments, *i.e.*, administration and centralised clerical work, testing, drawing office, transport, research and development, staff training and welfare, rents, rates and levies, legal and financial expenses. There is however a great danger of cost items that could easily be kept separately becoming merged under headings such as "management," "overheads," and "establishment charges," which convenient dumps soon swell to unmanageable proportions. Such a practice swamps the essentially centralised functions such as management and destroys the value of other well sectionalised grouping. (The variety of meanings attached to the word "management" by different undertakings is alone sufficient to nullify comparisons. The adjective "general" is another danger signal in this connection!)

All work centralised for convenience and economy but attributable to sections (*e.g.*, testing and transport) should therefore be split up according to the main functional headings of generation, transmission, etc. They will then reappear in the general cost statement under the heading "common services" in the appropriate section.

Costs not associated with processes and which therefore cannot be so allocated, such as management in the true sense (*i.e.*, overall management) and welfare, should be kept in some detail so that they may be distributed to sections in some uniform and rational manner for costing purposes. Such distribution must be to some extent arbitrary, which is another reason for keeping this residue as small as possible.

Within these main functional divisions of generation and the like there will be sub-divisions depending on the process and usually related to the type of plant employed. Thus, generation will have subdivisions for fuel, ash, steam and electricity ; and these may be further sub-divided, *e.g.*, steam into raising, utilisation, and recovery. Low-voltage distribution will have sub-divisions for sub-station plant, distribution mains, consumers' services, etc. Finally, whatever the process, the actual expenditures will be of various kinds and will be entered under the appropriate sub-heading in a column representing the type of expenditure (wages, materials, etc.) or type of work done (supervision, operation, repairs, etc.).

Inevitably, the sub-divisions used and the degree of division will vary greatly with the undertaking and purpose : what follows is inserted purely for illustration purposes. A uniform and scientifically grounded system for all undertakings would be an enormous advantage, and as a suggested basis for such a system an E.R.A. Report has been published (K/T 112) adequate for the needs of the largest undertaking.

Capital Expenditure.—This naturally falls under the same main heads, and many of the sub-divisions will be similar to those used for revenue account expenditure, though in general less categories will be

required. Instead of the further division under type of expenditure which is in the revenue account there will be a division according to the estimated length of useful life. For purposes of cost relating and allocation, these capital expenditures can be converted into annual charges and included with the revenue account expenditure in the appropriate section. This conversion will be effected by means of a percentage to cover interest and depreciation according to the plant life. If desired for costing purposes, this percentage can be increased to cover any other overhead costs that can be related to the capital expenditure, *e.g.*, insurance and financial expenses such as cost of loan sanctions or floatations, and the writing off of intangible assets.

Finally the whole process may have to be duplicated, or more, when it is desired to have different sets of costs for different areas or under different Acts or Orders. The following table illustrates a portion of the complete cost statement relating to primary transmission, including on-costs transferred from other statements.

	Salaries and Wages:		Materials and Contracts.	Total Direct Cost.	Capital Charges.	Common Services.	Total on-Cost.	Grand Total.
	Supervision and Operat'n.	Repairs.						
	£	£	£	£	£	£	£	£
H.V. Sub-stations	00,000	00,000	00,000	00,000	00,000	00,000	00,000	00,000
Overhead Lines	00,000	00,000	00,000	00,000	00,000	00,000	00,000	00,000
Underground Mains	00,000	00,000	00,000	00,000	00,000	00,000	00,000	00,000
—	—	—	—	—	—	—	—	—
Total Primary Tr'sm'n and Interconn.	00,000	00,000	00,000	00,000	00,000	00,000	00,000	00,000

Relating: General Considerations.—The next step, called cost relating, is to relate these costs to appropriate variables. Its purpose is to provide a denominator—" costs *per* something." The difficulty is twofold: firstly, to find variables to which the cost can be rigorously related and which serve the practical purposes for which the cost analysis is intended, and secondly, to establish what the relationship is and to mould this into a workable formula. Inevitably, there will be an insoluble residue from the experiment which obstinately refuses to be related to anything, and one must then devise some method for loading this residual cost.

To give an example,* the capital cost of a transformer sub-station is clearly related to the kVA output of the transformer and therefore to the maximum demand in kVA. The cost of the transformer active materials, and other items proportional to cubical contents, will be roughly proportional to kVA. Items proportional to its surface area such as land, single-storey buildings, casings and tanks, will be proportional to $(\text{kVA})^{\frac{2}{3}}$, whilst linear items such as fencing of site will be proportional to $(\text{kVA})^{\frac{1}{3}}$. The resultant will therefore probably lie between direct and cube-root proportionality, and if capital cost or annual capital charges are plotted against M.D. of load they will give a curve of the type shown by the full line in Fig. 23.

If a known point on the curve is, say, £1,000 for an M.D. of 1,000

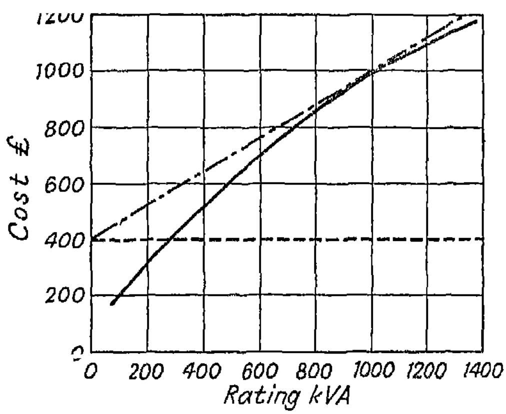


Fig. 23. Cost Components; Constant and Proportional

kVA, a straight line can probably be drawn through this point which would be sufficiently near to the actual curve over the range of variation in question. Such a line in this case is given by £(400+0·6 kVA), shown chain-dotted. The first step, therefore, in simplifying any cost relationship will usually be to express the cost as the sum of two components, one of which is directly proportional to the chosen variable and the other independent of that variable. The independent portion can then perhaps be related to some other variable, and the process repeated.

It is evidently not enough to find a variable to which the cost can be related: the variable must be one likely to be practically useful. The investigation must therefore have close regard for the use to which the results will be put. This use will in many cases be either the formulation or criticism of tariff figures or else the guidance of action, *e.g.*, in finding which is the most economical way of carrying out some

* This example was suggested by W. A. Carne.

distribution function. An example of a variable which though technically appropriate yet may have to be disregarded, is that of *distance* in connection with distribution costs. If the purpose of the investigation is to provide values for a tariff, and if the particular form of tariff (as is usual) contains no element varying with distance, there is little use in employing distance as a variable in the cost analysis, however attractive it may be otherwise.

Choice of Variable.—It has been seen that the purpose of the exercise is to relate the costs to the service rendered by the undertaking. This service can be assessed in a number of different ways, much of it being measurable in engineering and numerical quantities such as the kWh and the kW or kVA sent out, the number of consumers served, the area covered, or the capacity of the apparatus electrified. Combinations of these variables, such as kW-miles or load densities (consumers per mile, units per consumer, etc.), are also possible indices. Other attributes of the service have a more intangible nature and could only be assessed in terms of the satisfaction obtained from the uses to which the electricity is put.*

In practice there will probably be quite a number of variables with respect to which a uniform relationship can be established, although with some of them the correspondence may be purely fortuitous. When an undertaking expands, whilst keeping its general proportions unaltered, the kWh, kW, number of consumers, number of staff, total revenue, etc., all expand together. A statistical investigation may then indicate a relationship between quantities which have no direct connection whatever. Costs such as taxes and administration expenses may both be thus related, say, to total revenue whereas in fact the former is associated only with a particular element of the revenue (the profit) and the latter is more a function of the number of staff or the number of consumers.

With so many possibilities an arbitrary choice may be made, at least pending more detailed investigation. One way of making the selection is to consider how the cost formula is to be used and therefore how many, and which, variables can conveniently be taken account of. If the cost figures are used in a tariff framework, the position is broadly as follows :—

A simple flat-rate tariff takes account of one variable only, namely kWh.

A block tariff can take account of two variables, *e.g.*, kWh and

* Some easily measurable quantities affecting the value of the service may yet be too insignificant to justify their inclusion in a cost formula. For example, the time-keeping services to consumers with synchronous clocks could be assessed, and the corresponding cost (that of keeping the frequency hovering around 50) could be related thereto.

consumers.* (With more than two blocks, the element of load density is also catered for.) A two-part or variable-block tariff which is "stepped" (*i.e.*, includes a non-proportional element in the first part) can take account of three variables, *e.g.*, kWh, kW and consumers.

As an example of the last-named, consider the tariff £7 per annum for the first kW or the first 1,000 sq. ft. of floor area and £5 per annum per kW or per 1,000 sq. ft. thereafter, plus 1*d.* per kWh. Since there is one account per consumer, this tariff can be re-stated as £2 p.a. per consumer plus £5 p.a. per kW or per 1,000 sq. ft., plus 1*d.* per kWh.

Law of Variation: kWh and kW.—The next stage is the quantitative one, namely, to establish the form and numerical value of the relationship. Actually, the form may well be of a somewhat complicated nature but in application it will usually be highly inconvenient to take account of anything but a linear relation. The curve must then (as in Fig. 23) be approximated to by a straight line, representing proportionality plus an unrelated constant.

Figures for the relationship should as far as possible be obtained statistically, *e.g.*, by plotting costs against kW or kVA† of demand, obtained either at different times (corrected for changes in the price-level) or for different but comparable areas. In many cases, however, statistical evaluation will be impossible and discrimination must be employed. Taking each cost, and each variable, in turn the investigator must ask himself: If (*a*) the units (*b*) the kilowatts, etc., were to double, other things remaining the same meanwhile, by what percentage would the cost increase? This is the figure which must be inserted in the table as indicated on p. 129, and if a number of experienced costing engineers could pool their discriminations a standard set of percentages might be evolved. Once these figures were agreed, cost relating would become a relatively routine matter, and the figures for the cost formula could be supplied from year to year by the cost accountant.

In the absence of a recognised set of percentages, the procedure in relating any given cost will be to consider each variable in turn, starting with those whose relationship is clearest. The costs proportion to kWh are generally the easiest to identify, and, having sifted them out, the remainder can be tested for demand relationship.

* Consumer costs can also be covered by the addition of a service charge, or a meter rent, or by the insistence on a minimum annual charge, and this can be done under any type of tariff, even the simple flat-rate. For example, a flat-rate tariff can include a meter rent of say 4*s.* 6*d.* a quarter, and assuming that 2*s.* a quarter covers the actual metering expenses, this leaves 10*s.* a year as a contribution towards other consumer costs. A minimum or service charge is a franker and probably less-unpopular device to the same end.

† Usually kW will be found best for generation costs and kVA for transmission and distribution costs. But the latter can often be treated on a kW basis also, on the consumption of a uniform power factor.

Assuming the form shown in Fig. 23, this means that these costs can be split into two components : one directly proportional to the demand (kW or kVA), and the other constant or basic, *i.e.*, invariable with demand—sometimes called the "zero capacity" cost. This latter, measured by the height at which the straight line cuts the *Y* axis (400 in Fig. 23), represents *inter alia* the cost of having a consumer on the books and connected to the mains even if he uses neither energy nor power. (The connection must be imagined to consist of a cable or line of infinitely fine cross-section.) The next step is to find some variable to which this basic portion can be related.

Other Variables : Consumers : Unrelated Cost.—Some light can be thrown on the problem as follows :—Suppose that the cost of supply (excluding kWh costs) in a particular area is related to kVA of maximum demand in the manner shown in Fig. 23, and approximately represented to some scale or other by the formula £(400 + 0·6 kVA). The supply to a second and similar area would cost the same, and if the two areas were brought under a single ownership without any change in operation the cost to the owner would be £(800 + 0·6 kVA). The constant portion is now double, and this is attributable to a doubling of the territory, not a doubling of the load.

It is clear that for a scientific relating of distribution costs it would be essential to employ a variable involving distance. Where this is not feasible it may be argued in respect of a cost associated with area or territory that, since the only reason for covering territory is to reach consumers who are spread over it, the constant in the above formula may legitimately be related to the consumers served.

As one proceeds down the distribution system an increasing number of the costs are seen to be directly related to the number of consumers. They will include the capital charges on the non-proportional element in the service connection, and the metering, billing and showroom expenses. Less obviously they may include sundry constant or residual items as described above. This fact, coupled with the ease with which a consumer charge can be embodied in a tariff or a cost-formula, suggests that after kWh and kVA the third most convenient variable will be the number of consumers.

In order to evaluate the figures, the whole of the "unrelated" cost left over from the kVA investigation can be re-plotted in the manner of Fig. 23. The base scale will now be the number of consumers, and the nearest straight line to the curve will express a directly proportional cost (£ per annum per consumer) and a basic unrelated cost.

When everything possible has been done to sift out the costs proportional to the above three variables, there will doubtless be a residue not varying with any of them, or at least for which no direct proportionality can be found. On the theory of marginal costing described in Chapter III, these should be covered in the tariff on a

use-value basis. They include the "common" costs (*i.e.*, those which are "escapable" but not "divisible" amongst the consumers). They also include such irrecoverable items as the initial expenses involved in giving a supply *per se* and expenditure on air-raid precautions which are not costs in the economic sense but should if possible be covered by the tariff.

The following is intended to show how the table of percentages might look. The figures are purely illustrative :—

Revenue Costs. l.v. Distribution.	Percentage Proportional to:			
	Consumption (kWh).	Demand (kVA).	Consumers (No of)	Residue.
Salaries and Office Wages.	15	25	—	60
Operating Sub-stations	15	25	—	60
Repairs and Maintenance, Sub-stations	10	50	—	40
Repairs and Maintenance, Mains	—	30	70	—
House Services : Repairs and Maint. and Testing Meters, Installations	—	—	100	—
Testing and Transport	15	25	—	60

On the basis of some such table, the actual expenditures can be multiplied by the appropriate percentages in order to give the costs related to each of the variables.

Example of Cost Relating : Generation.—It was seen in Chapter IV that generation costs are in two groups—those involved in owning and housing the plant, and those involved in operating it. The former (comprising capital charges, rents, insurances, etc). are dependent on the first cost of the installation and are therefore directly and wholly related to the maximum demand on the station. The latter, namely, the working expenses of generation, are related partly to demand and partly (the greater part, including most of the fuel) to consumption. The best known method of dividing these working expenses into fixed and running components is that which was employed in determining the cost of production at selected stations under the 1926 Act and described on p. 70.

A more elaborate formula was developed by a technical sub-committee of the Incorporated Association of Electric Power Companies. Instead of being based on a single mean load factor for the station, this was based on a composite figure taking into account the amount of plant actually in operation and the number of hours during which it was run.

Cost Formula including Distribution.—An extension of the above to include distribution costs encounters various difficulties. Generation is a concentrated process analogous to factory production. Materials

and labour are applied to machinery, and as a consequence a uniform assessable product flows out of the building. All the costs can be satisfactorily related to two variables only—consumption and maximum demand—both easily measured and free from ambiguity. (A third quantity, power factor, complicates the demand variable on the electrical plant but is usually sufficiently constant to be allowed for in the two-part formula.)

Distribution, on the other hand, is a dispersive process operating over a wide and varying area. Its " end product " is a service to the consumer. It is less uniform in character than generation and less easy to measure. Costs are incurred which require to be related to additional variables (*e.g.*, distance and number of consumers) and the distribution system must be sectionalised, not only because separate costs are involved but because the maximum demands may occur at different times.

A final difficulty affecting both generation- and distribution-cost allocation is that, as will be seen in the next chapter, the whole conception of maximum demand is liable to become upset by uncertainty as to time.

The following is an example of a possible cost-formula designed to represent the cost of supplying small industrial consumers from a low-voltage network. It is inserted merely to show what such a formula might look like, and the numerical values have no particular significance. (Formulæ of this character have been given by Messrs. Woodward and Carne* and revised by J. A. Sumner.†)

Consumer Charge—£2 10*s*. per annum.

Demand Charge—£5 10*s*. per annum per KW of effective demand on the high-voltage system and bulk-supply point, plus £4 10*s*. per annum per kVA of effective demand on the low-voltage network.

Running Charge—0·45*d*. per kWh with a fuel-cost adjustment.

In order to see how the above might work out in a given case, consider a small-power user on the low-voltage network, and assume he has a diversity of 1·4 relative to the low-voltage peak and 1·6 relative to the high-voltage peak. (These are similar to the assumptions made in a previous case.) Also, that the power factor is 0·75. The annual charge then becomes $\frac{5{\cdot}5}{1{\cdot}6} + \frac{4{\cdot}5}{1{\cdot}4 \times 0{\cdot}75} = 7{\cdot}72$ per kW plus 2·5. The correct industrial tariff to cover the costs on these assumptions would therefore be a *per capita* charge of £2 10*s*. per annum per consumer plus £7 15*s*. per annum per kW of metered demand, with a running charge of 0·45*d*. per kWh.

* *Journal I.E.E.*, 1932, **71**, p. 872.
Journal I.E.E., 1937, **81**, p. 457.

A more acceptable way of securing the consumer costs would be to levy a fixed charge of £8 per kW for the first 10 kW of demand and £7 15*s*. per kW for all demand beyond this. This would bring in a fixed sum of £2 10*s*. per annum per head from all but the very small consumers, and in the aggregate would satisfy very closely the cost-formula. The weakness of the above even from the costs point of view (*i.e.*, the necessity for " bias ") and its unsuitability as a tariff for consumers has been discussed in an earlier chapter.

CHAPTER VII

THE ALLOCATION OF DEMAND COSTS

Statement of Problem.—The importance of the work of this chapter arises from two facts. Firstly, that only about a quarter of the total cost of supplying the average low-tension consumer is proportional to his energy consumption, whilst more than half depends on the demand which he makes on the system. Secondly, that with the vast majority of consumers only this energy is metered, and in almost no case is any measurement made of the contribution to system demand. Any claim that methods of charging represent costs of supply must therefore reckon with the subject-matter now to be discussed.

It was seen in the last chapter that the simplest division of supply costs that can be made with any approach to accuracy is as follows :—

(*a*) *Running Cost*—related to energy.

Standing Cost { (*b*) Demand-related.* (*c*) Consumer-related.* (*d*) Residue.*

Furthermore, on the analogy of Fig. 23, it is possible to treat each of the components, (*a*) to (*c*), as directly *proportional* to its related variable over a limited range of variation.†

The present chapter deals with item (*b*). It aims at seeing how a cost proportional to *system* demand may be related to the demand (and possibly consumption) of the individual consumer or group of consumers. This element is the most difficult to deal with because of the uncertainty of the basis. One can easily allocate item (*a*) on a basis of kWh supplied, and (*c*) on a basis of number of consumers served. With item (*d*), since this is not obviously related to any simple quantity, or at least to any quantity that is to be employed in the cost-formula, this cost must be distributed more or less arbitrarily. But item (*b*) is difficult because "demand," more specifically maximum demand, is not a simple numerical quantity like kWh or

* The proportions of these cost elements will depend on the type of plant and process, and the position in the system : thus item (*a*) is chiefly incurred in generation, item (*c*) in distribution. For convenience, the total demand-related cost incurred throughout the year is considered as a single annual expenditure. There may, however, be monthly variations, due, for instance, to the shutting down of a power station or part of a station for a few months in the summer.

† [illegible] ...ards terminology, the looser term "demand-related" is frequently em... ...han "demand-proportional" because although the costs are presumed ...fted out so as to be directly proportional to the maximum demand ...here are other M.D's involved to which the relationship may be less

consumers in which $2 \times 2 = 4$. Maximum demand in fact is more of a vector than a scalar quantity.

It was seen in an earlier chapter that owing to diversity the collective M.D. on a supply system is usually much lower than the sum of the individual M.D.'s of the component loads involved. Moreover, the ratio between component M.D. and contribution to system M.D. varies considerably, and hence a simple numerical averaging-out process is admissible only among components of strictly equivalent characteristics : in practice, consumers of a certain description or class. The allocation between different classes of consumers requires something more complex.

In the early days of public electricity supply, the capacity of a generating station was determined by the demand made by the individual lighting-consumers after dusk, when the collective demand reached its peak. Originating from Hopkinson's realisation of the dualism of the cost of electricity supply (standing and running costs), the "peak-responsibility" method allocated demand-related cost according to the contribution of consumers to that peak. This worked satisfactorily until, owing to the appearance of peaks in the morning, or at midday, the concept "peak" became indeterminate.

Many different methods of allocation have been developed in the past, particularly in America and on the Continent, and a Report of the Electrical Research Association made a comprehensive survey of these methods.* In order to keep the problem within bounds, this chapter deals specifically with the allocation to groups or classes of load, not the allocation to individual consumers. The same principles can, however, be extended to the latter, as will be seen in a later chapter. These group classes will be referred to as the *component* loads which together make up the total *system* load.

Methods Based on Demand Only.—These are represented by two extremes ("Peak-responsibility" and "M.D.") and by an intermediate specimen ("Punga"). Each of these extremes is liable, on occasions, to give absurd results, and the intermediate one will then be only one degree less bad (on the principle that the mean of two wrongs makes a right).

The *Peak-responsibility Method* allocates annual demand-related cost in proportion to the demand of each class of consumers at the time of the annual collective M.D. on the supply system concerned. It can be illustrated by its treatment of three extreme cases : to a component making no demand whatever at the critical time, no share in demand-related cost is debited ("off-peak" loads) ; if the M.D. of a component coincides with the collective M.D. (system peak), the allocation is in direct proportion to the component M.D. ; and in the case of a continuous demand of constant magnitude (100-per-cent.-load-factor

* K/T 106. Critical Resumé.

load), which monopolises an equivalent amount of plant capacity whatever the time of collective M.D., the allocation is likewise in direct proportion to the component M.D.*

In supply systems with developing domestic loads the incidence of the annual M.D. (absolute peak) shows a tendency to shift from the evening to the morning or midday, even to the midday of a Sunday. If the orthodox peak-responsibility method were to be retained in these altered circumstances, a morning M.D. would result in commercial lighting becoming virtually an "off-peak load," and a predominating Sunday peak would extend this to almost all industrial and commercial demands.

Another anomaly inherent in this method is that when there are two peaks of almost equal magnitudes at different times, a very small load change may swing the absolute peak from one time to the other and thus produce an entirely disproportionate change in the cost allocation. In fact, the whole basis of the peak-responsibility theory is that the system has a single stable peak, and that the component whose costs are to be allocated is so small in proportion that it makes no difference to the position of the system peak.

At the other extreme, the allocation can be based on the M.D. of the component load, irrespective of when this M.D. occurs, or of its effect on the collective load. This *Demand Method* recognizes no off-peak loads, and gives credit for collective diversity even to loads that are inherently incapable of diversity, *viz.*, the 100 per cent. load-factor loads. It is simple to apply, since all the data required are the component M.D.'s.

It is easy to demolish the case for either of these bases by merely considering extreme examples. Thus the "demand" basis means allocating the same demand charges at 4 a.m. in June as at 4 p.m. in January. Since it is hardly conceivable that an increment of load at the former time can cost the undertaking anything whatever in demand-related expenses, such a method is too flagrantly a misrepresentation of cost except when allocating to individual loads (within a class) whose peaks all occur at the same time.†

The "peak-responsibility" basis looks equally foolish whenever there are approximating peaks at widely different times of day battling for supremacy. (For example, the aggregate grid load curve changed over during the war from a late-afternoon to an

* In the two latter cases the allocation per kVA or kW of component M.D. is a maximum; in all other cases it is lower in the ratio of "peak-responsibility" demand to maximum demand of the component load.

† Thus within a group of, say, lighting loads of different load factors but all having their peaks at about the same time, the demand method gives a correct allocation to the separate loads. It then becomes a sort of localised peak-responsibility method, only distinguishable from it in so far as the group peak does not coincide with the system peak. In general, the chief claim that can be made for the demand method is its convenience of application—it is cost-accurate only under the condition when it merges into the peak-responsibility method.

early-morning peak. Subsequently, the peak has sometimes occurred in late morning, and many people expect that it will again revert to late afternoon.) One has only to visualise the situation in which the Highways Department of a local authority purchased its street-lighting supplies from the Electricity Department " at cost." One year, the undertaking's peak was in the morning and the street lighting was " off peak " : the next year the peak swung to late afternoon and the street-lighting load came fully on to it. With the normal proportions of demand and running costs it might well be that the calculated " cost " doubled in a single year, and however satisfactory the calculation might appear to the economics pundits it would be very difficult to explain to the Highways Committee.

Mitigation of the shortcomings of these two extreme methods has been sought by combining them. *Punga* suggested a method of allocation based on the sum of the two quantities, component M.D. and component demand at the time of system M.D. While fluctuations due to changing times of system M.D. are this reduced in magnitude, off-peak loads receive a certain allocation. Moreover, the share allocated to a 100 per cent.-load-factor load tends to be lower than the apparently correct value resulting from consideration of plant capacity monopolised by such a load.

Methods Based on "Used" and "Unused" Capacity.—The *Lauriol Method* can be illustrated by a much simplified example. Suppose that the cost to be allocated relates to three equal units of a power station and is incurred by three non-overlapping consumers, *G*, *H* and *J* (Fig. 24, all days being presumed the same). Set 1 is used by all three consumers and its annual standing cost is shared equally between them. Set 2 is the joint responsibility of consumers *H* and *J*, whilst set 3 is used only by *H*, who is therefore allocated its whole annual cost. If each set costs £6,000 per annum, the allocation is as follows :—The standing cost of consuming between 0 h and 8 h is one-third of set 1, so *G* incurs £2,000 : the cost between 16 h and 24 h is one-third of set 1 plus one-half of set 2, so *J* incurs £2,000 + £3,000 = £5,000 : the cost in the 8 h-to-16 h period is one-third of set 1 plus one-half of set 2 plus the whole of set 3, so *H* incurs £2,000 + £3,000 + £6,000 = £11,000. Each hour of the day can, therefore, be associated with a particular rate per kW of demand.

If, now, *G*, *H*, and *J* are not separate consumers but separate blocks of load, each contributed to by several consumer groups or components, the same principle can be applied. A component load overlapping one or more of these periods may be deemed to incur costs at these hourly rates and according to the kW-demand in each hour.

The resulting cost allocation involves dividing the daily system load curve into, say, 24 vertical columns. Demand at any particular hour has a cost-allocation rate depending on the height of the load curve

at that hour ; hence demand-related cost can be allocated as an *energy* rate varying with the time of day. The appropriate rate can be found by rearranging the 24 hourly columns of the load curve in their order of magnitude, thus giving a load-duration curve. Dividing this load-duration curve into a number of horizontal slices corresponding to the sections of plant supposed to be employed, it will be clear that the cost of any plant section has to be borne by the corresponding load duration. Thus, set 3 in Fig. 24 has only 4 hours. of use and its cost per hour is correspondingly high, giving a high allocation to all demands during the 8-to-12 period.

If all days were the same the entire allocation could be based on one set of load curves, and the procedure would be comparatively

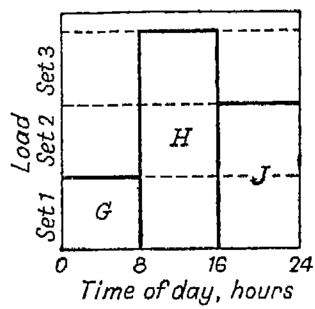


FIG. 24—Simplified Illustration of Lauriol Method.

simple. However, due to the weekly cycle of working-days and Sundays and the seasonal variations in hours of daylight and temperature, the Lauriol principle cannot be applied generally on the basis of the load conditions obtaining on a single day. Some approximation can be achieved by choosing a number of typical days.

While the methods referred to in the previous section relate the annual standing cost in respect of a system to the latter's capacity in terms of kVA or kW as determined by the M.D. on it, the Lauriol principle regards capacity as a product of power and time, the potential annual capacity of a system corresponding to M.D. $\times$ 8,760 hrs. This concept was subsequently further developed by the introduction of the terms " used capacity " and " unused capacity," the former represented by the area under the load curve (particularly the annual load-duration curve), and the latter by the remaining area.

Under the *Phantom-Consumer Method*, the unused capacity in respect of a system is supposed to be taken up by a phantom consumer, thus bringing the system load factor to 100 per cent. Then, it is argued, the only correct way of allocating cost is on the basis of kWh

consumption alone, *i.e.*, in proportion to the amounts of used capacity attributable to the individual parties concerned. As, however, the phantom consumer is unable to pay, its share (which is equivalent to the unused capacity of the system concerned) is redistributed to those parties who are deemed responsible for the existence of any unused capacity, *viz.*, those who, at the time of the annual system M.D., exceed their annual mean demand. The responsibility of such a party is assumed to be proportional to the ratio between its demand at the time of system M.D. and corresponding annual mean demand.

Since the Phantom-Consumer method incorporates—although in a modified form—the original peak-responsibility concepts, its operation is subject to the same limitations as the peak-responsibility method. Mitigation in this respect is achieved by the *Complete-Peak Method*, which extends the concept " peak " from the absolute annual peak to the whole portion of the system load which, on the day of annual M.D., exceeds the annual mean demand. While under the Phantom-Consumer method any component load that at the time of the system M.D. does not exceed its mean demand is treated like an off-peak load, the Complete-Peak method does this in respect of such component loads as do not exceed their mean demands during the period of the complete peak.

So long as the load on a particular system can be relied upon to reach its annual M.D. on a weekday—irrespective of whether in the morning, around midday, or in the evening—the Complete-Peak method is practicable. However, in distribution systems with a predominating domestic load, the complete peak may have to be associated with a Sunday, thus giving industrial and commercial loads the character of off-peak loads.

Consumption and Demand Method.—Peak-responsibility in any form is discarded by *Greene's Consumption and Demand Method*, which allocates some of the cost to M.D. (irrespective of when it occurs) and the remainder to energy consumption. The primary equation of Greene's method is $C = Kx + Dy$ when C is the total annual demand-related cost in respect of the system, K is the total annual consumption, and D is the sum of the annual M.D.'s of the component loads concerned.

This, as it stands, is a pure assumption : x and y are arbitrary coefficients which can be given any relative values provided they enable the equation to add up to the correct total cost. But a second assumption is made, namely, that a 100 per cent. load-factor component should carry in full its proportion of demand-related cost, *i.e.*, the annual cost of the system capacity which it monopolises. If P is the annual collective M.D., *i.e.*, the M.D. of the system (so that D/P is the diversity of the group of components) the cost per system kilowatt will be C/P. In the case of a 1-kW load at 100 per cent. load

factor the annual consumption will be 8,760 kWh, and the cost on the above equation should be $8{,}670x + y$. Equating this to the cost per system kilowatt (C/P) gives a second equation, and combining this with the first gives definite values to x and y for any given cost figures.

Having evaluated the coefficients x and y, the primary equation can be re-applied to each component load in turn. Allocation to a component load with a consumption k and M.D. d is given by $kx + dy$. (Since x is a cost per kWh and y a cost per kVA or per kW, this is in form the same as a two-part tariff consisting of a unit charge plus

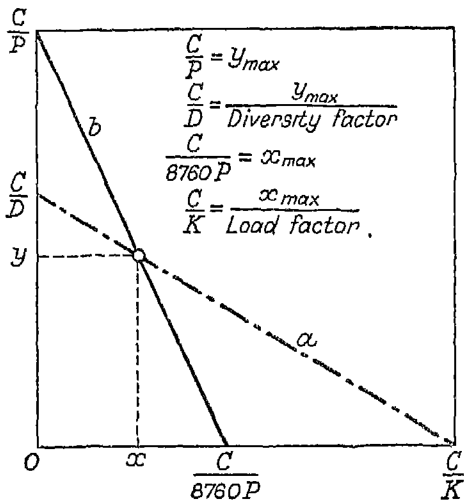


FIG. 25.—Greene's Consumption and Demand Method.

Equation a: $C = Kx + Dy$, i.e. $y = \frac{C}{D} - \frac{K}{D}x$

Equation b: $\frac{C}{P} = 8760x + y$, i.e. $y = \frac{C}{P} - 8760x$

a charge based on recorded M.D.) The total demand-related cost of the system will be the sum of the component allocations or

$$C = \Sigma(kx + dy) = (\Sigma k)x + (\Sigma d)y = Kx + Dy$$

A graphical representation (based, for simplicity, on unity power factor) will show the import of these two equations. In Fig. 25 the chain-dotted line depicts the primary equation (a), and the full line the equation (b). Taking the latter first, its intersections of the two axes (when $y = o$ and when $x = o$) will equal the maximum values which x and y can have for any given values of C and P. (By definition C is proportional to P, since C is the demand-proportional cost.) The chain-dotted line (a) is determined in a similar manner from the values of C, K and D. The actual values of x and y are those given by the intersection of lines (a) and (b).

It will be noted that the higher the diversity factor D/P the lower, relatively, will be line (*a*) and the coefficient y; whilst the higher the system load factor $K/8{,}760P$ the steeper will be line (*a*) and the higher the coefficient x. This illustrates the reciprocal relationship of load and diversity factors, and demonstrates how the allocations made are influenced by the likelihood of a component load contributing to the collective M.D.*

A characteristic feature of all the methods referred to above is that there is a certain minimum value for the allocation any load can receive. This is equal to its kilowatt-hour consumption multiplied by the quotient: system-cost divided by system-capacity in kWh (*i.e.*, M.D. × 8,760, corresponding to operation at 100 per cent load factor). It follows that an off-peak and a 100 per cent. load-factor load of equal annual consumptions are given the same allocation.

Summing up the foregoing portions, it would appear that most of the methods of allocation that have been described make use of some arbitrary assumption or formula in order to achieve a division that appears reasonable and consistent. They are in the nature of expedients to be judged chiefly by their convenience in application, although some check is given by their treatment of extreme cases. Excepting the peak-responsibility method, they hardly attempt to discover what costs the component demands are actually incurring. The following paragraphs endeavour to re-examine the problem from a strictly cost aspect and then to see whether a working compromise can be based on this cost examination.

True Cost Allocation.—As explained at the beginning, this chapter concerns only the demand-related portion of the standing cost, and this is assumed to be directly proportional to system M.D., and variable in either direction.‡

The first step is to distinguish between two rival bases of allocation, namely, "equity" and "economic cost." They both relate to the same problem, that of sharing round costs that have been incurred, but the view-points are somewhat different. The former regards the problem statically, whereas the latter presupposes mobility, and its criterion of cost is obtained by supposing small changes to occur.

* One should perhaps add *ceteris paribus* since, naturally, there cannot be universal correctness of results. A high-load-factor load may happen always to be off for a short while just when the collective M.D. occurs; and the M.D. of a low-load-factor load may just happen to coincide with the collective M.D. The theory is based on probabilities and assumes that the class of load in question has no special predilection for, or aversion to, the time of system peak.

‡ Since much plant, such as cables, has little or no recovery value, proportionality between demand and cost may appear to exist in the upward direction only. Fortunately, this difficulty is imaginary, since all supply undertakings are on a rising curve. Not only do extra demands cause increased cost, but reduced demands by one class save that same thing, since they are taken up by others. The postulated two-way mobility is, therefore, present, except for plant serving a single class only, and here the cost allocation presents no difficulty.

Endless confusion has been caused through not distinguishing between the two questions, "How ought I to distribute the expenses amongst the various classes of consumers?" and "What does it cost me to supply this class, *i.e.*, how are my expenses likely to be affected by changes therein?" To the former, many answers are possible: cost is relevant but need not be regarded as decisive. The latter should be capable of an exact answer, and that alone is what cost means. The distinction can best be seen in the extreme case of the expenses which do not vary with supply.

Thus the bottom rectangle in Fig. 23, on p. 125, represents a component of the standing cost which does not vary with the demand. If a certain group of expenses is the same however large or small the demand, it costs nothing (from this group) to provide extra supplies: the problem of how to distribute this group of expenses remains, but it is not a problem to which economic principles can be applied on a demand basis.

Considerations of equity or fairness are to some extent emotional and not purely intellectual and are, therefore, difficult to define, except negatively. Thus it may be said that many people's feelings of equity will be outraged by a method which, under any circumstances, makes no charge for plant used, even in a purely off-peak period; or which makes a very large change in allocation for a very small change in load.

When, however, one attempts a quantitative interpretation of the equity idea, the only criterion appears to be what costs might be incurred by the individual consumer classes if circumstances were different, *e.g.*, if each class were supplied by a private plant. This is really a "use-value" basis, and whilst it may be a consideration in the tariff, it is not an element in the cost to the suppliers.

At first sight, strict cost considerations would appear to lead directly to the peak-responsibility method in all its starkness.* Thus, if a

* Some authorities, such as Lauriol, deny that true cost considerations lead to the peak-responsibility method on the following grounds. Looking at the total of off-peak and the total of on-peak supply they say that the additional equipment required for on-peak supply is only the difference between the two. Therefore it is only this difference that can be regarded as the special cost of on-peak supply.

This is one of the most subtle fallacies in economics—the application of factors relating to totals where only factors relating to parts are relevant. In its earliest historical form we find philosophers comparing the total utility of water to society with the total utility of diamonds, and puzzled because the more valuable, water, has much the lower price. The solution is simple—we are concerned not with having water or no water, but only at the margin, comparing one gallon more or less of water (which is plentiful) with one diamond more or less (diamonds being scarce).

The principle that it is the margin and not the total that counts also disposes of methods based on the argument that on-peak users "hire" some of the plant whose cost they bear to off-peak users. (Such methods are described in the E.R.A. Critical Résumé already referred to.) On-peak users as a whole may be thought of as hiring in this way, but additional equipment put in for on-peak supply cannot be hired to off-peak users because there is already far more equipment available than off-peak consumers need. The whole cost of this additional equipment must therefore fall on the on-peak consumer.

system is just able to cope safely with the present load and another 1,000-kW load comes on, if this load occurs at the time of the annual absolute peak, extra plant must be purchased. If not, the existing installation will suffice and no extra standing cost is incurred. It seems a picture in black and white : the load either swells the peak or it does not. In the former case it costs the undertaking the full price of the extra plant, in the latter case nothing.

The position can, however, be better understood by reference to Fig. 26. If the peak-day load follows some curve A B C D, the cost† is a function of the height B. At any other point C or D, small load increases would appear to entail no cost since there is plant lying idle whose employment would not add anything to the fixed expenses. "But," it will be objected, "if at such times I make no charge in

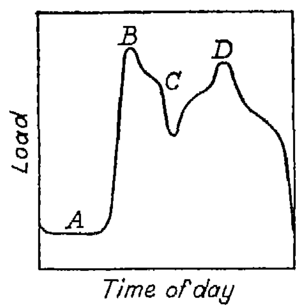


FIG. 26.—Variation of "Peak Potentiality".

respect of demand-related cost, demand may then increase at such a rate that it will form a peak, and I shall incur expenses in meeting it." This is exactly the point—there *is* a demand-related cost at C and D because there is a potentiality of an absolute peak at these times, and the true economic cost must be measured by this potentiality.

The cost of a kW at any time is then the cost liable to fall on the undertaking if an extra kW is taken, and is, therefore, related to the shape of the aggregate load curve at the time. This cost, since it is governed by the likelihood of having to install fresh plant, is related to the height of the curve at A, B, C, D, etc., and to the load responses at these points. It is a *certainty* at B (fresh plant needed immediately), it is a potentiality at D, less likely at C, and so on.

The aim of the engineer has always been to even out his annual load curves as much as possible and to secure as near an approximation to 100 per cent. load factor as circumstances will permit. He seeks to do this by offering such special tariff charges as will encourage

† For the sake of brevity, the term "cost" is here used to denote the demand-related (or more strictly the demand-proportional) component of the standing cost.

off-peak demand and fill up the valleys in the curves. This practice, which is as old as electricity supply, is securely founded on cost considerations. Moreover, the evaluation of these costs cannot be divorced from a study of the consumption/price relationship, since cost means cost liability, which depends on the consumers' response to price.

We have here the economist's "joint-cost" situation; for just as a sheep produces wool and mutton, or a cotton plant produces lint and seed, in a definite ratio, so a kW installed to cope with the annual absolute peak automatically makes a kW available during the rest of the 24 hours and 365 days.

Electricity supply engineers do not need to use the economist's examples because they have a perfectly good example in a sister service, that of gas supply. The destructive distillation of coal yields two major products, gas and coke, and under given manufacturing conditions they are produced in an approximately fixed ratio. If the coke is unsaleable, it is obvious that all the manufacturing cost will have to be debited to the gas. If, on the other hand (as has sometimes happened in the past in the manufacture of hard or metallurgical coke) the excess gas is of no value and is blown to atmosphere, all the cost must be debited to the coke. Between these two extremes of cost allocation the production expenses must be shared between the two, and the decision as to what proportion of the expenses is attributable to the gas and what to the coke is normally based on the prices at which consumers will take up the whole of both commodities.

Similarly in electricity supply, at any point short of the peak, the liability to plant expenses (and therefore the allocation of demand-related cost) can be measured only by the prices at which consumers are willing to take up the slack, *i.e.*, the figures which would tend to keep the load up to peak level

The importance of this price differentiation can be illustrated by another approach. The question which the commercial manager is asking is "How do my expenses vary with supplies?" Suppose, for example, that new machinery installed to meet the annual absolute peak involves an annual cost of £U. If there is no prospect of using this to increase supplies at some other time, the peak cost is £U. But suppose this makes it possible to take on, outside the absolute peak, some large new low-priced contract which could not be accepted before because there was not enough plant, and because by itself it could not pay £U—if this contract yields £V, then the peak cost is only £$(U - V)$.

Cost allocation, therefore, in its ideal should be based on a series of cost values, differing both as to time of day and as to day of the year, which, if translated into tariff charges, would bring the load factor as near to 100 per cent. as possible. The main yard-stick for this allocation would be furnished by the elasticity of demand at different times.

Peak Responsibility Assessment.—The above differs from the orthodox peak-responsibility theory chiefly in the different meaning attached to the word "peak." It might, in fact, be termed the potential-peak-responsibility theory. The traditional conception of the peak, as a single period of, say, half-an-hour's duration when the demand attains its annual maximum is altogether inadequate. We have to regard as "cost liable" all that part of the day and of the year when demand might reach maximum levels. The essential distinction is not between peak and off-peak, but between "potential-peak" and hours which are unlikely, however low the charge in respect of demand-related cost, to attain peak levels. In considering how large a period of time to include in the potential peak we have to keep an eye on the future, and include large parts of the load curve which, though now fairly safe, may be expected to grow within the time to which present decisions have to be related.*

Another reason why the half-hour of absolute annual peak is not the sole criterion of plant cost lies in the necessity for overhaul. Generating plant in particular requires to be taken down periodically, and an extensive overhaul programme is arranged for the summer months, and may have to be extended through other months during which peaks are unlikely. Peak responsibility must then have regard not only to the absolute system capacity but to the capacity temporarily available under the overhaul programme. As an example, during 1948, load on the grid had to be shed on 38 occasions between April and October, although the load was far below the winter peak. There was therefore a definite indication of demand-related expenses on account of generating plant even in the summer.

The ideal method of allocation may thus be summed up as follows: There will be certain times when, no matter how small a charge is made (even zero) in respect of demand-related cost, or whatever likely changes occur due to other causes, some plant will still be lying idle.‡ This is the off-peak period, and there is then no liability for plant expenses, and no demand-related cost must be allocated. During all

* Price is only one of a number of things whose alterations may cause the load to rise to peak values. Economic demand is also affected by charges in income level, by technical improvements in the utilization of electricity, changes in consumers' habits, effect of propaganda, and general changes in the competitive position of electricity compared with other supplies. Moreover, load conditions are not static, and a portion of the load curve which now causes no anxiety may well do so within the lifetime of the plant whose cost is being allocated. But whatever these other causes and changes, at any one moment price will always have some effect on demand, and this response to price gives a clue to the allocation of demand-related cost.

‡ The question "What are the limits of the potential peak?" is equivalent to the question "During what period can I increase supplies without increasing expenses?" The answer is that this depends on how much supplies are increased, and this in turn depends on the charge that is made. No section of the load curve can be treated as off-potential-peak if the effect of making, at that period, no charge in respect of demand-related cost would be so to increase demand as to necessitate installing additional equipment.

other times, there is some potentiality of an absolute peak arising, and cost should be allocated according to this liability. Throughout this potential-peak period there is a graduated peak-responsibility or potentiality, measured by the price differentiation which would tend to produce a constant-height load curve.

This is the only formally correct basis. It allocates the cost to all consumers whose decisions to consume more or less are liable to affect the undertaking's expenses, and only to such consumers.

Basic Features of Improved Method.—In attempting to translate as much as possible of the economic cost theory into a workable formula the first thing is to disregard all demands occurring outside the potential-peak periods, no demand-related cost being allocated thereto. The next stage would be to divide the potential-peak periods into zones according to the varying degree of peak potentiality. It would lead to a time-of-day allocation of the kind used by Lauriol but with different values. This, however, is impracticable for several reasons, one being that it assumes a far more elastic situation than actually exists. The manager of a utility undertaking has not the freedom for experiment of, say, a greengrocer. He cannot charge what he likes nor change his prices from day to day. Even if he could, electricity demand is not elastic enough to respond freely to such changes in the way that the demand for fruit or vegetables can be made to cope with a glut or a shortage.

The second reason is that the required zoning could only be based on known or predictable cycles as, for instance, regular " on " and " off " periods of factories, periods of daylight and darkness, peaks and valleys in domestic cooking, etc. However, one of the main factors, the effect of cold weather, is generally unpredictable (except for a vague association with a number of calendar months) and is thus not related to hours and days but only to actual temperature.

It therefore appears expedient to treat all zones within a certain potential-peak period as having equal chances of forming an absolute peak. This is admittedly a considerable departure from the cost ideal, but it is possibly as near as a really simple method is likely to get.

Another pre-requisite of simplicity is that, in order to apply the method, it should not be necessary to evolve detailed load curves relative to the components concerned. The method should require only such data as are likely to be generally available. The first requirement, and the one most readily satisfied, is the annual consumption. The second requirement is the annual M.D., which is sometimes obtainable by M.D. metering, or (particularly in the case of classes of consumers) can be estimated from the annual consumption by using an assumed load factor.

On the lines of the foregoing paragraphs these data must then be narrowed down so as to refer only to the potential-peak periods. This

will not affect the value of the M.D. (which will generally occur during the potential-peak period), but it will necessitate a knowledge or estimate of how much of the total consumption occurs during these periods. From these two quantities, in conjunction with the aggregate annual duration of the potential-peak periods, may be derived the "period load factor."

Relevance of Consumption.—The use of consumption or load factor as an element in allocating demand costs is justified by the inverse relationship which exists between load factor and diversity factor (see Chapter V, pp. 104 to 109). What follows is on similar lines except that it refers to period load factor.

It is clear that a component load having a 100 per cent. period load factor, and whose load curve consists of a series of rectangles of constant height corresponding to each potential-peak period, will have an absolute certainty of contributing fully to the system M.D. With any component load having less than 100 per cent. period load factor there is only a chance, not a certainty, of its M.D. coinciding with the system M.D. Moreover, the lower the period load factor, the smaller this chance becomes.

To this end, a uniform allocation per kVA or kW of component M.D. may be fixed at an amount lower than the total demand-related cost divided by the sum of the component M.D.'s, and the balance may be allocated by way of kWh-consumption. As was pointed out in Chapter V, something of this sort has long been practised in connection with M.D. tariffs for power consumers, where it has been described as "biasing."

Reviewing the various methods summarized above, it will be seen that Greene's method embodies several of the features just mentioned. It is based on both M.D. and consumption, and provides a bias according to component load factor. It has, however, one fundamental weakness: it ignores the fact that in public electricity supply there are regular periods in which there exists no likelihood of a system M.D. occurring. By excluding all the periods in which no potentiality is deemed to exist, *i.e.*, confining the computation to assumed potential-peak periods, this weakness is overcome and a workable approximation to the economic cost allocation can be obtained.* But in so doing, the character of the method is fundamentally changed and it thus appears justifiable to give the resultant modification a new name.†

* In a refinement of the method, which is described at the end of the chapter, the potential-peak period is zoned to correspond to different degrees of potentiality.

† After the report from which this summary was extracted had been drafted it became known that at the 1943 conference of the Electricity Supply Engineers' Association of New South Wales a proposal had been made by Mr. A. J. Cresswell to modify Greene's method by disregarding any energy taken during the "off-peak" period of 8 h to 20 h all the year round. (*Proceedings of the 18th Annual Conference of the Electricity Supply Engineers' Association of N.S.W.*, p. 32.)

The E.R.A. Method.—The "E.R.A. Method for Allocating Demand-Related Cost," as the improved cost-formula has been called, is based on consumption and highest half-hour demand during potential-peak periods. Consumption is disregarded during the remaining periods (off-potential-peak), these being defined as periods in which no likelihood exists of a system M.D. ever arising, even if for consumption during such periods no charge were made in respect of demand-related cost.

For convenience of reference, the symbols and equations are repeated below. These are as before, except that dashes are added to those symbols which now refer to the restricted (potential-peak) period, and a symbol T takes the place of 8,760 hours.

Symbols : C Total annual demand-related cost in respect of a system.
P Annual collective M.D. on the system.
d' Highest demand of a component load during the potential-peak periods.
k' Annual consumption of a component load during the potential-peak periods.
T Aggregate annual duration of the potential-peak periods.

Allocation to a component load : $k'x + d'y$.

Equation for determining x and y :

$$C = (\Sigma k')x + (\Sigma d')y \qquad . \quad . \quad . \quad (a')$$

$$\frac{C}{P} = Tx + y \qquad . \quad . \quad . \quad . \quad (b')$$

Assessment of Potential-Peak Period.—The simplest and most obvious way of deciding what period to exclude would be by reference to the hours of inactivity of the bulk of the population—say from 23 h to 7 h all the year round. The particular hours and seasons must, however, depend on the character and locality of the costs to be allocated. Whilst the above periods may be quite safe for generation costs, it is conceivable that a city-distribution network in which no demand-related cost were allocated to the night hours might run the risk of developing an absolute peak due to thermal-storage heating. On the other hand, there would here be a continuous off-potential-peak period throughout the summer.

Usually this assessment could with perfect safety and with little greater complication be narrowed by taking (as peak-liable) from 7 h to 23 h of each day in the two winter quarters only. (See Fig. 27, areas shaded "liberal.")* Then T has a value of 2,910 hours, and

* This must, however, be qualified as regards power-station plant owing to the necessity for periodic overhauls, complicated by the post-war gap between installed capacity and maximum demand. At present the overhaul programme has to be so planned that the margin between available generation capacity and likely demand is hardly any greater in summer than it is in winter. It is impossible to say whether this will prove to be a continuing situation.

the determination of the values k' becomes relatively simple, since the consumption during the remaining night hours is generally low and easily estimated. The value k' for each component load then consists of the energy sold during the two winter quarters, less such a portion as is estimated to have been taken during the nightly periods of from 23 h to 7 h.

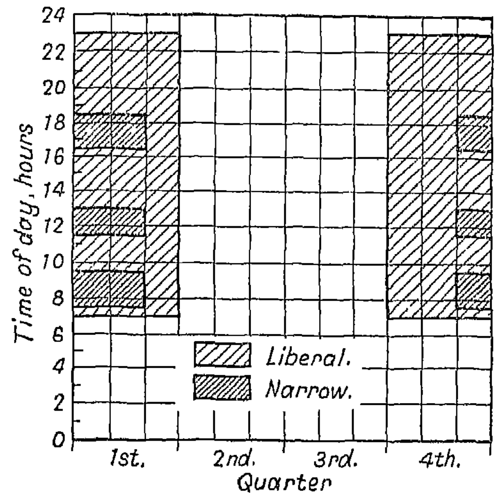


Fig. 27.—Potential-Peak Periods.

In many cases (particularly for short-term decisions), a narrower assumption can be made for the potential-peak period, leading to a closer cost estimate. One such assumption, illustrated in Fig. 27 (area shaded " narrow "), is for the hours of 7.30–9.30, 11.30–13.00, and 16·30–18.30 of the months of December, January, and February. The disadvantage is that a comprehensive load analysis is then needed to determine what proportion of the consumption takes place during the period.

As regards the demand d', if the component load is not segregated so that an M.D. reading is available, this can usually be estimated without a detailed knowledge of the shapes of the load curves concerned by assuming a value for the annual load factor and working back from the known annual consumption. For component loads of given character, load factors do not vary greatly, and some assistance in this estimation is afforded by an E.R.A. publication.*

* *Technical Report* K/T 126.

Practical Illustration.—In order to illustrate the application of the E.R.A. method an undertaking of moderate size is taken, in which there is unlikely to be any specialised staff to carry out detailed load analyses. The potential peak period is taken at 7 h to 23 h of the two winter quarters, giving a value for T of 2,910 hours. The total system load is made up of five component loads, namely, domestic, commercial, public-lighting, industrial low-voltage and industrial high-voltage, for each of which is known the consumption quarter by quarter and the annual M.D. Separate demand-related costs would be required for the high-voltage system (including the generation or bulk-supply cost) and for the low-voltage system, since the industrial high-voltage component does not participate in the low-voltage costs. For simplicity of illustration the two sets of costs are aggregated in the following summary. Figures for the complete system are as follows (In order to simplify the equations, the consumption is expressed in megawatt-hours and the time in 1,000 hours) :—

Annual Consumption in Potential-Peak Period* . . .	8,340,000 kWh (Σk)
Sum of annual M.D.'s of component loads . . .	8,500 kVA (Σd)
Annual system M.D. . .	6,650 kVA (P)
Annual demand-related costs .	£42,800 (C)

Applying the two formulæ, namely—

$$C = (\Sigma k)x + (\Sigma d)y \text{ and } \frac{C}{P} = Tx + y$$

gives $y = 4{\cdot}33$ and $x = 0{\cdot}726$, *i.e.*, £4·33 per kVA and £0·726 per 1,000 kWh consumed during the potential-peak period.

The application of these two coefficients will give the allocation of demand-related costs to each of the component loads. For example, the data for the domestic load is as follows :—

Annual units sold in potential-peak period .	2,760,000 kWh (K)
Annual M.D.	2,200 kVA (D)

The component load allocation is given by $C = Kx \times Dy = 2{,}760 \times 0{\cdot}726 + 2{,}200 \times 4.33 =$ £2,000 + £9,530 = £11,530.

The above calculation must be regarded as a whole, namely, as a mechanism for allocating the expenses proportional to system demand amongst the component loads responsible for it. Although the mechanism operates on a dual basis, the two costs when calculated must be added together again to form the total demand-related cost of the component load. Moreover, whatever is done with the two

* Estimated as follows. By taking six typical daily load curves, one for each month of the two winter quarters, the proportion of kWh sent out between 23 h and 7 h averaged 10 per cent., and this figure was used for the complete system. Within this total an adjustment was made on account of the public-lighting load (in which the proportion was known and higher) and a figure of 8 per cent. was assumed for each of the other component loads.

components, they are only a device for conveying in conjunction the demand-related cost. They are not a tariff, or even part of one.

If further sub-division is desired, this total of £11,530 can be re-allocated amongst sub-groups making up the domestic-consumer group, *e.g.*, between consumers on the flat rates and those on the two-part tariff.* In both the division and the sub-division illustrated in this example, the range of load factors is somewhat limited and the diversity factors comparatively low. As a consequence, the resulting allocations do not differ very much from those obtaining under the straight demand method. (It will be noted that, both in the E.R.A. and in the demand method, since the factors used depend on the sum of individual M.D.'s, the results vary according to the degree of sub-division of the collective load.)

Advantages and Refinements of E.R.A. Method.—This method is not put forward in any way as a perfect solution to the allocation of demand-related costs but as a practical improvement on either of the two methods (demand and peak-responsibility) chiefly used in the past. It is also easier to apply with accuracy than the peak-responsibility method because the field of guesswork in the load analysis is more limited. This is particularly noticeable when separate costs for the high-voltage and the low-voltage systems have to be dealt with. The peak-responsibility method then requires a knowledge of the component loads at the times (not necessarily identical) of the peaks on the two systems.

The E.R.A. method is intended to be applied in situations where there are certain known periods during which a system peak is extremely unlikely to arise, and where during all other periods there is some degree of peak likelihood although the actual time of the absolute peak is unknown and unpredictable. Under these conditions of uncertainty (conditions which exist in many undertakings at the present day) demand-related costs are to some extent fortuitous and there can be no exact method of allocating them. The E.R.A. basis then offers the most scientific approximation.

On the other hand, if a load is expected to come fully on the absolute peak whatever its load factor (*e.g.*, when a radiant-heating load in a cold snap will cause the absolute peak to emerge) this principle would obviously be inapplicable: to such a load, the peak-responsibility allocation would apply. Again, if at some future date the conditions stabilised on a single outstanding and predictable yearly peak time, the E.R.A. method would virtually merge with the peak-responsibility method.

In its present form the method is imperfect because it assumes that all times within the potential-peak periods have an equal degree of

* An example of its allocation to the *individual consumers* in the group is given on Page 228.

peak potentiality. Only two sorts of period have been postulated, namely, off-potential-peak, during which an absolute peak is highly unlikely, and potential-peak, during which a uniform peak-likelihood is assumed. This simplification has been justified on the ground that it makes possible a workable scheme of cost allocation, superior to those hitherto employed.

That all times within the potential-peak periods are not, in fact, equally dangerous can be seen by considering the fringes of such periods. It might be a moot point whether in some given case the period should start at 7 hours or at 7.30 hours. One could infer from this uncertainty that the interval from 7 hours to 7.30 hours is not so dangerous regarding peak-potentiality as, say, from 8 hours to 8.30 hours. The method, to this extent, gives only an approximation to the economic cost allocation.

An obvious refinement of the E.R.A. method is, therefore, to zone the potential-peak periods so as to correspond with the different degrees of peak liability. As a first approximation, areas can be delimited within the potential-peak periods during which the absolute peak is most likely to occur. These, which may be termed "imminent-peak periods," can be given a higher allocation than that given to the remaining potential peak-periods. If, for example, the potential-peak periods were represented by the two large shaded areas in Fig. 27, the imminent-peak periods might be the smaller, closely-shaded ones. No formula can be suggested, but even though the proportions of the two allocations were quite arbitrary such a subdivision would be an obvious step nearer to the economic-cost principle.

In conclusion, the following table sums up the salient features of the three methods which have been principally discussed.

Method.	Data Required.	Correspondence with Cost.
Peak-responsibility	Detailed load curves .	Only when (*a*) there is a single peak at known time, or (*b*) when the load is known to come fully on the absolute system peak.*
Demand .	M.D. readings or estimates.	Only when (*c*) all component loads have absolute peaks at a similar time.
E.R.A. .	M.D. readings or estimates. Estimate of consumptions occurring outside potential-peak periods.	Nearer than the above methods when(*a*), (*b*), and (*c*) are absent.

* Thus, on a particular system the absolute peak may arise during cold snaps. Although the time is unknown, a load which is specifically associated with cold weather may be correctly allocated on the peak-responsibility method.

CHAPTER VIII

LOAD STUDIES

Introduction.—The study of load curves is of importance to cost estimates in a number of ways. It was seen in the last chapter that demand-related costs can be most accurately allocated by considering peak liability, and this can only be estimated from the load curve. In applying the cost-allocation method there described, estimates are required of the consumptions during specified periods of the year, and the estimates given at the end of the chapter are based on typical load curves. A study of load curves, and an analysis of load factors, is also important in estimating the effect of certain tariff devices such as off-peak rates and seasonal price variations.

On the other hand the analytic study of load curves, with a view to splitting them into their component parts and causes, is so large and distinct a subject that its treatment on an adequate scale would have been out of place in a book on electricity tariffs. This short chapter has however been included, outlining very briefly the main principles underlying the practice of load-curve analysis. The cognate subject of load-factor analysis is also shortly described.

Load Variations.—If a load curve extending over several years is examined, the difference between the loads at different times can be regarded as compounded of the following variations :—

(*a*) Yearly Variations.—*I.e.*, the broad secular change between each year and the next, due to the general long-term trend of the load (usually a growth).

(*b*) Seasonal Variations.—*E.g.*, between summer and winter.

(*c*) Day-of-Week Variations.—*E.g.*, between working-days and week-ends.

(*d*) Time-of-Day Variations.—*I.e.*, the changes from hour to hour of the day which are repeated with the clock times.

(*e*) Variations due to climatic fluctuations.

All but the last of these variations are fairly uniform and predictable, and all but the first are cyclical. Thus, changes dependent on the time of day, or day of year, repeat themselves with each day or each set of seasons : changes due to climatic fluctuations, whilst they have no regular cycle, will describe a closed loop of some kind and (other things being equal) must return to their starting-point when the temperature and other affecting conditions do the same.

The study could be approached from the other end by considering the load at some particular moment, say 4 p.m. on Tuesday, December 20th, 1949, and asking why the load would be different—

(*a*) at the same time on the corresponding day in December, 1950;

(*b*) at the same time on a Tuesday at the end of June ;

(*c*) at the same time on the previous Sunday ;

(*d*) at some other time on the same day.

(*e*) had the weather been colder or warmer.

(It will be obvious that these comparisons cannot be entirely clear-cut. Comparisons (*a*), (*c*) and (*d*) all assume that there is no change in respect of item (*e*), whilst comparison (*b*) must be adjusted for the six-months' growth according to the results of comparison (*a*).)

Load-Curve Analysis.—The method followed in analysing a load curve is to eliminate all but one of these variables at a time so that the effect of that one may be studied. Thus, if all the variations (*a*) to (*d*) can be neutralized or compensated for, the remaining load changes must be due to climate, and these will give a clue to the magnitude of the heating load. Similarly, if all the variables but (*c*) are eliminated, a comparison between Saturday and mid-week afternoons gives an indication of the industrial load, whilst fluctuations of the afternoon loads in different districts according to early-closing days can give useful information regarding the shop load.

The following notes outline the steps that might be taken in a particular case, *e.g.*, when it was desired to isolate the heating load. The notes are lettered to correspond to the above table.

(*a*) *Yearly*. If the undertaking has shown a steady annual increase in energy sold for several years in succession, and there has been little change in the load factor, it will probably be sufficient to compensate on this basis in any comparison extending over a period. Thus if the annual increase in sales is 12 per cent., a comparison between two days which are a year apart could be compensated by allowing 12 per cent. for the regular load growth. Similarly, a comparison between days separated by a month or more could be adjusted by adding 1 per cent. for each month's growth. It will be noted that the percentage employed should be the *average* recorded over several years. Any one year may show a different result : thus a cold year might well show a bigger increase than usual, but only part of this would be attributable to the variations under (*a*) and the rest to those under (*e*).

(*b*) *Seasonal*. These variations are avoided by comparing times which are either close together or else are in the same season in different years.

(*c*) *Weekly*. For many purposes, it will be sufficient to compare all

mid-week days together, Sundays with Sundays, and so on. For greater accuracy, it may be necessary only to compare the same days of the week and this must obviously be done where early-closing and other mid-week holidays are involved.

(*d*) *Hourly*. This is the largest and most rapid of the variations, and the most difficult to compensate for. Elimination can, however, be done by an ingenious device employed by P. Schiller and others.* The method is to plot the load from day to day *measured at a particular time each day*. Thus, whilst the load curve may show violent changes every morning as the populace gets to work, sufficient to mask the changes from all other causes, if the load at 7.30 a.m. is plotted for each weekday in turn it will be a relatively smooth curve whose variations can be attributed to other changes than merely those associated with time of day.

Having successively eliminated all the recurrent variations in the manner described, what is left can be attributed to climatic changes and used as an indication of the heating load. Thus if the fluctuations in such a " one-clock-time " load curve are plotted to a base of days, and if temperature fluctuations are similarly plotted (inverted), a correlation between the two can be established. With a knowledge of other climatic conditions besides temperature, departures from the temperature relationship can often be explained (*e.g.*, by a strong east wind).

Once the relationship between outdoor temperature and load variations has been established for that time of year and at each hour of the day, these fluctuations can be allowed for ; and the modified curve can then be re-examined for other variations. Changes due to a sudden dark cloud or high-level fog in the daytime, or a one-hour displacement occurring when clocks are changed due to " daylight saving ", can be used as a measure of the lighting load. Subtracting this, the remaining curve can be analysed for its other components.†

Difficulties of Application.—Although much can be done to analyse a load curve into its components by isolating the variables in this way, there are certain dangers to be guarded against. One cannot, for example, assume that the change associated with one variable when a second variable is held constant will be the same when that second variable has some other (constant) value. Thus it might be found that at a particular time of day (say the morning peak time) the load in January was greater by 100 MW for each 1° F. drop in outdoor temperature. One would not expect the same law to be followed in

* "An Analysis of the Load on a Modern Electricity-Supply System", by P. Schiller, *Journal I.E.E.*, 1944 ; 91, Part II, p. 433.

† When a dependent variable (in this case the load) is substantially affected by two independent variables (*e.g.* temperature and daylight), the method of multiple-regression analysis can be employed. This is illustrated in the *E.R.A. Report*, K/T 115.

July. In the same way, the day-of-week variations are not likely to be the same throughout the year.

Another difficulty arises from what might be called "reciprocal variations". Thus in analysing the load on a residential network, an obvious line of approach is to compare winter and summer loads with a view to separating the components which are fairly uniform (cooking, water heating, etc.) from those which are heavier in the winter (heating and lighting). But there may be on this network consumptions which are heavier in the summer, as when immersion heaters are used in conjunction with solid-fuel systems. The result of this on the total load curve would be to neutralize the effect of some of the winter load and produce a deceptively large "uniform" or "basic" component.

In spite of these and other difficulties, load-curve analysis on the above lines has been carried out with considerable success, and the results (within the fairly wide margin of error which must be anticipated) can be accepted with considerable confidence. The results of some of these analyses have already been made use of in the chapter on diversity, and the references given on pp. 89–93 should here be consulted.

Water-Heater Load Curve : Telecontrol Experiments.—There have been a number of tests in which the usages or demands of individual domestic appliances or groups have been actually measured either by recording-instruments on the premises or by taking pilot wires back to a convenient measuring centre. On any considerable scale such experiments are naturally expensive, whilst if on a small scale, or performed on particular consumers or specially equipped premises selected for observation purposes, it is difficult to be certain that the usages are representative. There are, however, situations in which it is possible to make tests on quite large groups at little expense, for example when the group is remote-controlled.

The experiments described below were carried out at the author's instigation on two distribution networks to which were connected some five hundred to seven hundred immersion-type water heaters. Each heater was controlled by a relay on the consumer's premises, and could be switched off and on by means of audio frequency signals superimposed on the network at the substation.*

The normal purpose of this ripple-control installation was to enable the heaters to be cut off during peak periods so as to conserve generating capacity and reduce the bulk-supply charges. For the purpose of the experiments, the control was operated abnormally by switching the heaters off and on again every few minutes throughout the day.

* "The Measurement of Water-Heater Diversity by Superimposed Control". *E.R.A. Technical Report K/T* 105 ; *Journal I.E.E.*, 91, Part II, No. 19, February, 1944. (The suggestion for such a test was first made by R. B. Rowson.)

At each switching operation the change in the total feeder current was noted, and thus the water-heater load existing at that instant was measured. By plotting these figures, it was possible to separate out the water-heater load curve from the rest of the network's load.

The results were of course only representative of the particular type of installation tested, namely, "circulator" immersion heaters with hand-switch control, in tanks connected to back boilers. Nevertheless, they are quoted here, both as an illustration of method, and for the light they throw on diversities and load factors. In each case, the network was supplying a load between 60 per cent. and 90 per cent. domestic, and the water-heater load curve proved to be very similar in general shape to that of the total network. In each case both heater peak and network peak occurred between 8 and 9 a.m., and there was very little diversity between them.

There was, however, very considerable inter-group diversity among the heaters themselves, this being about 4 at the critical peak period and very much more at all other times. This could be put another way by saying that, whilst it is likely that almost every heater would be used several times every day, the largest number in use at any one time was only about one-quarter of the total connected. Had the network peak occurred in the later afternoon (due to the addition of industrial and commercial loads) the number of heaters then in use would have been only 1 in 8 to 1 in 13 of those connected.

The heater loadings were 2 kW per heater in one case and nearly 3 kW in the other, but the average effective load factor (two tests on each) was almost the same in the two cases, namely 32 per cent. (By effective load factor is meant the quotient—

$$\frac{\text{kWh consumption per heater per diem}}{\text{after-diversity demand} \times 24}\text{)}$$

Load-Duration Curve.—If a load curve is divided into a large number of vertical slices, and if these slices are then rearranged in order of magnitude, the resulting graph is called a load-duration curve. Such a curve has the advantage over a load curve that it can be plotted over a whole year if necessary and still remain a simple figure, whereas a load curve for a year would be a hopelessly complicated affair. On the other hand a load curve shows the actual sequence of events whereas the load-duration curve merely shows their summation. The area is the same under the two curves (representing kWh), and this area, divided by the surrounding rectangle, gives the load factor.

The main use of the load-duration curve is to give a picture of the utilisation made of the supply system over a period of time. For this purpose, the two scales can conveniently be given as percentages rather than in absolute values. Fig. 28 shows the load-duration curve on the system of the British Electricity Authority for the year 1947/8

based on the estimated potential demand. It shows that a base load of about 11 per cent. was maintained throughout the year, whilst, at the other end, the top 20 per cent. of load lasted only about 5 per cent. of the time (440 hours).

If the maximum load equals the total plant capacity the load-duration curve will show the time-usage of this plant. The bottom rectangle will then show what percentage of the plant could be

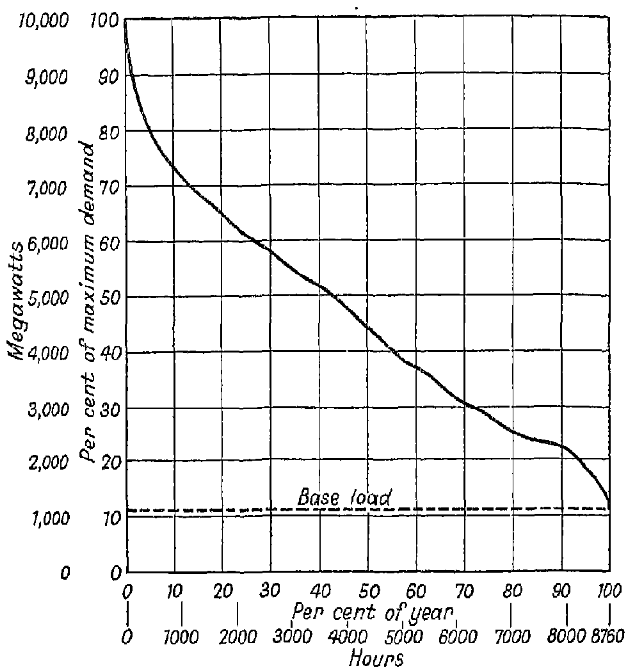


FIG. 28.—Load-Duration Curve.

utilised continuously throughout the year, whilst the top part of the curve will show peak utilisation.

These figures, of course, represent the theoretical full-load usages of these plant capacities, not the actual usages of specific plants. As regards the base portion, no actual plant can be run at full load indefinitely. As regards the peak portion, it must be remembered that the load-duration curve does not portray the sequence of events. Any part of the curve represents the sum of all the occasions when the load has had that value: thus the portion between 90 and 100 per

cent. of M.D. (some 150 hours duration) includes not only the single absolute peak of the year but a large number of separate local peaks, often of very short duration. On either side of each of these occurrences, plant to the full capacity had to be in readiness, and it follows that plant is needed (though not fully used) for longer hours than is indicated in the diagram, particularly at the upper end.

If the total plant capacity is greater than the maximum demand, the vertical scale of the load-duration curve can be extended to a height equal to the plant capacity and can be scaled as a percentage of this capacity. The curve will then terminate at a point short of the top, and the diagram will represent time-usage of plant capacity, some of which is shown as not used at all. The mean height divided by the maximum height of the diagram will now represent, not the load factor, but the *plant load factor*, *i.e.*, the ratio of the average load to the aggregate rated capacity of the generators which supply it.*

Load Restrictions and Load Factor.—During the post-war years, generating capacity was frequently insufficient to meet peak demands, and output was compulsorily restricted. The result was an artificially high load factor due to the decapitation of the peaks, and the following gives some examples :—

In diagram 1 of the *1948 (Clow) Committee Report*, curves are given for a day (1st December 1947) on which there was extensive load-shedding. One curve shows the demand actually met, and the second curve shows the demand which it is estimated would have been met had there been ample plant available. The shaded area between two curves represents the energy lost due to load-shedding, and frequency and voltage reductions.

Measurement of the curve shows that the loss of megawatts due to load-shedding and other restrictions was $12\frac{1}{2}$ per cent. of the MW demand met, whereas the loss of kWh was less than $2\frac{1}{2}$ per cent. of the kWh supplied. The effect on the curve as a whole was to raise the daily load factor from a hypothetical or corrected figure of 65 per cent. to an actual load factor for the day of over 71 per cent. Similar curves in the *British Electricity Authority First Report and Accounts* showed a loss due to load-shedding and frequency reduction on the day of estimated annual peak (4th February 1949) of 5 per cent. in kW, and less than 1 per cent. in kWh.

The effect on the yearly load factor might be expected to be even more marked. Yearly load factor expresses the ratio between the mean height of the load curve throughout the year and the maximum height on the day of the annual peak. Since on most days of the year there are no restrictions, the mean height of the curve (yearly energy $\div$ 8,760) will be proportionally little affected whereas the maximum

* A combined load-duration and plant operation diagram was given in the paper on "Load Despatching" by A. R. Cooper : *Journal I.E.E.*, 1948, 95 ; Part II, p. 719.

height is substantially affected. The system load factor for 1948–49 is given in the *B.E.A. First Report* as 47·3 per cent., based on maximum potential demand, and 49·0 per cent. based on actual demand met.

Load Factor Components.—With load factor, as with diversity factor, the first step is to break the problem down into its component parts. Linked with this is the question of what one should aim at in load factor improvement, *i.e.*, what " target " is to be set. The term " load factor ", when used without qualification, commonly means

$$\text{annual load factor} = \frac{\text{kWh of the year}}{\text{peak kW} \times 8{,}760}$$

This annual load factor is often spoken of as though it were a single unit—one and indivisible—and that the aim should be to bring this up to 100 per cent. or to some lesser " optimum " value. In fact, the annual load factor can usefully be dissected into at least three widely different components, and separate targets should be set for each of them. Each of these components plays its part in reducing the annual load factor, and it might be said that the lowness, or imperfection, of any annual load factor is the product of the lowness of each of these components.

In finding this component, one can approach the subject in the same way as when analysing the load curve. (The headings (*a*), (*b*), and (*c*), below, correspond roughly to (*d*), (*c*), and (*b*) on pp. 151 and 152). Starting with the half-hour in the year in which the system load is a maximum—say, 8.30 to 9 a.m. on a cold Monday morning—one can examine the reasons why this level is not maintained throughout the year. The main reasons are as follows :—

(*a*) the load is less at other times of the day on that particular Monday because, *e.g.*, of reduced activity at night. Say, the mean load over the 24 hours is 60 per cent. of the maximum. (Daily load factor then = 0·6.)

(*b*) the mean load during the week is less than the mean load on the Monday, because, *e.g.*, of reduced industrial activity at the week-end. Say, the mean load during the week is 90 per cent. of the Monday load. (Day/week factor then = 0·9.)

(*c*) The mean load during the year is less than the mean load during that week, because, *e.g.*, most other weeks in the year are warmer and lighter. Say, the mean annual load is 80 per cent. of the mean load that week. (Week/year factor then = 0·8.)

The yearly load factor is clearly the product of these three components, and putting these in symbolic form we have the following:

(*a*) Load factor of peak day $= \dfrac{\text{kWh of peak day}}{\text{Peak kW} \times 24} = 0{\cdot}6$ in above case.

(*b*) Day/week factor $= \dfrac{\text{Peak week kWh}}{\text{Peak day kWh} \times 7} = 0{\cdot}9$ in above case.

(*c*) Week/year factor $= \dfrac{\text{Year kWh}}{\text{Peak week kWh} \times 52} = 0{\cdot}8$,,

Thus, the yearly load factor—

$L = (a) \times (b) \times (c) = 0{\cdot}6 \times 0{\cdot}9 \times 0{\cdot}8 = 0{\cdot}43$, *i.e.*, 43 per cent.

Intermediate factors can be similarly calculated. Thus the peak-week load factor $= 0{\cdot}6 \times 0{\cdot}9 = 0{\cdot}54$, *i.e.*, 54 per cent.

In order to give some idea of possible values, the following table shows an analysis of five different cases. The first refers to an independently operating station in the East, where the maximum variation in sunset time between summer and winter is only one hour. (Seasonal temperatures variations were also slight.) The others refer to individual British stations, except the last, which is an estimate for the aggregate Grid load for the year 1948–49.

A feature of these particular figures is that the daily load factors are all very similar, only varying by 6 per cent., although the overall

LOAD-FACTOR COMPONENTS (PER CENT.)

Factor.	(*a*).	(*b*).	(*c*).	L.
Expressing the Ratio of mean kW →	Peak Day . Peak Half-hour (Daily L.F.)	Peak Week : Peak Day	Year : Peak Week	Year . Peak Half-hour (Yearly L.F.)
Undertaking* 1	59½	94½	95	53½
,, 2	61½	81½	68	34
,, 3	65	85	48	26½
,, 4	60¼	94½	45¾	26
,, 5	65½	88	82	47.3
Possible Target	[100]	[95]	[84]	[80]

yearly load factor shows a two-to-one variation. This is well shown by a comparison between undertakings 1 and 3. The former has more than twice as big a load factor as the latter, achieved not by night-load tariffs (the daily load factor is actually worse) but by the relative absence of seasonal climatic variations and by the seven-day week of native industries.

This analysis must be kept in mind when taking steps to improve the annual load factor, or it may be found that improvements, say,

* Except for Number 5, the undertakings are fairly small and the figures are pre-war. They are inserted for illustration purposes, in order to show the sort of values which may arise, and they are not regarded as intrinsically significant. As regards Number 5, whilst the overall load factor is obtainable from published figures, the break-down is an estimate made on the lines of p. 163.

of (*a*) make (*b*) and (*c*) worse, and so nullify much of the advantage. Thus, suppose that the peak-day load curve discloses a predominant domestic demand. The obvious corrective would be an increase in the industrial load. The latter, however, might prove to be highly seasonal in character with a peak load in certain months or on certain days of the week. The effect on the peak-day curve may be excellent, yet the result on the year's load factor might be disappointing.

Optimum Values.—A still greater reason for splitting the load factor into these components is that the target of load-factor improvement is not necessarily 100 per cent. in each factor. In dealing with apparatus such as cables, and probably also switchgear and transformers, there may be nothing to prevent these from carrying their rated load all day and every day. From the cost point of view, then, the higher the load factor the better, since the bigger will be the number of kWh over which the capital charges can be spread. But with power-station plant (such as boilers and turbines), regular overhauls are necessary, and if the load factor were 100 per cent., stand-by plant would have to be purchased merely to enable the other sets to be taken off periodically for maintenance.

Apart from involuntary " outages," the plant-overhaul programme is planned so as to leave as far as practicable the same margin at all times between anticipated demand and available capacity. Excluding rapid load fluctuations which cannot be taken advantage of for this purpose, the ideal load factor is one which just permits the necessary yearly overhaul to be completed. This magnitude of load factor is here referred to as the " optimum " value, and strictly this differs with each class of plant concerned. It is possible, however, to express rough figures for a generating station which will give some idea of what to aim for.

When this is attempted, it is found that any such optimum figure for the yearly load factor as a whole would be very misleading. Instead, it is necessary to examine the three load-factor components, since the target position is quite different for each of the three. Little or nothing in the way of routine overhaul can be done in the night on major plant which has to be on full load again early next morning. Hence the optimum figure for the daily load factor will in all probability be 100 per cent. Some overhauls can be done at week-ends, and the station engineer might therefore welcome some degree of week-end lull, provided he can rely on it for a clear spell of 30 hours or so. Finally, the week/year factor is clearly the one where departures from 100 per cent. are most likely to be of value in the overhaul programme.

In order to illustrate the point (and not with any intention of supplying firm information) some guess-work figures for optimum values are inserted in brackets at the foot of the table. These figures are actually

based on the following, quite arbitrary, assumptions, *viz.*, that the *ideal* 24 *hours* would be one in which the load was absolutely steady; the *ideal week* would be one in which there was a reduction in load, represented by the equivalent of a Sunday load of two-thirds of the weekday load; the *ideal year* would be one having a summer load reduction such that the summer week averaged two-thirds of the winter week. (These descriptions are deceptive, because such average loads would not mean anything like six months' consecutive running at the two-thirds value.)

The significance of any such figures in relation to tariff devices for load-factor improvement will be obvious. The load shift from the day to the night-time, whether accomplished by low off-peak charges or other inducements, will be sheer gain. However successful, such devices will still leave the night load well below the day load, and since the night-time is too short for overhaul, the target here is 100 per cent. daily load factor.

Improved load-spreading through the week, whereby Saturday and Sunday are brought nearer to the working-day level, is not likely to exceed the optimum value; moreover there are few tariff arrangements which specifically affect it. An improved load-spread throughout the year is, however, far from being an unmixed boon, and the optimum value for the power station must be put considerable below 100 per cent. This limits the usefulness of a device such as the seasonal tariff variation which tends to shift load from winter to summer, but does nothing to shift it from day to night. On the other hand, for the distribution system (in which so much of the capital investment occurs) the optimum values would probably be much higher.

Further Examples of Load-Factor Analysis.—The following table shows the daily load factor of a number of load curves of interconnected systems. Curves numbered (1) to (8) show the load on the Grid on certain selected days in 1947 and are taken from the article "Facts and Figures" by E. R. Wilkinson.* Curve (2) is not representative for the reasons given previously, namely, that it has been artificially decapitated by load-shedding. Omitting this, all but one of the remaining Grid curves have daily load factors lying between 64 and 67 per cent.

Curves (9) and (10) are for two Continental† systems and are inserted by way of contrast. Curve (9) refers to a day on the interconnected Austrian system, where great encouragement has been given to off-peak consumption in order to use energy from run-of-river hydro stations. The minimum night load is nearly half as great as the maximum day load, and 7 of the 46 per cent. is directly attributable

* *Electrical Industries*, September, 1949. Based on a lecture given at the B.E.A Summer School.

† *Elektrizitatsverwertung*, July/August, 1948.

to storage space- and water-heaters. The last curve represents something of an extreme in the opposite direction: the combination of a steep late-afternoon peak with a relatively small night load gives a load factor of only 51 per cent. and a minimum load of 17½ per cent. of the maximum.

DAILY LOAD CURVES

Number	Curve Description	Date	Load Factor: Per Cent.	Minimum Night Load as a percentage of maximum day load
(1)	Cold winter weekday— Potential demand .	1/12/'47	65	
(2)	Ditto— Demand met .	1/12/'47	71	
(3)	Cold winter Sunday . . .	30/11/'47	65	
(4)	Mild winter weekday . . .	22/12/'47	66	
(5)	Mild winter Sunday	21/12/'47	65	
(6)	Summer week-day	30/ 6/'47	64	
(7)	Summer Sunday	29/ 6/'47	53	
(8)	Typical winter weekday . .	1948/49	67	31
(9)	Austria: interconnected system .	—	74	46
(10)	Paris network (pre-war) . .	20/12/'38	51	17½

The information given by these daily curves can be extended in the following manner:—

A hypothetical winter week under severe conditions can be built up by assuming five winter-day curves as tabulated as (1), one Sunday curve as at (3) and one curve having an area midway between the weekday and the Sunday curve. (Naturally, any such estimate can only be a rough approximation: no two days and no two weeks will be identical even under steady climatic conditions.) The resulting "week factor", *i.e.*, the ratio of the highest weekday consumption to the average over the week comes to 0·92. A similar build-up for a mild winter week using curves (4) and (5) gives a "week factor" of 0·93. A similar calculation for a summer week using the curves indicated at (6) and (7), gives a value of 0·89.

It will be seen that there is no very great difference between any of these results. Probably, in respect of the absolute peak of the year, the ratio between the mean weekly consumption and the consumption on the day of absolute peak would be slightly lower unless the peak day was part of an extended cold snap. For purposes of an overall estimate,

N.B.—The above figures are scaled from diagrams, and the maxima represent absolute maxima, not half-hourly integrations. The load factors calculated therefrom are therefore slightly lower than they would be on the usual basis of reckoning.

it may be assumed that the figure is about 0·88. The daily load factor for the absolute peak day might also be slightly below that of curve (1)—say, 64 per cent.

The load factor on the Grid system, based on the maximum potential demand (*i.e.*, the " natural " rather than the artificially high values) is given in the *B.E.A. First Report* as 44·7 per cent. in 1947–48 and 47·3 per cent. in 1948–49. Using the foregoing estimates for daily and day/week factors, the yearly load factor for 1947–48 can be taken as made up of the following components :—

Daily load factor	64 per cent.
Day/week factor	0·88
Week/year factor	0.79
Yearly load factor	64 × 0·88 × 0·79 = 44·7 per cent.

The load factor for the following year has been similarly analysed (Undertaking 5 in previous table). This breakdown is, of course, only an estimate, but it serves to indicate the order of magnitude of the figures involved, and will give a basis on which load-factor improvement might be aimed.

These figures again illustrate the limits to the usefulness of the seasonal tariff in reducing generation costs, assuming that the only effect of such a price differential is to shift some load from winter to summer. If the seasonal (week/year) factor is already of the order of 80 per cent., and if generating plant is so short that there is no margin for all-the-year-round servicing, any increase in the figure may well upset the summer overhaul programme.

These remarks do not apply in the same way to the distribution system. There is, moreover, the possibility that the long-term effect of the seasonal tariff will have some influence on the extent and type of installation, *e.g.*, by putting a bias in favour of summer water-heating. The most helpful results of a seasonal tariff may therefore arise from its long-term effect on installations.

PART III

RETAIL TARIFFS

Chapter IX deals generally with tariff structures as examplified in United Kingdom practice. The next two chapters cover industrial and domestic tariffs, and the last chapter in this section deals with time-varying tariffs and restrictions.

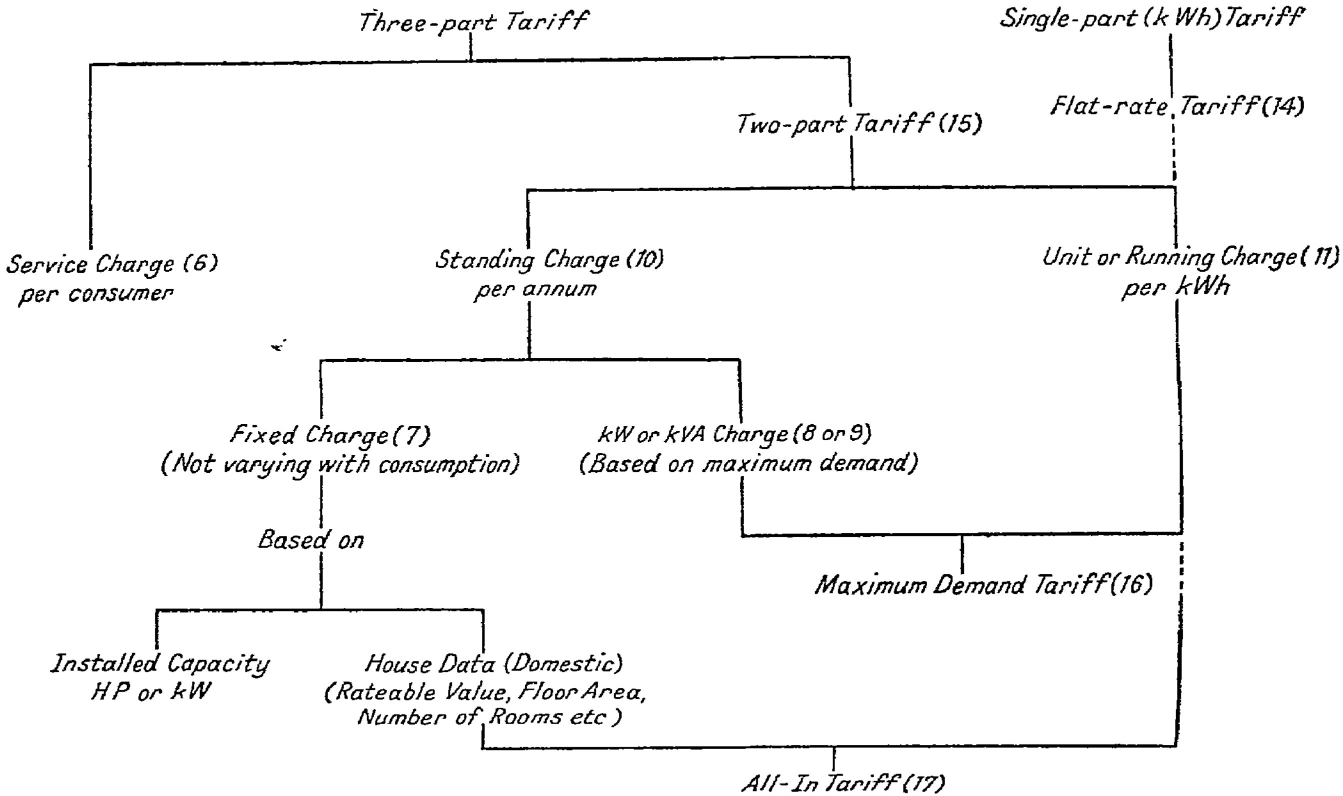


Fig. 29.—Tariff Chart (Numbers refer to B.S.I. Glossary.)

CHAPTER IX

TYPES OF STRUCTURE: GENERAL SURVEY

Introduction.—This part consists of a critical description of actual tariffs as they existed in Great Britain in March, 1948. A sharp distinction will be drawn between the form or structure of the tariff and its magnitude or money content. The present study concerns chiefly the former, and where money values are employed this is largely for illustration purposes. It deals only with retail tariffs, *i.e.*, those offered to consumers: bulk-supply tariffs covering electricity for resale have been dealt with under costs.

Tariffs can be classified in either of two ways, namely, according to the construction or type of the tariff itself, or according to the class of consumers to whom it is offered. Both classifications must be considered, since both enter into most tariff schedules: no undertaking offers the same tariff to all classes of consumer, and generally there are alternatives open to users within a class. The present chapter starts with a classification according to type of structure, and proceeds to a theoretical survey designed to bring out the underlying structural resemblances. Classes of user are then considered, and subsequent chapters deal with the two main classes, and go into greater practical detail.

General Characteristics.—Broadly, the effect of almost every tariff (except the very simplest flat-rate energy charge) is to reduce the mean price per unit of electricity as the annual consumption increases, other things remaining the same. These "other things" may be merely the consumer's house size or installation, or they may be his maximum demand. Hence a graph plotting the mean price in pence per kWh to a base of annual consumption shows a falling curve like that in Fig. 30.

This falling characteristic can be brought about in a variety of different ways, *e.g.*, by a two-part tariff having a standing charge either fixed or variable, or by a block tariff having a single part based on kWh but priced in descending steps. There is a corresponding difference in the conditions which have to be satisfied in order to secure a falling price. With the normal domestic two-part tariff the mean price falls when the consumption increases relative to the house size or value. In the maximum demand (Hopkinson) tariff the reduced price is only obtained when the kWh consumption increases relative to the kW of demand—*i.e.*, when the load factor increases. In a block tariff a mere increase in the consumption per consumer lowers the mean price.

Tariff Components.—Before listing the tariffs it is necessary to define certain of their components and the corresponding combinations (see Fig. 29):

(*a*) *Running or Unit Charge.* A charge per unit* (kWh) supplied.

(*b*) *Standing Charge.* An annual charge independent of the kWh supplied. It may be either a fixed charge (*i.e.*, the same each year) or a demand charge varying with the kW or kVA.

(*c*) *Service Charge.* An annual or capital charge per consumer or per supply-point independent of the electricity actually supplied.

Single-Part Tariff. This term, though not generally employed, is used here to indicate a tariff in which the charge is based on a single element, usually the consumption in kWh, rather than on two or more elements such as M.D. and consumption, or house size and consumption. Such a tariff will contain the single component (*a*).

Two-Part Tariff. One containing the two components (*b*) and (*a*).

Three-Part Tariff. One containing the three components (*c*), (*b*) and (*a*).

Tariff Schedule.—The following is a list of the principal tariff types. The terms used are intentionally loose and descriptive, and for precise definitions reference should be made to p. 298.

I—Energy (or kWh) Tariff (single part), comprising the *flat-rate tariff* (single charge in pence per kWh) and the *multiple tariff* (separate flat rates for different purposes, *e.g.*, power, lighting, heating, etc.).

II—Block (or Sliding-Scale) Tariff (single-part). First block of consumption per annum at one flat-rate price, the next block at a lower flat rate, and so on. Sizes of blocks fixed, independent of consumer.

III—Two-part Maximum-Demand (Hopkinson) Tariff, consisting of a (variable) standing charge based on maximum demand, and a running charge.

IV—Two-Part All-In Tariff, consisting of a (fixed) standing charge based on the consumer's house size or value, and a running charge.

(*Note.*—As will be seen in subsequent chapters, tariffs *I* and *IV* are the ones usually offered to domestic consumers, *II* and *III* being offered to industrial and commercial consumers.)

* Although the word "unit," to indicate the unit of energy (kilowatt-hour), now has the sanction of the British Standard Glossary, it is still, in the author's opinion, somewhat of a slang term and he has avoided it as far as possible—particularly where ambiguity might arise between this specialised use and its more general employment as a small element of anything.

IIIa—Variable-Block Maximum-Demand Tariff. *IVc Variable-Block All-In Tariff.*	Alternatives to the two-part tariffs. The standing charge, instead of being levied outright, is spread over the first x kWh and added to the running charge, thus forming a first block of high-priced units. Additional units pay the running charge only. Magnitude of x varies with consumer (M.D., house size, etc.).

Tariffs III and IV may be "blocked" or "stepped" in either or in both parts, *i.e.*, the first portion of either the standing or the running charge (or both) may be at a higher rate than the subsequent portions.

Tariff I may be subject to discounts for quantity giving a progressive reduction in price with increase of consumption. Usually this is only a few per cent., but in extreme cases it may merge into Tariff II.

There may also be variations on account of fuel cost and power factor.

Special Types.—The following, less common, tariffs should also be mentioned. The first three are special forms of multiple flat rates.

Time-of-day (or Two-rate) Tariff.—Lower price at certain times each day.

Equated Rate Tariff. Flat rate which includes rent of apparatus, coupled with minimum consumption.

Load-rate Tariff. Lower price when rate of consumption exceeds a predetermined magnitude.

Fixed Charge or Service Tariff. Payment only for demand or service, *i.e.*, like III or IV but without running charge.

Formulæ and Equivalents.—The above tariffs will now be expressed in symbol form together with the overall price (in pence per kWh) which results therefrom.

Tariff I.—p_1 pence per kWh for domestic lighting.
p_2 ,, ,, ,, ,, heating, etc.
p_3 ,, ,, ,, ,, industrial power.

In the case of a domestic supply consisting of n_1 kWh of lighting and n_2 kWh of heating, the overall price will be

$$\frac{n_1p_1 + n_2p_2}{n_1 + n_2}.$$

Tariff II.—p_1 pence per kWh for the first n_1 kWh.
p_2 ,, ,, ,, ,, ,, next n_2 kWh.
p_3 ,, ,, ,, ,, ,, next n_3 kWh, etc.
p ,, ,, ,, ,, all additional consumption.

For any large consumption n the overall price will be

$$\frac{n_1p_1 + n_2p_2 + \text{etc.} + (n - n_1 - n_2 - \text{etc.})\, p}{n}$$

(portion in brackets to be positive). For smaller consumptions not all the blocks will be utilised, and the formula must be modified accordingly.

Tariff III.—£q per annum per kW of maximum demand plus p pence per kWh.

Tariff IV.—£q per annum per unit of house valuation plus p pence per kWh.

Overall price $\dfrac{q}{36.5L} + p$, or $\dfrac{240q}{n} + p$.

L is the annual load factor (for Tariff III), n is the number of kWh per kW of demand (Tariff III) or per unit of house valuation (Tariff IV).

Tariff IIIA.—p' pence per kWh for the first h hours per annum use of maximum demand (or the first h kWh per kW of demand).

p pence per kWh for all additional consumption.

Tariff IVA.—p' pence per kWh for first h kWh per annum per unit of house valuation.

p pence per kWh for all additional consumption.

The overall price will be the same as that given for III and IV provided that $(p' - p)h = 240q$. (The reason for this relationship is that in the A tariffs the fixed charge per kW, $240q$, has to be paid for by the excess price $p' - p$ of the first h kWh.) But this price formula will only hold so long as the load factor L exceeds $\dfrac{h}{8{,}760}$, or the number of units n exceeds the size of the high-price block ($h \times$ M.D.). Below this load factor the price will be constant at p' instead of rising as on Tariffs III and IV. (*N.B.*—the quantities q and h, although here given annually, are usually expressed monthly or quarterly with corresponding changes of magnitude. Note also that all the above symbols represent constants except n and L which refer to the annual load of the particular consumer.)

Variable-Block Tariffs.—The alternatives IIIA and IVA may be described as " disguised two-part tariffs ", and they are an attempt to temper the east wind of a fixed charge to the shorn lamb of a consumer. The fixed charge, instead of being levied as such, is spread over the first batch of units and added to the running charge. As soon as these first high-priced units have been consumed the fixed charge will have been liquidated, and all subsequent units pay the running charge only.

For all consumptions beyond this minimum block, Tariffs IIIA and IVA will therefore give identical results to III and IV.

It will be seen that, in form, these tariffs are block-rate, but in effect they are two-part. Their difference from a simple (*i.e.*, fixed) block tariff is the essential one that the size of the block varies with the consumer instead of being constant as in the pure block tariff. (Also there is usually only one high-price block.) They differ in their effect from the normal two-part tariff only in one respect, namely, that if the actual consumption in any year is less than the magnitude of the high-price block the consumer will not pay the full amount of the fixed charge. The difference may be psychological rather than material, since the tariff should be so offered and proportioned that the consumer is almost bound to exceed the minimum block. But whether he does so or not, the tariff certainly overcomes any objection that may be felt to paying " something for nothing."

There are several advantages in such a method. It is more popular with the consumer since it omits the standing charge, which is always the stiffest part of sales resistance. Moreover, with careful design this concession to the consumer can be made at little or no risk to the undertaking. It can also simplify the tariff schedules, particularly in the domestic field. With the normal two-part all-in tariff a flat-rate alternative is usually offered, in the absence of which the price per unit will become excessive on very low consumptions. But with a variable-block tariff the first, high-price, block itself constitutes a flat-rate " ceiling " above which the price per unit cannot rise. It can therefore take the place of both the two-part tariff and its flat-rate alternative. For this purpose it can be so constructed as to give precisely the same results for all consumers except those whose consumptions are very small in proportion to their demand or house size. The following section will show how this can be done.

There is one other possibility with the variable-block tariff that must be mentioned. In equating its operation to that of a two-part tariff, the correspondence only applies when the variable-block tariff has two blocks only—a first block at a high price, together with a low follow-on rate for all additional consumption. This two-block arrangement is the usual one, but there have been exceptions particularly in domestic applications : thus Glasgow Corporation had a three-block variable-block domestic tariff and the North-Eastern Electric Supply Co. even had one with four blocks. A more recent example is the North of Scotland Hydro-Electric Board tariff described on p. 223.

The operation of a variable-block tariff with more than two blocks lies somewhere between that of a two-part and that of a fixed block tariff. These operations are not studied in detail here, and the descriptions of the variable block-tariff throughout this book must be taken as referring to the two-block variety unless otherwise stated.

Construction.—In Tariffs IIIA and IVA the fixed charge is in effect, spread over the first block of units. The length of this spread-over in the case of an industrial tariff should as a rule be fixed so that almost all consumers will exceed this point and come on to the cheaper rate for some of their consumption. If the spread-over period is too short, the price of the first block will appear excessive ; if it is too long, many consumers will not reach the end of it and so will not pay for all the fixed charges they have presumably incurred. Moreover, if the consumer regularly falls considerably short of the first block quantity he will have no incentive either to spread his load or to increase his consumption, since he will feel there is little hope of getting on to the cheaper rate.

The usual basis for this spread-over in the case of the variable-block M.D. tariff is 600 to 800 hours-a-year use of the maximum demand, corresponding to a load factor of 7 to 9 per cent. (the actual average of the industrial tariffs of this type gave $h = 680$ hours, *i.e.*, a load factor of $\frac{680}{8,760} = 7\frac{3}{4}$ per cent.). When the consumer's load factor is is less than this figure (*i.e.*, if his consumption is less than this number of hours multiplied by his maximum demand) his mean price will be lower than it would be under the corresponding two-part tariff. Such a departure from apparent costs can, however, be fully justified because of the high diversity probabilities of such low-load-factor consumers.

The first block price of these tariffs is generally three to four times the follow-on rate. (An average for the country was $3\frac{1}{3}$.) But when it is desired to line up with a particular two-part tariff, the figures must be adjusted according to the values and ratio of the two parts. In the maximum-demand example given below, there is a two-part tariff of £5 per kW plus $\frac{1}{2}d.$ per kWh (III). To get the same effect with a variable-block tariff (IIIA) it is evident that p must equal $\frac{1}{2}$. Furthermore $(p' - p)h$ must equal $240q$ so $(p' - \frac{1}{2})h = 240 \times 5 = 1,200$. As it was desired in this case to make $p' = 2$ so as to resemble Tariff II, it follows that $h = 800$.

In a domestic tariff of this type (IVA) the size of the first block should correspond with the probable or minimum lighting consumption for that size of house, so that all uses other than lighting come on to the cheaper rate. The price ratio between the first block and the follow-on rate can be bigger than with industrial tariffs, corresponding to the much higher utility value of the first block. (The average for the country disclosed a ratio of 8.) Apart from this, the construction is exactly the same as for the industrial tariff, and the same formulæ apply. In the case illustrated below, the first block price has been taken at $2d.$ in order to show the resemblance to the other tariffs, but a more practical value would have been $3\frac{1}{2}d.$ or $4d.$ In the same way the fixed charge of Tariff IV would more usually be 10 per cent. of the rateable value.

Examples and Comparison.—The simplest basis of comparison between the different tariffs is that of overall price per kWh. For this purpose the following tariffs have been constructed, representing each of the types described above. The particular values and proportions have been selected so as to bring out the general similarity of all the types, but most of them are representative of actual practice (pre-war figures). The exceptions to this are that the lighting (or equivalent lighting) price in Tariffs I and IVA is disproportionately low, and the number of blocks in Tariff II is low for an industrial tariff (usually there are three or four).

Tariff I. 2*d.* per kWh for lighting.
½*d.* per kWh for all other purposes.

" II. 2*d.* per kWh for first 8,000 kWh.
½*d.* per kWh for remainder.

" III. £5 per annum per kW plus ½*d.* per kWh.

" IIIA. 2*d.* per kWh for first 800 hours' use of M.D.
½*d.* per kWh for remainder.

" IV. 5 per cent. of R.V. (or £1 p.a. per 1,000 sq. ft.) plus ½*d.* per kWh.

" IVA. 2*d.* per kWh for first 8 kWh per £1 R.V. (or per 50 sq. ft.).
½*d.* per kWh for remainder.

It will be found that with the above figures Tariffs IIIA and IVA are the same as III and IV provided the minimum load factor is exceeded. This load factor is 9·1 per cent., or 800 hours' use of the M.D. in IIIA and 8 kWh per £ of rateable value in IVA. Below this figure, the A tariffs give a flat rate of 2*d.* instead of a rate which rises as the load factor declines. Comparing Tariffs I and IVA, these will be identical for consumers who take 8 kWh of lighting per £ of rateable value. The pure block-rate Tariff II is fundamentally different from others since it gives a reward for the size of the consumption independent of its electrical characteristics. It will resemble the others only for a consumer of a particular size, and in the present instance Tariff II will equate to Tariff IIIA when the maximum demand is 10 kW.

Summary and Graph.—Surveys show that all but an insignificant minority of the tariffs in the country can be classified into four types, two of them being subject to a simple variation. These types have been analysed in the foregoing sections, and before leaving this theoretical portion of the work a graphical illustration will be useful. The purpose of this graph is to show what results each tariff will give, and under what circumstances each will give the same result. It will be realised that the whole study is an attempt to trace the broad lines and underlying consonance of the various rates, leaving to later chapters the elucidation of their details and differences.

In general, any two-part, two-rate or two-block tariff can be represented graphically by the same type of figure. Such a figure may conveniently plot the mean or overall price per kWh to a base of load factor or annual consumption. With a uniformly scaled base-line, the

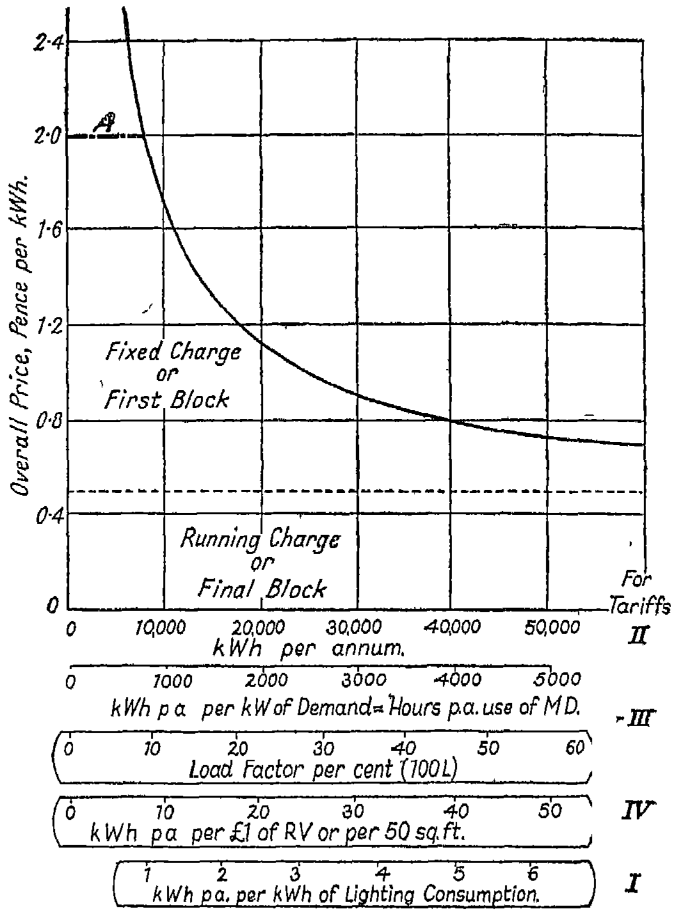


FIG. 30.—Tariff Types and Prices.

figure will consist of a rectangular hyperbola (price inversely as consumption) representing the incidence of the fixed charge or high-price block, standing on a rectangle representing the running charge. With a base scaled inversely the hyperbola becomes a straight line, as illustrated in the next chapter.

Fig. 30 shows such a graph having a uniformly divided base-scale,

and the scale markings represent the tariffs listed above. The A tariffs are identical except at low consumptions, where they follow the chain-dotted line marked *A* on the graph (Tariff II also follows the *A* line). Taking the base-scales in order from the top, it will be seen that the first row of figures will represent kWh per annum for any consumer under Tariff II, whilst the second one shows kWh per annum per kW of demand under Tariff III. An alternative, load-factor, scale is shown for the latter tariff. The lower sets of figures refer to Tariffs I and IV, and will be sufficiently clear from the diagram itself. (*N.B.*—Figs 35 and 36, on pp. 193 and 194, shown the same tariff values carried to 100 per cent. load factor.)

Consumer Classes.—The second method of classification mentioned at the beginning of the chapter is by classes of user. This is the method followed in the detailed examination made in subsequent chapters. Before proceeding to this it is necessary to see what these classes are, and in what proportions they make up the total consumption. Equally important is the proportion in which they contribute to the total revenue, and this is therefore an appropriate point at which to bring in the magnitudes of the tariffs as well as their structures. (The figures used for magnitudes in the preceding sections have been hypothetical and only for illustration purposes.)

The tables below show the consumer classification which was employed by the Electricity Commissioners and has been followed since by the British Electricity Authority and the North of Scotland Hydro-Electric Board. The first table gives a detailed split, by consumer, for the year 1937/38. It will be noted that roughly one-third of the revenue came from the industrial load, one-third from the domestic, and the remaining third from commercial, traction and public-lighting loads. These values are illustrated in the diagram (Fig. 31). The base distances represent units sold whilst the heights show the mean price per unit. Each area therefore represents the revenue received from that class of load. Unfortunately, in later years only the units sold have been segregated to this extent, not the revenue or mean prices.

A less detailed and slightly different classification has been maintained subsequently and is illustrated in the second table. This is on the basis of usage rather than consumer, and whilst the heading "lighting, heating and cooking" corresponds very closely to the sum of domestic (including farm) and commercial, it appears that some 3 per cent. of domestic consumption was entered as "power" and included in the latter category. In this table, public lighting has been grouped with domestic, and traction with industrial, in order to show the eleven-year changes in the two groups.

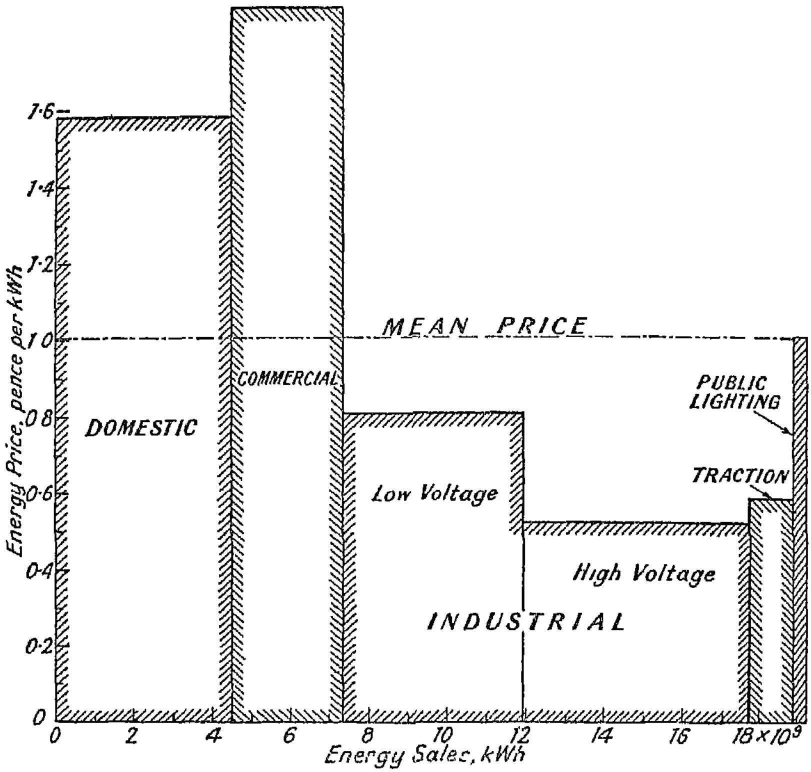


FIG. 31 —Electricity Sales 1937/8.

CONSUMER CLASSES AND LOAD PROPORTIONS : PRE-WAR

	Percentage of Units.	Percentage of Revenue.	Percentage of Consumers.	Mean Price : *d.* per unit.
Domestic and Farm . .	24	35½	87	1·5
Commercial : Shops, offices, and Institutions . .	15	26	11½	1.8
Industrial : Up to 650 V. (23½%) Above 650 V. (29½%)	53	33	1½	0.64
Special : Traction (6%) . Public lighting (2%)	8	5½	—	0·59 / 1·06
	100	100	100	1·05

LOAD PROPORTIONS (MAIN GROUPS): PRE-WAR AND POST-WAR

	1937/8:			1948/9:			Changes in Eleven Years:	
	Per Cent of Units	Per Cent of Revenue	Mean Price	Per Cent of Units	Per Cent of Revenue	Mean Price	Units.	Mean Price.
Lighting, Heating and Cooking* .	41	63	1·67	47	58	1·45	+144%	−13%
Industrial Power and Traction . .	59	37	0 64	53	42	0 93	+ 79%	+ 45%
Overall Mean Price	..	..	1·05	..	..	1·18	..	+ 24%
Number of Consumers (millions) .	9 36			12·39			+32%	

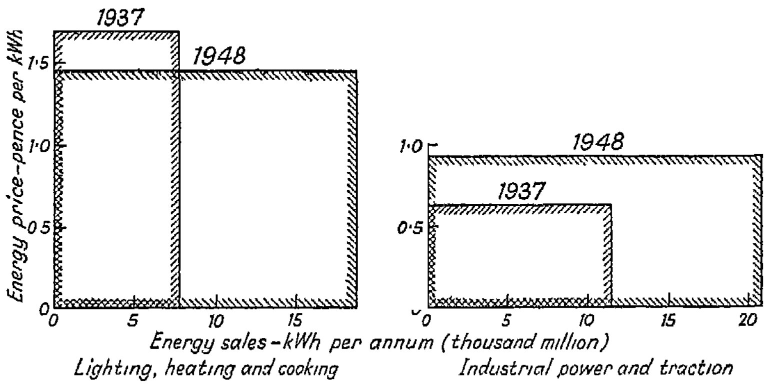


FIG. 32.—Revenue from Two Groups: Pre-War and Post-War.

These figures are illustrated in the diagram, Fig. 32. It will be seen that the two groups are now about equal in terms of consumption but the non-industrial group contributes somewhat more revenue. This could be explained on cost grounds, since the distribution and consumer costs (per unit) are likely to be appreciably higher with smaller consumers (almost all low tension), though this is partially offset by a better effective load factor due to greater diversity.

Analysis of Load Growth.—The studies of previous chapters have shown the importance of breaking down complex quantities such as elasticity and load factor into their component parts, and the same

*This line is roughly the sum of the first two and the last lines of the previous tables (domestic, commercial and public lighting). The correspondence is not exact for the reason stated on p. 175.

is true of consumption and load figures, and even of prices. Statistics are notoriously misleading, and few more so than the figures so often quoted of consumption per head of population, or percentage increase in load, or the change in the price per unit. Such figures are useless until analysed in some way.

Consumption per head means nothing without some knowledge of how the electricity is used. Short of such information, the inhabitants of some remote village in Scotland or Norway may appear to possess far higher electrical development than the citizens of London or Paris merely through the proximity of a large electro-chemical works. Instead of an amorphous omnibus concept like total consumption per head one must at least know the *domestic* consumption per head, and if this can be further broken down into consumption per consumer and consumers per head of population the pattern of domestic electric development begins to emerge.

Similarly with load increases, an overall statement that electricity consumption has doubled in 10 years, however cheering (or alarming!) it may be, is little use without some analysis, first into the different classes of consumption and second into the two components of the increase, namely increase in number of consumers and increase in consumption per consumer. Such an analysis is attempted in the following table.

TEN YEARS' GROWTH

Value in 1947 expressed as a Ratio of the Value in 1937
(Third column = product of the first and second columns.)

	Number of Consumers.	Units per Consumer.	Units.
Domestic and Farm .	1·39 (10,500,000)	2·11 (1,200)	2·94
Commercial . . .	1·15 (1,200,000)	1·24 (3,300)	1·43
Industrial . . .	1·15 (150,000)	1·63 (115,000)	1·86

The figures, which are only estimated, show the proportional increase in the three quantities (consumers, units per consumer and units), *i.e.*, the ratio in which they have grown in ten years. (1937 = 1). Thus the number of domestic and farm consumers has gone up in the ratio 1 : 1·39, an increase of 39 per cent. The consumption per head of these consumers has gone up in the ratio 1 : 2·11, an increase of

111 per cent. Finally, the increase in domestic sales is the overall result of these two increases, and the ratio is therefore the product of their ratios, *i.e.*, 1·39 × 2·11 = 2·94—almost a threefold increase. The commercial and industrial consumers both increased by 15 per cent. and the units per consumer by 24 and 63 per cent. respectively. (Certain of the absolute values are also given—in brackets. These figures are only approximate.)

Commenting on the values of these increases, it will be noticed that in every case the growth in sales was due more to a greater consumption per consumer than to an increase in the number of consumers. This has been a large factor in bringing down the cost of production per unit (or in preventing it from rising so much). It has its counterpart in the price charged under block and two-part tariffs (other than the maximum-demand type).

Load Growth in Two Decades.—It is interesting to compare the character of the development in the two successive decades 1927–37 and 1937–47. This is shown in respect of the domestic and commercial load (lighting, heating and cooking) in the table below. In the first ten years the increase in load was due chiefly to the taking on of new consumers. The number of consumers per head of population actually increased three-and-a-half times (250 per cent. addition) whilst the consumption per consumer increased only 1·2 times (*i.e.*, 20 per cent. addition). In the second ten years the positions are reversed: the number of consumers increased only 1·2 times, whilst the consumption per consumer increased 1·7 times. The combined effect of the two increases gives a total increase per head of population of 4·2 times in the first ten years and 2·1 times in the second ten years. (By adding the affect of a small increase in the population of the area of supply, the overall increase in units is obtained.)

The reason for the difference in character of development of the two ten-year periods is presumably that a condition approaching saturation in the number of consumers per head of population was being reached in the later period. The consumers per cent. of population in 1947 (24½) may be taken as consisting of 21½ domestic and 3 commercial. This means one domestic consumer per 4½ persons. Allowing for the more distant premises whose connection is not immediately practicable, the figure is such that signs of saturation would not be unexpected. On the other hand, saturation of consumption per consumer is contra-indicated, since the second ten-year period shows a greater rate of increase than the first.

The overall effect of the twenty-year growth is analysed in the last column. Calling the 1927 figure unity, the 1947 figure shows the increase ratio. The total increase is the product of three factors, namely, increase of units per consumer (2·04) increase of consumers per head of population (4·2) and a small increase of population (1·08).

LIGHTING, HEATING AND COOKING : CONSUMPTION AND CONSUMERS

	1927–8.	1937–8.		1947–8:		20-Year Increase (Ratio).
			10-Year Increase (Ratio).		10-Year Increase (Ratio)	
Units per Consumer .	666	797	1·2	1,360	1·7	2·04
Consumers % of Pop.	5·8	20·1	3·5	24·4	1·2	4·2
Units per Head . .	38·5	160	4·2	332	2·1	8·6
Population (millions) .	44·3	45·8	1·03	48·0	1·05	1·08
Total Units . .	1,708	7,348	4·3	15,982	2·2	9·3

In calculating these figures, it has been assumed that the proportion of non-industrial consumers has remained at the same figure as in 1937, namely, 98½ per cent. of the total consumers, since a later breakdown was not available. It would be interesting to trace the corresponding development in the industrial load, but without knowing the number of consumers this is not possible. If, for example, the proportion of industrial consumers, which was 1½ per cent. of the total in 1937, had increased to 2 per cent. or dropped to 1 per cent. in 1947, it would not change the non-industrial calculation appreciably but it would entirely upset any analysis of the industrial consumption on the above lines.

Analysis of Price Change.—Perhaps the most surprising fact revealed in the table on p. 177 is that, in spite of the enormous increases in the price of coal and of almost all other items entering into the cost of supply, the mean price per unit for lighting, heating and cooking has actually gone down. The reasons are somewhat complex and before examining them it is necessary to consider what is meant by price change.

A change in price may mean either of two things, namely, a change in the price ticket (*i.e.*, the tariff) or a change in the average price paid per unit. Strictly, price should mean the former not the latter, but in the case of electricity it may be somewhat unrealistic to consider price only in its narrower sense of the amount paid for the same quantity of electricity because (*a*) very few people are, in fact, taking the same quantity as before and (*b*) most domestic electricity sales are on a steeply-sliding scale. The difference between the two definitions is clearly seen in the case of domestic-electricity prices in the decade before vesting-date,* because in this period the two 'prices'

* Vesting date is a convenient landmark in electricity prices as marking the end, for the time being, of certain downward trends. A variety of reasons conspired to delay the inevitable increase in domestic electricity prices during the war and immediately post-war years, and it was left to the Area Boards as one of their first task to put in order the houses vested in them.

actually moved in opposite directions, *i.e.*, the marked price went up but the mean price went down.

Although precise figures for the former change are not available, it is known that in this period a very considerable number of undertakings, possibly about one-third of the whole, increased some or all of their domestic tariffs, whilst only a very few of them made decreases. In the aggregate, domestic electricity prices, as strictly defined, certainly went up in this period even though, at the end of it, the average unit was cheaper than before. This agreeable piece of magic resulted from the two facts already mentioned, namely, that most consumers increased their individual loads, and that most supplies were furnished on promotional types of tariff giving a progressively lower overall cost per unit with increasing numbers of units.

A quantitative examination of this magic may be helpful. The figures in the table on p. 178 show that in ten years the average consumption per domestic consumer increased by 111 per cent., and the problem is to estimate the probable numerical result of this under the tariffs generally ruling. The E.R.A. sampling survey referred to at the beginning of Chapter XI indicated that only about one-fifth of domestic consumers are on single flat rates, and that they only account for about 3 per cent. of the total sales. Almost all other tariffs (multiple flat rates, two-part, block and variable-block, load-rate, etc.) are of a promotional character. Broadly speaking, therefore, almost the whole domestic consumption is charged for on tariffs with a falling-price characteristic. (Consumers changing over from flat rates to two-part produce a similar effect.)

From a typical curve for such tariffs, and making some assumption as to the probable proportions of fixed and running charges, it may be estimated that an increase in consumption of 110 per cent. would result in a drop in the price of about 50 per cent. Comparing this with the actual drop in mean revenue per unit over the period in question, it would seem that actual domestic tariffs in the aggregate rose in this period by some 20 per cent.

Mean Price Changes: Domestic and Industrial.—The foregoing may be summed up as follows in terms of the average cost of lighting, heating and cooking supplies. In the ten years 1937–47 the number of units purchased went up by 113 per cent. and the money paid for them went up by 76 per cent. Hence the mean price per unit went down somewhat. This resulted from the difference in the operations of two opposing factors, namely, (*a*) increased consumption per consumer under promotional types of tariff and (*b*) some increases in tariff prices.

In the case of industrial power and traction the units went up by 71 per cent. and the expenditure thereon by 143 per cent. Hence, the mean price per unit went up considerably. Exactly the same two factors operated, but in this case (*b*) was greater than (*a*). This is because

industrial tariffs tend to be less promotional in character than domestic and because price-increases have been more general, largely through the automatic operation of a fuel-cost variation.

Tariffs for Special Purposes.—In the past, many undertakings have had special tariffs for a number of particular usages beyond those comprised in the three categories of domestic, commercial and industrial. These special rates may be regarded as, on the one hand special bids to secure particular loads, and on the other hand as reflecting the special nature of the costs involved, whether below the average (*e.g.*, night-baking) or above the average (*e.g.*, welding). Reference could be made to one comparatively small undertaking which had thirty-one printed tariffs, and this, although unusual, was probably not unique.

In general this special character resolves itself merely into a greater or less degree of off-peak nature or diversity probability, and it seems preferable to design the standard tariff to take proper account of these factors, rather than to develop special *ad hoc* tariffs for each usage. The future tendency, therefore, will probably be away from such multiplicity, particularly in view of the requirements of the 1947 Act enjoining simplification and permitting special agreements only when the published tariff is not appropriate.

Except in the single case of welding, these special tariffs are not described here ; but the following, far from exhaustive, list will give some idea of the commonest of them ; shop-window and display lighting, cinema supplies, night baking, battery charging, large-scale cooking.

Statutory Requirements regarding Electricity Prices.—These are described below in a series of sub-paragraphs setting out the statutory position before and after nationalisation.

Pre-Nationalisation Price Control.—Prior to April 1st, 1948, there were various statutory provisions affecting electricity prices, most of them in the nature of protection to consumers. Maximum flat rates of varying magnitude were embodied in most of the Special Acts and Orders governing distribution rights, and these put limits to the amounts that could be charged to consumers on such tariffs. In recent years this protection had come to have very little effect because the actual price (owing to progressive reductions in charges and the increasing use of two-part tariffs) was usually well below the statutory maximum.

As a counterpart to this there was protection to the undertakings because, with a few exceptions, they were entitled on all flat-rate tariffs to make a minimum charge should energy to that value not have been consumed. The particular figure was specified in the original Order, but in 1942 a uniform minimum charge of 25*s.* per

annum (and *pro rata*) was substituted for the varying charges specified in the Orders. In 1944 the figure was reduced to 10*s*. per annum, thus giving a substantial consumer protection. This ceiling for the minimum charge continues to apply until the Order is rescinded.

A further protection to consumers was provided by the fact that when two-part tariffs were put forward these were almost invariably optional and there was a flat-rate alternative available. Finally, there was a provision for government appeal either by consumer or undertaking. The Minister, following representation by a group of consumers, by the undertaking, or by the Local Authority (if it was not also the undertaking), could make an Order varying the prices or methods of charge stated in the original Order.

There was also some degree of financial control, or at least influence, operating against excessive electricity prices. In the case of local-authority undertakings, there were limits to the amounts that could be transferred from the electricity undertaking and used in relief of rates, and annual surpluses (after meeting costs, capital charges and contributions to a limited reserve fund) in excess of 5 per cent. of the capital expenditure had to be used in the reduction of charges. In the case of companies, there were sliding-scale Regulations in a number of Acts and Orders, (relating principally to power companies and to the companies in the London area), linking dividend-increases with price-reductions. In general, these were ineffective because the prices actually charged were in most cases substantially below the figures which would have caused the sliding scale to operate.

Present Position—British Electricity Authority Area.—The above provisions and inducements (except the minimum-charge ceiling which is a piece of independent, but temporary, legislation) have been superseded by the Electricity Act, 1947. Under this Act, the 14 Area Boards took over some 540 separate distribution undertakings and inherited nearly this number of different tariff schedules. Each Area Board can charge prices according to such tariffs as it may fix from time to time, but until this fixing has been done the tariffs in force immediately before Vesting Date in any part of the Board's area have to remain in force.

When tariffs are fixed they must be framed so as to show the methods by which, and the principles on which, the charges are to be made, as well as the prices which are to be charged. They must be published and given adequate publicity These last requirements must also be satisfied by the tariffs which the Central Authority fix for the supply of electricity by them to the Area Boards.

The Area Boards also have power to make special agreements with particular consumers where the tariff in force is not appropriate owing to special circumstances. Both in fixing tariffs and in making agreements the Boards, like the undertakings before them, must not show

undue preference to any person or class of persons nor exercise any undue discrimination against any person or class of persons. Tariffs can include charges for fittings. As under previous legislation, the Boards can also make a charge for service lines, namely the cost of so much of the line as lies on the consumer's premises or lies more than 60 feet from the distributing main.

The Central Authority may give specific as well as general directions to an Area Board regarding tariffs, and in addition they have the general duty of securing that the combined revenues of all the Electricity Boards taken together are not less than sufficient to meet their combined outgoings properly chargeable to revenue account, taking one year with another. Within this fairly wide framework, the Area Boards would therefore appear to have the legal right to impose what tariffs they will, both in form and magnitude, and without the necessity for alternatives.

There is nothing to take the place of the financial inducements or checks, but it may be argued that the new set-up does not require them. Since the whole industry is now operating under publicly-owned, non-profit-making bodies, there is no incentive to overcharge, and there is no purpose outside the industry (whether rate-relief or dividends) to which any surplus monies can be diverted. While there is nothing to prevent excessive prices being imposed to balance unnecessary wage or salary payments, the same could be said of the position prior to nationalisation. (In the company-operated sector there was doubtless less motive for such dissipation.)

Present Position—North of Scotland Area.—In the area administered by the North of Scotland Hydro-Electric Board, the prices to be charged by the Board are to be determined by them in accordance with Regulations made by the Secretary of State, after consultation with the British Electricity Authority. Under Regulations made in 1946, the price to ordinary consumers must be such that the average charge for each unit does not in any year exceed 6*d.*, subject to a minimum annual charge of 10*s.* per consumer.

The same Regulations also provide that the Board *shall* fix a block tariff or tariffs and *may* fix an alternative flat rate or two-part or other tariff, subject to the maximum price already mentioned. The Regulations as amended in 1948 further provide that the tariffs fixed for different classes of ordinary consumers shall be uniform throughout the North of Scotland District unless a difference of tariff (within the maximum price) is specifically approved by the Secretary of State where circumstances justify it.

Consultative Councils.—In place of the specific protection to consumers afforded (on paper if not in fact) by such provisions as the maximum-price regulations, and in place of the power of appeal to the Minister there is a more general provision by the way of Consultative Councils.

These Councils are set up, one in each area, with the general function of representing the consumers' interests, and one of their duties is that of considering any matter affecting the distribution of electricity in the area, including the variation of tariffs. A consultative Council, if it cannot obtain satisfaction either from the Area Board or from the British Electricity Authority, can, as a last resort, make direct representations to the Minister. He, if he is satisfied that a defect is disclosed, can notify this defect to the Central Authority, who must then direct the Area Board to rectify it.

General Provisions.—In addition to these precise requirements, the Act also lays down certain general provisions under which the Electricity Boards have to secure as far as practicable the cheapening of supplies, to avoid undue preference, and to promote the simplification and standardisation of methods of charge. There are also provisions whereby the Minister may give directions of a general character to the Central Authority (and the Secretary of State to the North of Scotland Board), but it is not clear how far such powers could appropriately be used in relation to electricity prices.

Survey of Undertakings.—It is extremely difficult to get a clear and proportioned picture of the tariff situation as it existed prior to March 1948. In the first place, there were about 560 independent tariff-making authorities, one-quarter of whom sold 90 per cent. of the energy. If an inclusive survey is attempted, the result is a vast medley of tariffs, some of which were employed on so small a scale as to be not worth considering. Even if all the smaller undertakings are omitted, the position is by no means simple. Some undertakings quoted three or four alternative tariffs for the same service, and there is no way of finding whether these were all of equal importance or whether some of them were mere relics applying to only a handful of consumers.

A survey was, however, made of the pre-war situation by examining only those undertakings selling more than 10,000,000 units per annum. They numbered only 173 out of a total of 600-odd in the year considered (1936) but they were responsible for 91 per cent. of the energy sold to consumers. Two of these were power companies selling to a few big consumers and without published tariffs, so that actually only 171 undertakings were examined. Only two groups of load were considered, namely, industrial and domestic, thus omitting all special rates for shops, cinemas, etc., and all restricted-hour rates. The result was that a certain order emerged from the chaos, and crystallised into the four types defined at the beginning of the chapter.

The results of this survey, so far as industrial consumers are concerned, are made use of in the next chapter. Corresponding results were obtained for domestic consumers, but these have been superseded by figures given in the *B.E.A. First Report* and the latter are employed in the following chapter. In this case there was a later survey on

different lines by the Electrical Research Association, using a sampling technique, and use has also been made of these figures.

Notes on Nomenclature and Present Position.—Since on April 1st 1948 the sale of electricity to consumers was transferred from about 560 " authorised undertakers " to 14 Area Boards, it is necessary to adopt nomenclature in the descriptive portions which will span over these very different phases of management. Actually, the change was gradual in respect to tariffs, and many areas of supply much smaller than the territory of a complete Area Board have continued to be operated as a tariff unit pending equalisation over the whole territory. The looser term " undertaking " is used here in preference to the more precise (and somewhat more gruesome) term " undertaker ", and the same word applied to the post-nationalisation period may signify a sub-area or district temporarily operating as a tariff unit and corresponding to the area of supply of one of the vested authorised undertakers.

The position at the time this book went to press was that, whilst the Area Boards had made considerable changes in the money-value of the tariffs, they had made no considerable changes in form. Consequently, they were still operating what was in effect some 560 different tariff schedules. In the descriptive work which forms the greater part of the present section, the tariff position will therefore be described in pre-Vesting-Date terms as though the 560 undertakings' areas were still operating as separate and individual retailers. In this description the present tense will be used.

Committees and Recommendations.—A number of committees have been appointed from time to time to make recommendations on electricity tariffs, particularly domestic. The chief of these are listed below. When referred to elsewhere in the book they are indicated by the date, etc., as italicised here. The first two Reports were originally obtainable from H.M. Stationery office :

1925 Committee. (Report issued 1927).—Advisory Committee appointed by the Electricity Commissioners on Domestic Supplies of Electricity and Methods of Charge.

1929 Committee. (Report issued 1930.)—Advisory Committee appointed by the Electricity Commissioners on Uniformity of Electricity Charges and Tariffs.

1935 F.B.I. Committee.—Tariffs Sub-Committee of the Federation of British Industries on Industrial Supplies under the chairmanship of J. S. Highfield.

1946 Committee.—Committee appointed by the Electricity Commissioners on Uniformity of Electricity Tariffs. It was stated that an

interim report on domestic tariffs was made to the Electricity Commissioners in 1947, and that the Committee did not continue its work owing to the changes brought about by the 1947 Act. Whilst the report was not published, it is believed that the recommendations followed very closely those of the 1929/30 Committee, *i.e.*, in favour of a two-part tariff with a variable-block tariff alternative, the fixed charge or its equivalent to be based on floor area.

1948 (Clow) Committee appointed by the Minister of Fuel and Power to study the Electricity Peak Load Problem in relation to Non-Industrial Consumers. (Cmd. 7464, H.M. Stationery Office.)

B.E.A. Retail Tariffs Committee.—This internal Committee was set up by the British Electricity Authority in 1948 in pursuance of the requirements of the 1947 Act to promote the simplification and standardisation of methods of charge. A number of Sub-Committees dealt with industrial tariffs, rural tariffs, etc.

Whilst no published report has been issued, it has been stated in the Press that the Committee urged the Area Boards to press ahead with unification of tariffs within each area, and recommended the two-part or the variable-block tariff (preferably the latter), or the two combined as alternatives. Furthermore, that the fixed charge or number of units of the first block be based on the size of the premises determined by the number of rooms or (failing that) by the floor area.

CHAPTER X

INDUSTRIAL TARIFFS
(Including Commercial)

National Survey.—The following table gives the results, for the industrial load, of the survey described at the end of the last chapter.

	Number.	Out of a Total of—	Proportion. per cent
Undertakings selling over 10 million kWh . .	173	643	27
Total kWh sold by these undertakings (millions) .	9,262	10,210	91
Undertakings offering flat rates (of these, four gave discounts of 20 per cent. or more for quantity) .	55	171	32
Two-part M.D. tariffs (of these, half based their fixed charges on kW, half on kVA . . .	58	171	34
Sliding-scale reductions in these tariffs were given in	25	58	43
Two-part tariffs based on connected load . .	9	171	5
Total two-part tariffs	67	171	39
Variable-block tariffs (block size based on M.D. or H.P.)	25	171	14
Total two-part and variable-block . .	92	171	53
Block tariffs based on quantity alone (average number of block, 3·7)	79	171	47
Total block and variable-block tariffs . .	104	171	61
Tariffs varying with summer and winter . .	6	171	3½
Tariffs varying with time of day	1	171	0½

As already explained, the survey covered the published tariffs of 171 authorised undertakings distributing about 90 per cent. of the energy sold to public consumers. Of these undertakings, about one-third offered simple flat rates, either as their sole tariff or alternative to others. Approximately the same number offered two-part maximum-demand tariffs with the fixed charge based either on kW or kVA (half of each sort). The mean of the numerical ratio between annual fixed charge in £ and running charge in pence was 11½. (For this purpose the kVA charge was multiplied by 1·25, on the assumption of a power factor of 0·8.) In addition, there were nine two-part tariffs in which the fixed charge was based on connected load, either H.P. or kW, instead of on metered demand. (For brevity, this will be referred to as the H.P. basis.) Taking the M.D. tariffs only, 22 per cent. employed a sliding scale for the fixed charge, 14 per cent. did so for the running charge, and 7 per cent. did so for both. Thus, nearly half the M.D.

tariffs were combined with a certain amount of sliding-scale reductions for quantity.

A further group of tariffs were of the variable-block type. These took the form of a block tariff, usually with only two blocks, one very much more expensive than the other. The size of the first block depends on the consumer's M.D. or H.P.,* so that it becomes the equivalent of a fixed charge without the actual levying. Altogether, the M.D. principle formed the basis of just over half the tariffs listed.

The next line refers to the pure quantity block rates, in which the size of block does not vary with the consumer. This forms the largest single type examined, totalling 47 per cent., and the average number of blocks was 3·7. If the variable-blocks are included, the total number of tariffs of the block form becomes 61 per cent.

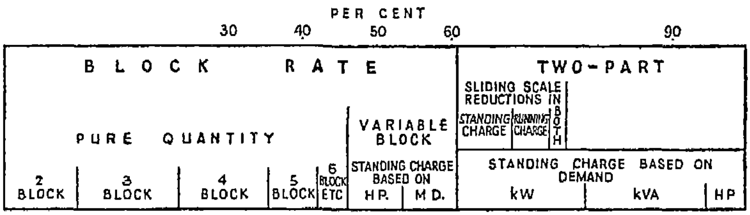


FIG. 33.—Diagram of Industrial Tariffs.

In addition to these main types there were six tariffs in which the charge (or the running portion thereof) varied between summer and winter, whilst in one case there was a time-of-day variation.

The chief results are shown diagrammatically in Fig. 33. The total number of block and two-part tariffs together just equalled the number of undertakings considered (represented by the percentage figures at the top of the diagram). This is because almost all the undertakings offered one or the other, and the few who did neither are just balanced by those who did both.

Two Main Types.—It will be seen that, leaving out the flat-rate tariffs, there are two main principles employed in charging for industrial electricity, which may be called the "power and energy" principle, and the "discounts for quantity" principle. Broadly speaking, the former represents costs whereas the latter represents use-value and has an eye chiefly for the selling aspect. From the costs point of view, only about a quarter of the total expenses of supply are directly proportional to the energy consumption and a large amount of the remainder is demand-related, so that a standing charge based

*For some not very obvious reason, in the variable-block tariffs the tendency is to make the first block size dependent on the *installed* capacity rather than on the *metered* demand.

on maximum demand seems the only logical way of recovering a considerable proportion of the costs. (It was seen in Chapter V that this " logic " is not entirely beyond reproach.) From the selling point of view, the outstanding feature of industrial competition is that the large consumer has a better chance than the small one of economically generating his own supply, and must therefore be given better terms.

Both these broad statements require some qualification. On the one hand, the M.D. tariff, owing to the operation of differential diversity, does not represent costs so closely as it pretends, and it does give some advantage to the large user. On the other hand, the block tariff can claim to have some correspondence to costs. For, owing to the existence of that non-proportional residue of expenses called " consumer costs," the total expense of supply is less per kWh to a large consumer even of the same load factor, voltage, etc.

But since the true consumer costs are only two or three pounds per industrial connection, whereas the average first price of a block tariff is twice that of the final rate (representing a discount of many hundreds of pounds to the large user), the one can hardly justify the other on cost grounds. Moreover, when the aim is merely to cover consumer costs and not to give the large buyer a substantial market advantage, this can be perfectly well done under the M.D. tariff by means of a non-proportional element in the standing charge. Taking it by and large, therefore, the M.D. rate is designed to be a " costs " tariff representing production values, whereas the block rate is a " sales " tariff representing utility values in the competitive power market.

M.D. Tariff.—The next few sections refer to the maximum-demand tariff, number III in the chart. The suggestion was originally made by the late Dr. John Hopkinson in 1882 that the tariff should be made up of two parts, one to pay for the fixed costs of supply and the other to pay for the running costs. Associated with his name is that of Arthur Wright, who developed a simple maximum-reading ammeter with an inverse time-lag, which (on a known voltage) gives a measure of the effective power demand. (It is noteworthy that in America Tariff III is generally known as the " Hopkinson " and Tariff IIIA as the " Wright " tariff.)

The advantages of this tariff may be briefly summarised as follows: It is a very close representation of actual costs, provided there are no diversity vagaries, *i.e.*, provided the individual consumer's demand is a correct index to the system demand. It is logical in the respect that the two major elements of cost—power and work—are separately metered and separately charged for. Its character—that of a fixed rental or overhead charge plus a proportional or running charge—is now fairly familiar to the factory owner and is used in other services, *e.g.*, the telephone. When desired, bad power factors can be penalised

(and in fact over-penalised) by levying the fixed charge on kVA. It forms the basis of the B.E.A. tariff for bulk supplies so that Area Boards who use it for their clients can buy and sell on similar terms. Finally, the consumer can at any time check his readings, which are such as to encourage energy consumption (low kWh charge) whilst discouraging large power demands.

Unless the tariff includes sliding-scale reductions with size of load, it will be defective from the use-value or sales aspect. The large industrialist can compete with a public supply far more effectively than a small-power user can : probably, the cost per unit of private generation in a large works would be less than half that in a small one. If a similar ratio is to appear in the selling price of the supply authority, this can only occur (on a uniform M.D. tariff) when the fixed charge is high and the big user has a considerably better load factor than the small one. In the tariff described below, there is a numerical ratio of ten between the two parts. The big user would then pay half as much as the small one under the following conditions :—

Load factor of big user . . .	40	30	20	per cent.
Corresponding factor of small user .	11·5	9·7	7·3	,, ,,

Unless there is as big a difference as the above between the load factors of the big and small users, the M.D. tariff employing uniform values will not represent equal sales-value to the two.

Minor disadvantages lie in the necessity for two meters and meter-readings, which becomes a serious item when the account is only a small one. Furthermore, a standing charge is always to some extent unpopular and may be very difficult to justify—especially when a large bill is run up through spasmodic peaks which only occur very occasionally.

Example and Incidence.—The following is a typical two-part power tariff (pre-war values) : £5 per annum per kW of maximum demand plus ½*d.* per unit consumed ± 0·01*d.* per unit for each 1*s.* per ton rise or fall in the price of a specified grade of coal, now standing at 14*s.* a ton. The M.D. to be measured by a maximum-reading ammeter or other device taking not less than twenty minutes to come to its full reading, and over periods not exceeding three months. With such a tariff, the lowest overall price per unit will naturally be paid by a consumer whose load factor is 100 per cent., and will be $\frac{5 \times 240}{365 \times 24} + \frac{1}{2} = 0{\cdot}137 + 0{\cdot}5 = 0{\cdot}637d.$ per unit.

For a consumer whose load factor is L (L being fractional) the total cost per unit will be $\frac{0{\cdot}137}{L} + \frac{1}{2}d.$, so that it will become 1·87*d.* per unit when the load factor is 10 per cent., and so on.

In general, a charge of £1 per annum per kW amounts to $\frac{1}{36\cdot5L}$ pence per kWh, where L is the load factor as a decimal. This is worth noting and is easily remembered from the number of days in the year. If L is expressed as a percentage, the figure becomes $\frac{1}{0\cdot365L}$ or $\frac{2\cdot74}{L}$ pence per kWh for every £ per annum per kW of fixed charge.

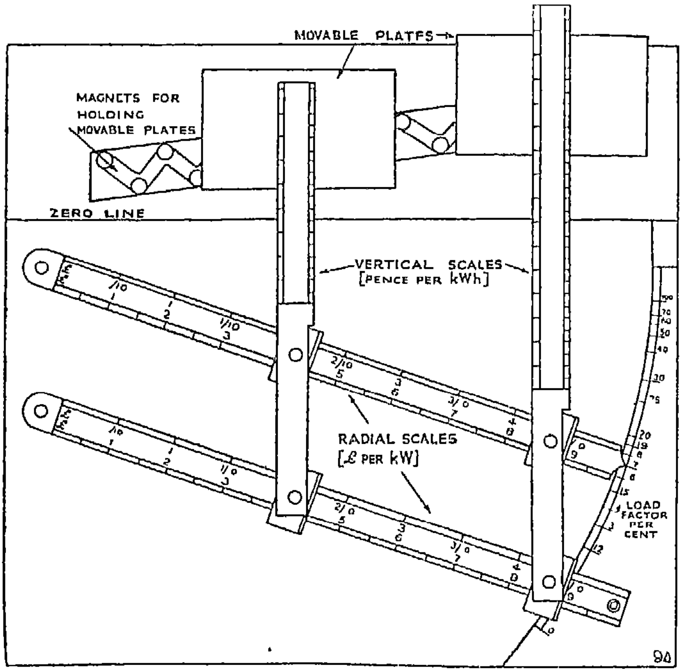


FIG. 34.—Two-Part Tariff Indicator.

Fig. 34 shows a two-part tariff indicator which has been devised by the author to demonstrate the effect of such tariffs as the above. It may be regarded as a working model of the two-part diagram shown in Figs. 12 and 13, in which the scale of the portion below the zero line varies with the load factor. The movable plate, held in position by magnets, is set to the unit charge, and the clips on the radial scales are set to the fixed charge. The intersections on the vertical scale then show the incidence of the fixed charge and the overall price per kWh at any desired load factor. In the model shown, the scales are duplicated so that two sets of tariffs can be demonstrated simultaneously.

Fig. 35 shows graphically the operations of the above tariff. The base is the annual load factor, to a uniform scale, and the curve is a rectangular hyperbola standing on a rectangle as explained on p. 174. Many power users do not appreciate the meaning of load factor, and for their assistance two further scales are appended. As an example, suppose that whenever the factory motors are in use they take, on the average, 70 per cent. as much power as they take at full-load periods.

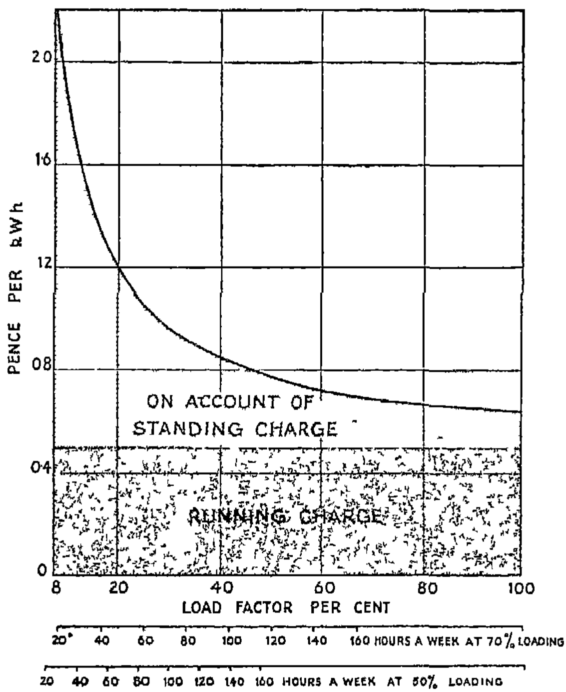


Fig. 35.—Two-Part Tariff. (Uniform base-scale.)

Then if they are in use, say, 50 hours a week the load factor will be be 21 per cent. The two lower base scales are constructed on the assumption of average loads of 70 per cent. and 50 per cent. respectively, and they will be some guide to the factory manager in gauging his probable overall price per kWh.

There are two objections to this form of graph. It is troublesome to draw owing to its hyperbolic shape, and it confines the really important load factors—say, those lying between 8 and 40 per cent.—to about a third of the picture. Both these objections can be overcome, at the cost of some trouble in scaling, by the employment of an inverse or reciprocal base-scale.

Fig. 36 shows the same values as the previous figure. The ordinate scale is identical, the only difference in arrangement (and this is not an essential one) being that the running-charge portion is placed on top of the other instead of beneath it. The base is scaled in load factor as before, but the distances of the scale-markings from the extreme right-hand end (marked ∞) are inversely proportional to the numbers marked thereon. Thus the 10 per cent. mark is twice as far from the right-hand end as the 20 per cent. mark, and so on.*

Since the standing charge when expressed as a cost per kWh is inversely proportional to the load factor, its graph will be directly

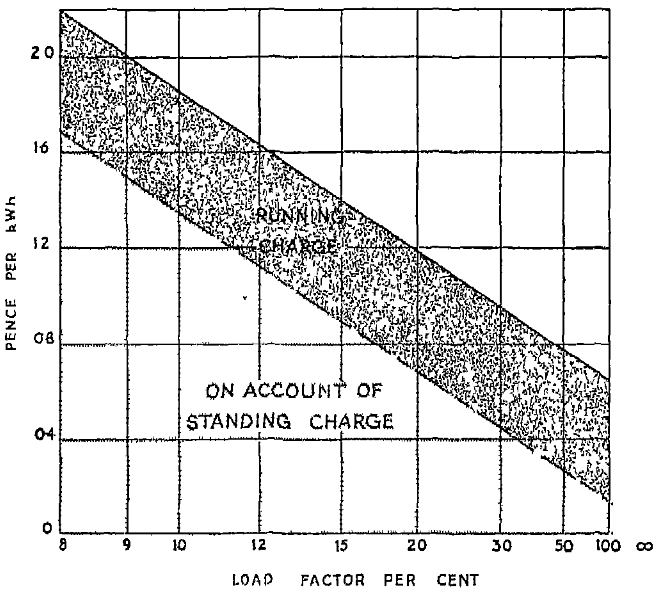


FIG. 36.—Two-Part Tariff. (Inverse base-scale.)

proportional to distances from the right-hand end, and therefore a falling straight line. Its value at 10 per cent. load factor is 1·37*d.* (see above), so that a straight line drawn through this point and the infinity point will determine the graph. The running charge is ½*d.*, and as the ordinate scale is a uniform one this gives a second straight line lying 0·5*d.* above the other.

Variable-Block.—It was shown in the previous chapter that almost identical results to those of the two-part maximum-demand tariff can be obtained with a variable-block maximum demand. Taking the two-part values used above, the effect can be achieved by the following:

* This method of plotting was suggested by Prof. R. O. Kapp, and special graph-paper can be obtained for the purpose.

2*d.* per kWh for the first 800 hours use per annum of the M.D. and ½*d.* per kWh for all subsequent consumption. The price curve will be the same as that shown in Figs. 35 and 36, except that it will have a flat top as shown at *A*, Fig. 30.

The diagram in Fig. 33 shows that such tariffs are extensively employed in industry, though chiefly for the smaller-scale user. It is doubtless for reasons of size that the number of units in the high-price block is frequently dependent on installed load rather than on metered M.D. Such an arrangement was particularly recommended by the 1929 Committee for small industrial users. In constructing such a tariff it is clear that the difference between the two prices multiplied by the size of the block must be equal to the equivalent standing-charge required to be collected.

Demand-Integration Periods.—When the Merz type M.D. meter is employed, the demand mechanism is meshed to the energy mechanism for successive periods such as half-an-hour (the demand-integration period) after which it returns to zero and is re-meshed, leaving behind it a " high-water mark " of its previous travel. The maximum-demand reading is then a corresponding multiple of (in this case twice) the largest number of kWh consumed in the period. Unless the load curve is absolutely flat-topped, the demand reading will be a maximum when the period is infinitely short (instantaneous M.D.) and will decrease as the period increases. In a number of cases investigated by the author,* the decrease per cent. of the instantaneous value was found to be approximately $ct^{0 \cdot 6}$, where t was the integration period in minutes and c was a constant varying from ½ to 1½ according to the " peakiness " of the load curves.

In this country the tendency is to standardise on half-hourly periods. The B.E.A., like the C.E.B. before them, use this in their bulk-supply tariff, and most of the separate undertakings in the past did the same in their retail tariffs, though fifteen-minute periods were not uncommon, and both 20 and 45 minutes have been used. A short period penalises the consumer unduly for his short-lived overleads, whilst a long period may mean that he evades some of the costs he is incurring : probably half-an-hour is a reasonable compromise.

As regards timing, it is usual to start the integration periods at particular clock-times, *e.g.*, 12.00, 12.30, 1.00, 1.30, etc., with half-hour periods. This introduces a fortuitous element into the recording, particularly of a very peaky load. Thus any peak lasting less than an hour has a chance of not being recorded at its full value, whilst if it lasts only half an hour it may exactly straddle the integration period and be recorded at only half-value. In practice, the actual record of a short overload will vary between these two limits. depending on the accident of its timing relative to the M.D. timing.

* *Journal I.E.E.*, 1942. **89**, Pt. II, p. 95

When a thermally-operated meter is employed (*i.e.*, a sluggish, maximum-reading ammeter), this gives an epitome of recent past history independent of particular clock times. It is, therefore, sounder in principle as a guide to peak-load responsibility, but its reading cannot be so precisely defined. It should not be impossible to develop a demand meter which would record the same sort of " floating half-hour " by mechanical registration. At intervals of, say, one minute the mechanism would add the kWh consumed during that minute and subtract the kWh consumed in the corresponding minute half-an-hour earlier. It would not be a perfect " mirror " of supply-plant heating, since it would take no notice of consumption which occurred more than half-an-hour back, but it would have the advantage over any thermal instrument of recording a precise and definable quantity, *viz.*, the largest consumption in any consecutive thirty minutes.

Demand-Assessment Periods.—A more important factor affecting the demand charge payable in respect of any given load is the length of the assessment period, or (to put the matter reciprocally) the number of times per annum the demand record is read and re-set to zero, and the demand-charge based thereon.* The less frequently this is done, the lower will be the load factor calculated therefrom, and the worse will things be for the consumer. In order to see this, let it be assumed that the daily load curves are all of the same shape and differ only in height. Then if the M.D. were read each day, and the load factor worked out, it would be the same for every day in the year. But if any two or more days are taken together (unless the days are identical) the M.D. reading will be that of the biggest day, whereas the energy consumption will be the average between this big day and others not so big. Hence the ratio of the average daily energy to the biggest daily power will get less as the number of days increases. (See example on p. 60 and Fig. 9.)

The B.E.A. bulk-supply tariff has an annual charge based, as was the C.E.B. Grid tariff, on the largest single demand recorded in the year,† and many distribution authorities in the past have taken the view that the same plan should be followed in the tariff they offered to their industrial consumers. In the author's opinion the comparison is a mistaken one, and based on an entirely wrong conception of the respective functions of the wholesaler and retailer. The terms of sale in Covent Garden may not be appropriate to the local greengrocer who has to serve the small casual shopper as well as the large regular one. The retailer of electricity should be equally ready to sell casual kilowatts on a reasonable price margin, realising that a single

* The record may be read monthly, but the demand charges based on the largest reading of the year. The assessment period is then a year, not a month.

† It is true that the blow is softened by averaging over two successive years and by a reduced rate for night readings, but the basis is still a yearly maximum.

half-hour demand does not in fact monopolise plant of that magnitude for a whole year.

Moreover, the B.E.A. bulk-supply tariff differs from retail consumer-tariffs in a number of ways, besides the obvious difference between large wholesale transactions and small retail ones. In the first place, it is not so much a tariff regulating the transactions of buyer and seller as a piece of internal book-keeping spreading the costs of a single operating unit (the supply industry of Great Britain) amongst its component parts. In the second place, as regards the kilowatt charge this is based on the aggregate effect of a very large number of readings taken at the various bulk-supply points, and therefore any anomalies such as those associated with M.D. timing, frequency of reading, off-peak demands, etc., are completely swamped. Finally, the comparative similarity of bulk-supply load curves means that the variations due to any of these factors are far less than they are when the load curves are more diverse.

Criticism of M.D. Tariff.—There are a number of anomalies in M.D. metering, some of which have been touched upon above and which become more serious the smaller and peakier the load in question. Probably the most serious of them all is that the same annual charge is made to a consumer who reaches 100 kW on a single half-hour in the year as is made to a consumer whose load reaches this value every working day in the year and for many hours each day.* Methods of compensating for this differential-diversity effect by way of reducing the standing charge and increasing the running charge have already been noted. Another modification which will help in this direction is a shorter assessment period.

By examining a number of load curves,† the author has estimated that if a single annual charge is to be replaced by four quarterly charges, or twelve monthly charges, the corresponding rate per kW may have to be increased by as much as 10 per cent. and 25 per cent. respectively in order to bring in the same revenue. Even so, probably most industrial consumers, especially the smaller ones, would welcome such a change. By thus shortening the assessment period, *e.g.*, by exchanging £6 per annum per kW for 10*s*. plus *x* per cent. per month per kW, where *x* is the percentage necessary to secure the same revenue, a much nearer approach to true costs can be obtained. Since M.D. consumers are generally served with monthly accounts, the change would not necessarily increase the meter-reading and billing work involved.

* A hardly less-serious flaw is that simple M.D. metering only indicates the magnitude of the load factor, not its character: i.e. whether or no the demand occurs at a time when it is likely to swell the system peak. This, however, raises a very fundamental issue and cannot be remedied by any simple modification. It implies a change in the nature of the tariff itself to something of a time-of-day character.

† *Electrical Review*, June 10th 1942.

Another method which gives an even better representation of true costs is to confine the M.D. readings to normal factory working-hours and to base the charge on the average of, say, the 4 or 6 winter months, disregarding all demands outside these periods. At bottom, the aim of any such discrimination is to give the consumer full credit for all haphazard diversity, *i.e.*, irregular variations not likely to coincide with those of other consumers, whilst charging him for regular variations likely to synchronise with those of others. Thus, a jobbing repair shop liable to have a short high peak at any time of the year should pay less than firms whose peaks are always in the cold months. If, in addition, the greater frequency of high demands during the period is also penalised by taking the average of a number of M.D. readings, much better cost representation will be achieved.

Reference may be made here to a Committee appointed by the Federation of British Industries which met in 1935 under the chairmanship of Mr. J. S. Highfield. Its purpose was to consider the possibility of obtaining greater uniformity in the basis of two-part tariffs for industrial supplies. In its conclusions, it agreed that a 30-minute integration period should be used, and that the instrument should record the maximum average load as measured during successive periods of 30 minutes.

As regards the assessment period, complete agreement could not be reached. A substantial proportion of undertakings (probably a majority) were willing to make separate maximum-demand charges in respect of separate monthly measurements. Others felt that since the Central Electricity Board metered and charged them on an annual basis, they should do the same with their consumers, although in fairness they should then only count the four dark months of the year.

Another recommendation of this committee concerned the proportion of lighting energy to be allowed at power rates. It was suggested that, provided the installed capacity of the lighting equipment did not exceed 20 per cent. of the capacity of the whole installation, there should be a uniform charge for both power and lighting.

The characteristics of the two-part M.D. tariff as normally operated on industrial supplies could then be summed up critically and somewhat unconventionally as follows. supplementing the discussion at the end of Chapter V: Although designed purely to represent costs, it can in fact be made to correspond very closely to use-values by the inclusion of sliding-scale reductions for quantity. On the other hand, as a cost vehicle it has serious defects.* These could be greatly reduced

* It is not suggested that the maximum-demand tariff, even with these defects is unfitted to represent costs and only reflects use-values. Actually, with industrial supplies, costs and use-values are very similar, and the maximum-demand tariff can make a fair show of representing them both. The industrialist's alternative to purchasing electricity is to generate his own power, and the costs of a private powerhouse follow much the same pattern as that of the public-supply system. The main difference lies in the lessened scope for diversity.

by reading the M.D. meter more frequently and confining its operation to potential peak periods. (The one change could be effected with the existing apparatus and staff, but the other would mean much additional equipment and probably remote-control installations.) Thus, if the demand meters were read monthly during the period mid-October to mid-April, and if they were cut out of action each day from, say, 7 p.m. to 7 a.m. a standing charge based on the average of these readings would be a much closer allocation of demand-related costs.

Installed Load.—With small consumers, the cost of two-part metering is a relatively large item. The standing costs may then be assessed on the capacity of the plant installed, instead of metering the M.D. (It does not, of course, follow that the same rate per kW will be charged. The actual demand may never equal the total installed capacity, and some allowance in the rate may be made for this likelihood.) The standing charge is then known as a *fixed* charge, as distinct from a kW or kVA charge, since it does not fluctuate from month to month or year to year.

The basis of the fixed charge may be the installed load (either in kW or kVA), *i.e.*, the sum of the rated inputs of the consuming apparatus, excluding stand-by and duplicate plant. Alternatively, it may be based on the *output* of the plant in mechanical horsepower. In the case of commercial loads, it may be based only on certain parts of the connected load, *e.g.*, the lighting installation.

It is clear that with a large consumer the computation of the installed load would be a matter of some difficulty and likely to raise a number of queries. This method is therefore more suited to small installations.

Block Tariff.—This type of tariff—numbered II in the chart—has been taken after the Hopkinson tariff because, although simpler in principle, it is complicated in practice owing to the large number of variables. In essence, the block tariff is a development of the simple flat rate per kWh with a progressive and extensive discount for quantity. The annual consumption is marshalled into a series of " blocks," usually about four in number, at progressively lower prices. As soon as the consumption overflows any particular block *the additional units* are charged at the lower rate appropriate to the next block.

The essential difference between the two-part and the fixed-block tariff does not lie in the presence of a standing charge—that can be avoided by a " spread-over " as in the variable-block tariff. The real difference is that in Tariffs III and IV (including the A variations) the standing charge or the size of the high-price block varies with the consumer, either according to his M.D., installation, or house size. But in the plain block tariff (II) the block sizes are fixed, so that the large

consumer automatically pays less per unit than the small one, whatever his electrical characteristics.

It has been seen that about one-quarter of the cost of supply is proportional to kWh, a certain fixed sum (two or three pounds per industrial consumer) is a *per capita* expense, and the rest is proportional to kW demand. The block tariff certainly covers (not to say smothers) " consumer costs," but otherwise its price differentiation cannot be said to have any direct relation to production costs. The particular character of the block tariff is therefore that it represents the value to the consumer rather than the cost to the undertaking.

Apart from the primary characteristics mentioned above, the chief merit of the block tariff is that it is simple to explain and to operate, since it involves only one quantity, namely, energy, and only one meter. It is usually more popular with the consumer than a tariff having a standing charge : the consumer knows where he stands and exactly what each unit of energy will cost him. Moreover, he is encouraged to consume more extensively when the added energy comes in at a cheaper rate.

One disadvantage of the tariff, apart from the major objection that it takes no account of demand or load factor, is that having so many variables it is almost impossible to standardise. In fact, it rarely happens that any two undertakings using it have exactly the same tariff. A further disadvantage is that power-factor penalisation is less simple than on the M.D. tariff, and quite spoils the simplicity and single metering.

Its sphere of utility in this country is virtually confined to the industrial load, though in America it is used also in the domestic field (*e.g.*, the Tennessee Valley Authority, p. 205). When applied to power loads which are fairly uniform in character and hours of use, it can give a fair approximation to costs owing to the operation of differential diversity.

Example and Incidence.—The following is a typical example of a block tariff, that of Halifax Corporation :—

3*d.*	per	kWh	for	first	1,000	kWh	per	annum.
2½*d.*	,,	,,	,,	next	1,000	,,	,,	,,
2*d.*	,,	,,	,,	,,	2,000	,,	,,	,,
1½*d.*	,,	,,	,,	,,	8,000	,,	,,	,,
1*d.*	,,	,,	,,	all subsequent consumption.				

At low consumptions this is simply a flat rate of 3*d.* When 1,000 units per annum are reached the cost is 1,000 × 3*d.*, which may be written 1,000 × 2½*d.* plus 1,000 × ½*d.* or plus £2 1*s.* 8*d.* From this point it operates exactly like a two-part tariff with a fixed charge of £2 1*s.* 8*d.* and a running charge of 2½*d.* When 2,000 units per annum are consumed the total cost is £22 18*s.* 4*d.*, which may be written

2,000 × 2*d.* plus £6 5*s.* Above this point it therefore operates like a fixed charge of £6 5*s.* and a running charge of 2*d.* Above 4,000 units a year the equivalent fixed charge becomes £14 11*s.* 8*d.* and the running charge is $1\frac{1}{2}$*d.*, whilst above 12,000 units a year the fixed charge is £39 11*s.* 8*d.* and the running charge 1*d.*

The following table gives the mean price formula for this tariff for any annual consumption *n* :—

Range of Consumption.	Price Formulæ (Pence per kWh).
Up to 1,000 kWh per annum . .	3
From 1,001 to 2,000 kWh per annum .	$\frac{500}{n} + 2\frac{1}{2}$
„ 2,001 „ 4,000 „ „ „ .	$\frac{1{,}500}{n} + 2$
„ 4,001 „ 12,000 „ „ „	$\frac{3{,}500}{n} + 1\frac{1}{2}$
Above 12,000 kWh per annum . .	$\frac{9{,}500}{n} + 1$

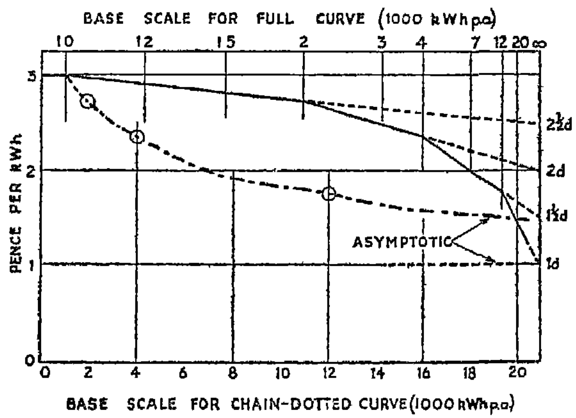


FIG. 37.—Block Tariff (Halifax).

The block tariff is therefore like a sequence of two-part tariffs, and when plotted to an even base-scale it becomes a series of hyperbolæ standing on different base lines. Fig. 37 (chain-dotted line and bottom base scale) shows the Halifax tariff on a uniform base-scale. It begins with a horizontal portion at 3*d.* (as in the A tariffs in Fig. 30), and then falls in a series of curves. The start of each new curve is indicated

by a small circle, and the final portion is asymptotic to the horizontal line at 1*d.*, *i.e.*, it would meet this line at a consumption of infinity.

Apart from the difficulty of drawing the curves, this graph suffers from the objection that the lower consumptions, from 1,000 to 4,000 kWh, are difficult to read off. Both these objections can be overcome by using an inverse base-scale, which results in a purely straight-line figure.

Inverse Base-Scale.—The upper (full line) graph and top scale of Fig. 37 shows the same tariff plotted to an inverse base-scale (ordinate scale unaltered). The method of constructing this graph is as follows: Any convenient base line, say 10 inches long, is taken, the right-hand end being marked ∞ and the left-hand end marked 1 (to represent 1,000 units per annum). Then the distance of any scale-marking from the right-hand end will be $10/x$ inches where x is the scale-marking (in thousands of units). It follows that the scale-mark for 2 will be 5 inches from the right, that for 4 will be $2\frac{1}{2}$ inches from the right, and so on.

Using this base-scale, the graph will have a height of 3*d.* at the base point 1. From 1 to 2 it will be a falling straight line such that, if continued, it would reach the height $2\frac{1}{2}$*d.* at the infinity ordinate (see dotted continuation). At the base point 2 a new block rate sets in, and the line takes a new downward inclination. If continued, this would meet the infinity ordinate at the height 2*d.* At 4 thousand units another block price sets in, and the line inclines in the direction $1\frac{1}{2}$*d.* At 12 thousand the final price sets in, such that the line would reach the height of 1*d.* at a consumption of infinity. Consumptions below 1,000 are at a uniform price of 3*d.*, and are shown by a horizontal line in both graphs.

It will be noted that the graph when plotted to a single inverse base-scale (upper curve) suffers from precisely the opposite fault to that previously noticed. Instead of compressing the early part of the curve it unduly compresses the later part. This can easily be remedied by extending the base-scale to the right by means of a new infinity point, and an example is shown in Fig. 40. When the graph is used to illustrate the operation of one particular block tariff, a useful plan is to let the graph consist of a single descending straight line. This can be arranged very simply by the following construction, the values referring to the particular tariff illustrated above.

Draw a descending straight line of any convenient slope, as shown in Fig. 38. Where this passes through the heights corresponding to the block prices 3*d.*, $2\frac{1}{2}$*d.*, etc., drop perpendiculars, *A*, *B*, *C*, *D* and *E*. Then *A* will represent a consumption of 1 (thousand units per annum) and *B* will represent infinity. The first part of *AB* can then be scaled inversely in the manner already described. When the point corresponding to 2 thousand consumption is reached, which will be at the

point X mid-way between A and B, a new scale is started having C for its infinity point. The distance XC is then divided up inversely until the point corresponding to 4 thousand is reached at Y (mid-way between X and C). A new scale starts here, having D for its infinity point : this is used until the 12 thousand mark is reached at Z, where ZD is one-third of YD. The remainder of the base line ZE is scaled inversely, Z representing 12 and E representing infinity.

When a single chart is to be used for a number of different tariffs, the best plan is to arrange the base in two or three consecutive sections, in the manner shown in a later example. Whenever the scale change coincides with a block-price change the results will be simplified.

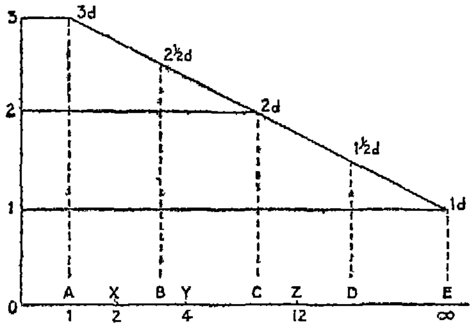


FIG. 38.—Diagram of Scale Change.

Further Examples.—Possibly the charm of the block tariff to the supply undertaking (if not to the consumer) is its infinite variety. Not only the number of blocks but the size, sequence and price of each block are all capable of independent variation—a truly terrifying number of permutations. Supply engineers have shown themselves fully alive to the endless possibilities of being " different," and the fact that one area has found certain values to be satisfactory seems to have been an adequate reason for using different values in the adjacent area.

The survey described above was certainly not very encouraging as to the prospects of unifying this type of tariff. Out of 171 published schedules, block rates were put forward for power sales by 104 undertakings. But when these 104 tariffs came to be examined they were found to consist of 102 different patterns. If the whole 600-odd undertakings had been examined, there would probably be found to be 300 or 400 block tariffs—all different.

Further examination did not disclose any one cause for all this variation or any definite trend or mean. The number of blocks varied from two to nine, although three and four appeared to be the most popular numbers. In some cases all the blocks were of the same size,

whilst in other cases (and these were the great majority) the block sizes increased progressively. The range of price between that of the first block and the final follow-on rate was also a very wide one, the average ratio being two to one.

The task of collating these more than 57 varieties of block tariff is work for the superman. Even to compare any two of them is by no means easy, and if one were to ask which tariff is cheaper, Liverpool or Colchester, East Ham or Loughborough, probably the engineers themselves would be hard put to for a reply. The most satisfactory

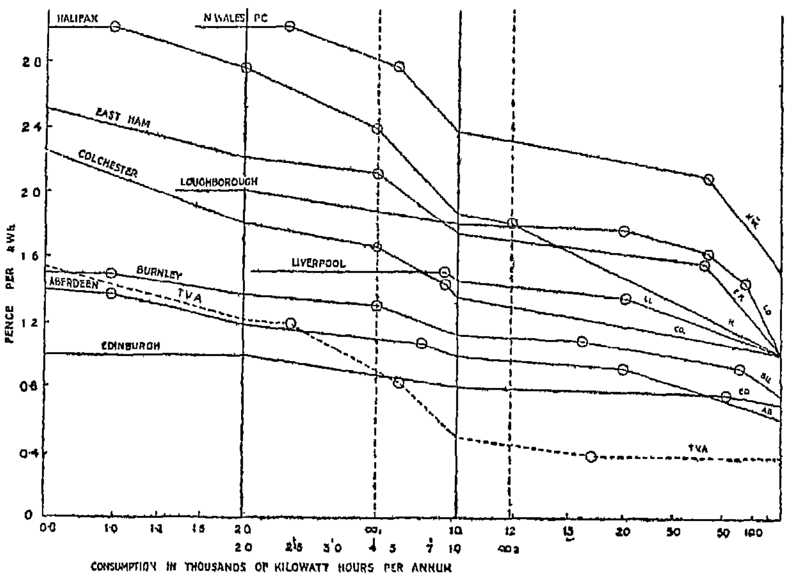


FIG. 39.—Comparison of Block Tariffs.

basis of comparison would be the mean price per unit at various consumptions, and here the inverse-scaled graph comes in useful. (The alternative basis of comparison, namely, the total revenue corresponding to different consumptions, can be illustrated by a "polygon" diagram as in Figs. 43 and 44.)

Fig 39 shows nine different block tariffs, taken almost at random from the published tariff data. They all refer to industrial loads, and the points of rate-change are indicated by small circles. The base is in three sections, scaled back from the three infinity marks shown, the changes occurring at 2 and 10 thousand. The scales for these are shown on different levels: this clarifies the construction but makes the finished graph look more complicated than it really is. On the

same graph is shown, dotted, one of the domestic tariffs for the Tennessee Valley Authority.

It may be felt that the base-scale is unduly complicated, but it is doubtful whether any other scaling would be so effective in unravelling the intricacies of this type of tariff. In the examples shown, there are some thirty blocks of various sizes and prices. On a uniform base-scale, this would result in thirty small hyperbolæ—very troublesome to draw and still more troublesome to compare and interpret.

Step Tariff.—The step rate is exactly the same as the block rate except that, when any given block size is exceeded, all the units and not merely the additional ones are charged at the lower rate. If such a tariff were offered without qualification there would be a discontinuity between the steps, and at certain points a small increase in consumption would mean a reduced bill. This has to be prevented by some further provision, and as a result the tariff schedule is more complicated than that of a block tariff. Its use is therefore rare and is not recommended.

The general effect of a step rate is similar to that of a block rate, namely, a reduction in price with increased consumption. There is this difference, however, that with the block tariff the mean price is continually falling even within the confines of each separate block (except the first), whereas with the step tariff the price is constant throughout any one step. This last result can be obtained for the last stage of a block tariff by stepping the final price *up* instead of down. This occurs in the Tennessee Valley Authority tariff shown dotted in Fig. 39. After a block at 0·2*d.* the price is stepped up to 0·375*d.* for all subsequent units, resulting in an overall flat rate of this magnitude.

Mean Throughout Country.—The following is an attempt to construct tariffs representing the mean of pre-war practice in this country. Such an attempt may be of service in several ways. In the first place, it is useful to know what were the average values and how far any particular tariff differed from the average. It is also important to know the mean proportions of such tariffs, *i.e.*, ratio of parts, composition of blocks, etc. Finally, such a construction forms a scheme round which may crystallise future unification.

Only three types of industrial rate are employed on a sufficient scale to require treatment, namely, the flat rate, the block rate, and the two-part M.D., or Hopkinson rate. The variable-block M.D., or Wright, may be regarded as a special form of the two-part M.D. Of these, the flat rate is often merely quoted as an alternative, either to Hopkinson or to special tenders made to individual firms.

It is therefore difficult to quote a representative average, but it may

be said that the overall mean price for industrial supplies for all undertakings worked out at a flat rate of 0·7*d.* per kWh. (This includes special contracts to large users and is lower than would appear from the average published tariff. It must also be borne in mind that the average figures given below refer to the larger undertakings, and although these supply 90 per cent. of the energy, they do not quite represent the whole.)

Taking the pure quantity block rates (*i.e.*, omitting those dependent on M.D.), the average number of blocks, including the final "follow-on" one, was 3·7, the frequency distribution being indicated in the

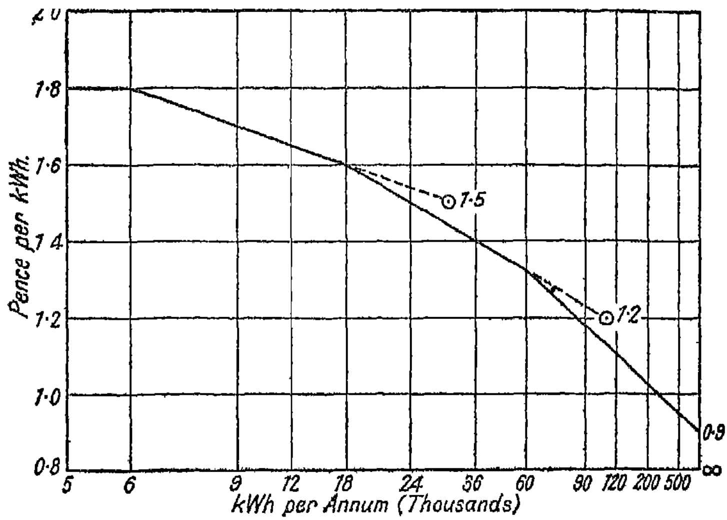


Fig. 40.—Average Block Tariff.

diagram of Fig. 33. The mean price for the first block was 1·8*d.* and for the last one 0. 9*d.*—just half as much. The mean size of the first block was 8,200 kWh per annum. Subsequent blocks usually increased 'n size though quite often they were equal and occasionally they deReased. Piecing together all these various mean values gives the follow-ıg as a sort of "average" block tariff (the figures have been rounded off ıs to give simple values per month and per quarter): first 6,000 kWh annum, 1·8*d.*; next 12,000, 1·5*d.*; next 42,000, 1·2*d.*; all over)00, 0·9*d.* Fig. 40 shows the operation of this tariff: the base is ed inversely from three different infinity points whose positions are icated by the numbers 1·5, 1·2 and 0·9.

n the case of the two-part M.D. tariff, the mean construction is a pler affair since there are far less variables. The average running rge was just over a halfpenny (0·507*d.*), and the average standing

charge was £5 15*s.* per annum per kW (or £4 12*s.* per kVA). Hence the numerical ratio between the two parts came to 11⅓. If, as suggested below, the tariff were only offered to the larger consumers the figures might be reduced slightly and rounded off to £5 per kW plus 0·5*d.* per kWh. This would have the advantage of a slightly smaller ratio between the parts, and hence a bigger allowance for differential diversity. (The operation of this particular tariff has already been fully discussed—see Figs. 35 and 36.) Alternatively, if it were desired to include consumer costs of, say, 50*s.* per connection this could be done by making the standing charge £5 5*s.* per kW for the first 10 kW and £5 per kW for all subsequent demand.

Possible Uniformity.—The possibilities of tariff unification may be taken a stage further, at least in a tentative fashion. Since the Hopkinson tariff requires a flat-rate alternative, and since the block tariff is itself a species of flat rate, it is clear that the simplest practical solution to the industrial tariff problem would be to quote a single tariff of the block type, as outlined above. Applied to the small and medium-sized power user of normal working hours, it is reasonably representative of costs, and very closely representative of market values. The larger consumers or those likely to have extended hours of use could then be offered an alternative of a Hopkinson tariff. Since this keeps more closely to the costs formula and the bulk-supply tariff, the price could be cut more finely (because more precisely) when it was necessary to compete with large-scale private supplies. Such a plan, *i.e.*, block rate for small consumers and Hopkinson for large, has been followed by a number of undertakings, *e.g.*, Leeds, Norwich and the Fife Electric Power Co.

With this possibility in view, it is interesting to compare the incidence of the " average " two-part tariff with that of the " average " block tariff. It is true that the former depends upon load factor and the latter on total load, but the two can be to some extent harmonised by means of the theory of diversity described in an earlier chapter. On this assumption, a very large power consumer having almost perfect diversity within his own working hours, and working, say, 50 hours a week, should have a load factor approaching $\frac{50}{7 \times 24} = 30$ per cent. His overall payment on the above two-part tariff will then amount to 0·96*d.* per kWh. By the block tariff the price would average this same figure when the annual consumption was about 400,000 units.

A small consumer having little internal diversity would be likely to achieve a much smaller load factor. If his load factor were of the order 10 to 12 per cent. his overall price on the two-part tariff would be 1·8*d.*—just the same as a small consumer would be paying on the block tariff. Thus the operations of these two " average " tariffs are sufficiently consistent for the one to be used as an extension of the

other without introducing any serious anomalies or jerks in their application.

Before leaving the subject, some contrary opinions should be quoted. Many authorities would disagree with the author regarding the operations of differential diversity, and would not trust a block tariff to cover costs even for groups of small and uniform power users. They would say that for an Area Board to buy from the B.E.A. on a two-part tariff and then sell at a flat rate is like buying by the ton and selling by the gallon a commodity whose specific gravity is quite unknown and highly variable.

On such a view, the best solution for the small industrialist would be a variable-block or Wright tariff. This can be made to merge into any desired two-part tariff values (employed for the large consumers) as described in the last chapter. Moreover, by making the block size dependent on the installed load instead of on the metered demand, a second meter is unnecessary and charging is somewhat simplified. This plan was recommended for small industrial loads by the 1929 Committee.*

Welding Tariffs.—In resistance welding, and still more in arc welding, very heavy currents may be drawn from the line for short periods of time. In spot welding, the welding time is usually only a fraction of a second and may even last less than one cycle. As a result, welding is liable to incur supply costs not adequately covered by normal charges, because the regulation is affected and adjacent circuits are disturbed thereby, and because on a two-part tariff, with metered maximum demand integrated for half-hour periods, the short-period welding currents may not be adequately recorded.

The matter has been investigated by a number of bodies. A Welding Committee of the Electrical Development Association were satisfied that extra costs are incurred not covered by ordinary metering and tariffs, but they made no recommendations on charges. Two E.R.A. reports have discussed the costs incurred and have made some reference to methods of charge.†

In practice much depends on the local situation, the sizes of the welders, and the proportion of welders to other types of load. In the past there has been no general practice, but a number of undertakings made extra charges based on the rated kVA of the welder, *e.g.*, a surcharge of 10*s.* per annum per rated kVA. Since there is a considerable diversity with a number of machines, an alternative plan is to vary the rate according to the number, *e.g.*, 15*s.* per annum per kVA for one or two machines, falling to 7*s.* 6*d.* per kVA when there are six or more. Another alternative is to use a formula for the quantity known as the " disturbance kVA."

* See p. 186.

† *E.R.A. Technical Report* :—K/T 110, " Electric Supply for Resistance Welding Machines " ; K/T 117, " Electric Supply for Arc Welding Plant ".

Commercial Tariffs.—This term covers the charges made for supplies to non-industrial, non-domestic establishments, such as shops and offices, institutions and places of entertainment. The types of tariff employed are generally similar to those for industry, the two commonest being flat or block rates, and two-part maximum demand. In the latter case the demand may be metered, but more usually it is based on the installed load. Instead of taking the whole of the load into account, a common practice is to base the demand charge on the lighting load only, with a correspondingly higher rate per kilowatt.

The main justification for this practice is the assumption that it is only the lighting load which is likely to come on to the peak. Such an assumption is probably out-of-date, and the practice seems likely to be more representative of use-values than of supply costs.

The following table is based on figures given in the *First Report* of the British Electricity Authority. It refers to numbers of consumers, not (like the previous analysis) numbers of tariffs.

PROPORTION OF COMMERCIAL CONSUMERS ON—

Flat and fixed-block rates		59%
Assessed basis	two-part, 23½ % variable-block, 9 %	32½%
Metered M.D.	two-part, 4 % variable-block, 3 %	7%
Other types		1½%
		100%

CHAPTER XI

DOMESTIC TARIFFS
(INCLUDING FARM)

National Survey.—The following table is based on figures given in the *First Report* of the British Electricity Authority. It refers to the numbers of domestic consumers on the various tariffs: when using it as a guide to the numbers of the tariffs, it must be realised that two or three alternatives were offered by most undertakings.

PROPORTION OF DOMESTIC CONSUMERS ON :—

Lighting flat rate	17½%	30½% flat rates
Multiple flat rate	13%	
Two-part	57%	66½% all-in
Variable-block	9½%	
Other types (fixed-block, load-rate, equated-rate, etc.		3%
		100%

A break-down of the 66½ per cent of consumers on all-in tariffs showed the following figures for the basis of the fixed charge or variable-block size :—

Rateable value	29%
House dimensions	21%
Number of rooms	12½%
Other bases (M.D., etc.)	4%
	66½%

An analysis of types of meter show just over 70 per cent. credit and nearly 30 per cent. pre-payment.

It will be noted that two-thirds of consumers are on all-in tariffs, which have largely superseded flat rates for all except the very small users. Numbers of consumer are, moreover, not a complete guide to relative importance. In a sampling survey made by the Electrical Research Association* it was found that one-fifth of all domestic consumers in the areas surveyed took lighting supplies on a single flat rate. But, on the assumption that these are the consumers with

* *Technical Report* K/T 125.

the lowest consumptions, the "Lorenz curve" plotting the cumulative values of consumers and consumption indicates that these lowest fifth only account for about 2 per cent. of the total domestic consumption.

Significance of Domestic Tariff Types.—Comparing the above survey with the survey of industrial tariffs, it will be remembered that industrial tariffs were based on one or other of two main principles, namely, "power and energy" and "discounts for quantity." With domestic tariffs, the former principle (as exemplified by the M.D. tariff) only appears to a very minor extent, whilst the latter (as exemplified by the fixed-block tariff) appears hardly at all. Instead, there arises an entirely new principle which seems to be based, like the vote, on a sort of residential qualification.

In the popular all-in tariff the consumer is "assessed" for his fixed charge, once and for all, on some basis connected with his house and mode of living, after which he has a low running charge whatever the character and amount of his consumption. It might be thought that this is merely a disguised M.D. tariff, but a moment's consideration will show that it is not so. The all-in tariff makes no attempt whatever to limit the consumer's demand or to relate the standing charge to the demand. In fact, its whole aim and effect is to increase household consumptions of every sort and therefore necessarily to increase demands (though not, it is hoped, in the same ratio). The rationale of the all-in tariff must therefore be sought in some other direction.

In order to understand the domestic tariff situation and where it differs from the industrial one, it is necessary to consider the facts of utilisation. It has already been emphasised that every tariff has its demand (*i.e.*, use-value) as well as its supply aspect, and must take account not only of what the electricity costs but what it is used for, and hence what it is worth to the user. In general, domestic tariffs differ from industrial ones, first in taking less account of costs and more of use-value, and second in having a much bigger complexity of values to deal with.

The central feature of industrial supply is that its use is largely for one particular purpose—mechanical power production. The cost of the competing alternatives is a fairly uniform one, and the margin between this and the basic electric cost is too small (at least with a big consumer) to leave much room for choice in tariff fixing. Costs are more precisely determinable because the hours of use are more uniform and the diversity less than in domestic loads. The consequence is that either the tariff takes a definitely "costs" form (power and energy charges) or else, when use-value is considered in the tariff form, it appears as a lower charge for the large-scale user (block rates).

In the domestic field, not only are the costs less certain (owing to the much bigger diversity range) but the utilisation is far more varied. Electricity in the home is put to many diverse uses, and the various

alternatives compete at a number of different price levels. For some purposes the service rendered is unique or very nearly so; and the utility value of such amenities as electric lighting, cleaning, time-keeping and wireless reception bears no relation to the energy consumption. For other purposes electricity is highly advantageous, but not indispensable: for still other purposes it is closely pressed by well-qualified competitors. The net result is that domestic tariffs, both in form and magnitude, keep less strictly to costs and pay more attention to selling considerations.

An extreme case of this tendency can be seen the "load rate" type of tariff which is levied by a meter constructed to record at a lower rate whenever a given power (or rate of consumption) is exceeded. Such a plan results in an undertaking buying and selling its supplies on precisely opposite rates—the buying tariff charging more when the demand is high and the selling tariff charging less. Too much should not be inferred from a single (and not usual) example, but it is safe to say that only in the domestic field could occur so extreme a case of neglecting costs and considering only the use-value side.

Competing Costs.—In the first chapter some attempt was made to schedule the various uses of electricity in the order of their consumption value. Such a list should start with those services which are most nearly essential and unique, and conclude with the large-scale and relatively "crude" services which can almost as well be satisfied elsewhere. Actually, there is a continuous gradation of use-values from the very high to the very low, but for practical reasons drastic simplification is necessary. The most that can be done in the domestic field is to compress all the uses into two groups, namely, a high-yield group characterised by a high competitive price and an inelastic demand, and a low-yield group having the opposite characteristics. The words "lighting" and "heating" will be used to describe the two groups, since these typify the corresponding uses.

Taking electricity's closest urban competitor, namely, gas, it will be well to get some idea of the ratio of the competing cost for these two services. Under given conditions of cold and darkness a fair-sized sitting-room could be lit by one or two gas mantles each consuming 5 cub. ft. per hour, and warmed by a gas fire taking, say, 30 cub. ft. per hour. At a flat-rate price the heat would therefore cost about four times as much as the light. The same room, lit and heated by electricity, would probably use a 100-watt lamp and a 2-kW fire, although certain special features such as portability might make $1\frac{1}{2}$ kW a nearer equivalent. The energy-consumption ratio of heating to lighting would then be 15 or 20 : 1. Comparing gas and electricity, if the lighting prices were level and if gas were at a flat rate for light and heat, electricity should be priced at 4/15 to 4/20, *i.e.*, $\frac{1}{4}$ to $\frac{1}{5}$ as much per unit for heat as for light in order to be equally competitive.

But the lighting prices are *not* level because of the kudos which electricity has in lighting to an extent which does not exist in heating. If electric light were preferred to gas to the point of willingness to pay twice as much for it, then the sales value of electricity for lighting would be eight or ten times its value for heating.

What is not fully realised by many of those who employ the two-part domestic tariff (and still less by those who imitate it) is that, in the case of electricity, the fixed charge is payment for a service (primarily electric lighting) having a very high sales-value out of all proportion to its energy content. Such a tariff, if applied to supplies which had a more uniform selling value, would lose most of its meaning and become merely a device for making the price appear lower than it really is.

Lighting and Heating Costs and Charges.—The common factor of domestic tariffs is the high lighting rate or its equivalent, and in the above sections this has been explained in terms of use-value. It must not be thought, however, that cost is entirely neglected, or that cost considerations could not also (at least partially) serve to explain the tariff situation.

On the score of cost, lighting is expensive to supply because it combines in an unusual degree a low load factor with a low diversity factor. Unlike the spasmodic loads considered in Chapter V (in which load and diversity factors are reciprocally connected), lighting is associated with particular times of day, and therefore likely, if not certain, to come on to the peak. It will be noted that, comparing lighting and heating,* it is in respect of diversity rather than load factor that the lighting load is so bad. On the score of individual load factor *per se*, any single lamp is likely to be used for almost as many hours a day as a single cooker or heater (equivalent full-load period), and the same may still be true if the total lighting installation be compared with the total heating. In both cases the load factor of a single household is likely to be low, and even the heating circuit will often not run into double figures. In the following paragraph the difference in cost between lighting and heating supplies is referred entirely to diversity factor, but it must be understood that in this case the figure is intended to include the combined effect of diversity and relative load factor.

In order to get some idea of the price ratio which the above facts would justify, compare a domestic lighting and a domestic cooking or water-heating load. Assume that the former necessarily comes on to the peak and has a diversity therefore of unity, whilst the latter is credited with a diversity of 3. Assume further that, out of the total expenses of supply, three-quarters of the cost is dependent on maximum demand

* The word "heating" here refers to all non-lighting domestic uses. So far as space-heating is concerned the case is even stronger, since a great deal of this comes as much on the peak as lighting does.

and one-quarter on consumption. The effect of a diversity of 3 will be to reduce the fixed costs in this ratio ; so that, if the correct price for the lighting energy is 1*d.*, that for the heating will be $\frac{1}{3} \times \frac{3}{4} + \frac{1}{4} =$ 0·5*d.*, *i.e.*, exactly one-half. A diversity of 5 for the heating load would bring its costs down to 0·4*d.*, giving a ratio of $2\frac{1}{2} : 1$.

The above estimate is a very rough one, but it is sufficient to indicate that, on purely cost grounds, the lighting price per kWh should usually lie between two and three times the heating price. This hardly applies to very small loads, since, owing to consumer costs, the expenses of a small lighting connection are disproportionately great and necessitate a high lighting rate even to cover mere costs.

Turning to the actual tariffs, the flat rates in a pre-war survey made by the author showed an average price for lighting of 4·44*d.*, and for heating and cooking of 1·0*d.* The ratio of the two was therefore about $4\frac{1}{2}$. In the case of the variable-block tariffs, the average ratio between the first-block price and the follow-on rate was exactly 8. The two-part tariffs are more difficult to assess, but the following will give some idea. Consider a typical tariff of $12\frac{1}{2}$ per cent. of the rateable value plus $\frac{3}{4}d.$ (The first portion of the rateable value may be charged at a higher percentage, but this can be taken as offsetting the meter rents on the flat-rate systems.) Let this be applied to a £30-rateable-value house, and assume that the lighting consumption is given by the expression 50 + 4 R.V. = 170 kWh in this case. The price paid for this will be $12\frac{1}{2}$ per cent. of £30 + 170 × $\frac{3}{4}d.$ = 1,027*d.* or 6·04*d.* per kWh. The ratio of equivalent lighting price to heating price is then 6·04/0·75 = 8.

The position then is that, whilst the cost of supplying lighting energy is two to three times that of heating energy, the price charged is five to eight times as much, and corresponds fairly closely to the relative use-value (assessed by comparison with the competing alternatives). It may therefore be concluded that even when the *forms* of the existing domestic tariffs, whether flat rate or all-in, can be justified by arguments of cost, any such explanation is inadequate to explain the tariff values.

There is one consideration which should be mentioned here and which may tend to qualify the above conclusion. An appreciable part of the cost of supply is independent of kWh, kW of demand or number of consumers. This residue of " common " costs incurred by the undertaking *qua* undertaking may be regarded as resulting from the determination of a body of consumers to have an electricity supply of some sort. In so far as these are domestic consumers (which most of them are) this cost can then be regarded as part of the basic lighting and radio load, since every consumer, whatever else he wants, will want this service. The fact of his becoming a consumer can therefore be regarded as a consequence of the lighting load, and a large part of these common expenses can be debited thereto on cost grounds. To

this extent, therefore, the high lighting costs (or its fixed-charge equivalent) can be regarded as a device not only for distributing common costs on market-bearing lines but also for using the "market" as a criterion of true divisible costs.

Multiple Tariffs.—Having examined the reasons (both of cost and of utility) for a high lighting rate, the various domestic tariffs can now be considered in turn. Only two types and one sub-type (namely, Nos. I, IV and IVA in the list on p. 173) need be separately considered, the others being of lesser importance and sufficiently covered by the notes on industrial tariffs. It will then be evident that each of the different types can be regarded as a method of levying, directly or indirectly, a high charge for the basic services (*i.e.*, lighting, etc.).

Tariff I is the simplest and most obvious method of doing this. It consists of a set of flat-rate prices, a high one for lighting, a lower one for heating, etc., and sometimes a still lower one (possibly with time restriction) for water heating or other special purpose. With this tariff the house must have two (or more) separate installations, each running back intact to the intake point. Two separate meters record the two consumptions, although of course both are fed from the same incoming cable. The consumer is restricted in his use of the one circuit to certain specified purposes such as heating and cooking, or he may be permitted any use except lighting. Such a prohibition, although difficult to enforce strictly, is assisted by the fact that the heating circuit is probably only provided with 15-amp. socket points whilst the lighting circuit installation is either to fixed apparatus or provides only lamp socket or 5-amp. plug outlets.

This is the simplest and most straightforward type of tariff possible, and complies with legal requirements without further alternatives. It reflects with fair accuracy the utility values, and allows full scope for non-lighting development. Its only serious disadvantage is the necessity for two sets of wiring and metering, and the consequent inflexibility of installation. It means that every room to be adequately served, must have two separate plug points as well as its permanent lighting points; and in practice this means that many rooms are not fully served. A further slight disadvantage is that the discrepancy between lighting and heating rates, being open and undisguised, sometimes becomes the target of public criticism. This may lead to a reduction of the lighting rate below its market value, thus dissipating resources which might be used to build up the non-lighting load.

Consumer costs can be covered by having discounts for quantity or a certain amount of blocking in the lighting rate. Another very common plan is to charge a meter rent considerably in excess of the actual capital charges on the meter. This helps to pay for the billing and reading and other *per capita* expenses. Consumer costs are also to some extent covered automatically (wherever the lighting rate is higher

than the proportional costs of supply) because any lighting installation, however small the premises, is likely to have a certain minimum consumption (*i.e.*, the constant 50 in the above formula).

All-in Tariffs.—The meaning of this title is that all the consumption on a given premises, for whatever purpose, is taken through one meter and charged at the same rate. In addition there is a standing charge or its equivalent, and this varies with the consumer but not with time or consumption. This means that the consumer's house or installation is assessed (usually once and for all) for the standing charge, and the only variable element in the bill is the energy charge.

The all-in tariff takes two forms. The most popular in the past has been the two-part tariff consisting of an annual fixed charge and a low running charge. The other form is the variable-block tariff in which the equivalent of a fixed charge is obtained through a first block of high-price units. In the following discussion the term fixed charge will be used to denote either the actual fixed charge of a two-part tariff or the equivalent fixed charge of a variable-block tariff.

The purpose of the fixed charge*, as of the high lighting rate which it has largely superseded, is as follows. On the cost side, to cover some of the demand-related costs and in particular the disproportionately high cost of supplying the basic minimum of essential " lighting " energy. On the use-value side, to represent the high utility of this basic consumption, and so be the means of spreading the common cost where it can best be borne. For any given sized house there is a certain minimum, or probable, lighting consumption which will or should be used. This is a relatively fixed and inelastic quantity—individuals will vary, but only by a few per cent. instead of many hundreds per cent. as they do with non-lighting consumption. The main object of the fixed charge is to secure a payment (approaching its market value) for this basic minimum of lighting service.

In the case of the £30-rateable-value house illustrated below, it may be presumed that to light such a house at all adequately will require some 170 kWh per annum. For this service the consumer is paying $0{\cdot}10 \times 20 + 0{\cdot}075 \times 10 + \frac{1}{2} \times \frac{170}{240} = £3\ 2s.\ 1d.$, or $4{\cdot}4d.$ per kWh. It is obvious that if the householder uses electricity at all he will use it for lighting; and it is probable that he will be quite willing to pay the above sum for this service, since there is no alternative way of satisfying his requirements that even approaches it in cost or convenience. Once this scarcity value has been paid for, further consumption for all purposes cost only $\frac{1}{2}d.$ per unit—at which price

* These remarks refer to that portion of the fixed charge which is directly proportional to the house data. The non-proportional element is to cover consumer costs.

it is competitive with other sources for these purposes. (N.B.—These are pre-war values.)

A rough but fairly sound guide to the value to the consumer of this basic minimum is provided by the size or value of his premises, whether measured by rateable value, floor area, or number of rooms. This is particularly true of the value of electric lighting which has a social virtue apart from its intrinsic merits. House size or value is also a very fair measure of the consumer's ability to pay. The pricing of a service according to the user's means has been practised by the medical profession very successfully for years, and although the practice has a faded look thanks to the N.H.S.* an element of this kind in the tariff has obvious practical advantages.

The advantage of the all-in tariff is the cheapness and flexibility of its installation, and the fact that it imposes no restrictions or taboos on the user. It requires only one meter, and it can easily be used to cover consumer costs by a grading of the fixed charge. It reflects utility or market values even better than the multiple tariff for the following reason. The multiple tariff treats the "lighting" and "non-lighting" categories as though they were synonymous with "high-yield" and "low-yield" respectively, although this is not altogether the case, and modern uses are making it less so. Many small non-lighting services, such as cleaning and wireless, have high utility values, and on the other hand the lighting load itself is a composite, having an inelastic and an elastic component. There is the basic minimum of lighting necessary to a particular sized house, and there is a further possible expansion in the way of indirect and architectural lighting. The all-in tariff contrives to secure the market value for the former component (and all other special services) without strangling the latter. The multiple tariff, on the other hand, charges heavily for all lighting, both the irreducible minimum and the elastic additions, whilst getting no special return for the other high-grade services.

The disadvantages of the all-in tariff in its usual two-part form are the unpopularity of the fixed charge and the virtual necessity for a flat-rate alternative. Both these objections are overcome in the variable-block form. Other advantages of the variable-block tariff are its greater flexibility, through the use of additional blocks when required, and the fact that it has a promotional effect even on the smallest consumers.

The only serious objection to the all-in tariff is the anomalous nature of the fixed charge, or its equivalent—the number of units in the first block. There is considerable disagreement as to how this shall be determined, and the fact that so many bases have been tried (and are still in use) is sufficient evidence that no one of them is satisfactory. This difficulty applies as much to the variable-block as

*Also, could such an element be *identified* it would doubtless be held to constitute "undue preference."

to the two-part tariff, although the objection to any particular basis of assessment is likely to be less acutely felt when there is not an actual lump sum dependent on it.

Basis of Fixed Charge.—In the above comparison it has been assumed that the fixed charge of the all-in tariff is based on rateable value. This has been done for simplicity, since it is a better-known quantity in most cases and conveys more idea of the probable size and wealth of the house than a statement of floor space, or number of rooms. But the same reasoning is applicable to any type of two-part domestic tariff, no matter how the fixed charge is assessed. The basis should, however, bear some relationship to the probable basic lighting consumption. The following is a brief statement of the pros and cons of the bases chiefly used in this country, and their relative popularity in the past is indicated in the table on p. 210.

The advantages of rateable value are that it is a precise figure by an independent authority, and easily obtainable from works of reference without the necessity for an inspection of the consumer's premises. The potential income from any given area is quickly estimated, and the tariff achieves a certain fairness as between old, rambling houses and compact, modern dwellings. Technically it is probably the best basis of all, since it corresponds most nearly to the real (as distinct from the ostensible) functions of the fixed charge.

The disadvantages are that the valuation is liable to be changed every five years and is by no means always uniform and fair, even within a given area. The tariff cannot be equitably employed by an undertaking operating over several local-government districts, and it is usually unsuitable for rural areas. It is also inapplicable to sub-divided houses unless these have been separately assessed by the local authority. The uniform basis of assessment soon to be applied throughout the country, may, however, remove many of these disadvantages.

The house-size basis may be operated either by an actual measurement of room floor space or estimated from the external dimensions and numbers of floors. This requires a visit to each consumer, but, once determined, no further revision becomes necessary. It is more flexible and more generally applicable than the rateable-value basis, since different rates can be applied, if necessary, to different floors, different classes of room, etc. The number-of-rooms basis may be regarded as a variant of the house-size basis with the disadvantage that it does not discriminate between different sizes of room. It does, however, correct a weakness of the plain house-size tariff, in that two rooms of 100 sq. ft. should be rated rather higher than one of 200 sq. ft. A better plan would be to employ a minimum fixed charge of so much per room plus a *pro rata* fixed charge of so much per square foot.

The installation basis usually takes the form of a fixed charge based

on the aggregate wattage of lamps installed. It is open to the very serious objection that the basis is a constantly varying one, and its continuous accuracy depends on the consumer's honesty unless frequent and unexpected inspections are to be made—a very objectionable procedure. It penalises generous lighting and it is expensive to administer.

The maximum-demand basis is different in principle from any of the others, and may be said to give a somewhat different character to the tariff itself. The M.D. may be metered in the same way as in the Hopkinson industrial tariff, and this is fairly common with commercial tariffs but is now almost absolete with domestic. The chief reason for its disuse is that the domestic consumer's maximum demand, particularly if he uses electric cooking, frequently occurs right outside the peak period (*e.g.*, on a Sunday), and the consumer's metered demand is therefore little indication of his effective demand on the supply system (see figures on p. 240). What is frequently described as a maximum-demand charge, both in domestic and commercial tariffs, is a fixed charge based on the installed load, either of the whole of the apparatus or of the lamps only. Some objections to this were noted in the previous paragraph.

Summing up the position, it is generally considered that the balance of advantage lies with the house-size. The 1925 and 1929 Committees (p. 186), which considered the matter in some detail, came to the conclusion that, although no basis was entirely without its defects, the size of house was open to considerably less objection than any other. They therefore recommended it for general adoption, provided that the consumer was given a flat-rate option. The 1946 Committee is believed to have made a similar recommendation. Subsequently, the British Electricity Authority has recomended Area Boards to use a basis connected with house dimensions, with a preference for number of rooms rather than floor-area.

Accounting or Assessment Period.—The demand-assessment period has already been dealt with under maximum-demand tariffs. The corresponding period in connection with all-in domestic tariffs is here called the assessment or accounting period. It may be defined as the length of time (within a year) over which the fixed charge is levied, or during which the first block of consumption is reckoned.

It is the almost invariable practice, in domestic tariffs for credit consumers, for the period of account to be three months ; *i.e.*, the fixed charge is levied, and the account is rendered, quarterly. In a similar way, the first block of a variable-block tariff is quoted as so many units a quarter, after exceeding which all subsequent units in the quarter are at a lower rate.* For purposes of comparison and uniformity, the

* When, in any quarter, the high-price quota has not been consumed the liability is sometimes allowed to run on, thus giving a cumulative debit in subsequent quarters.

figures in these examples have all been given per annum, and must be divided by four to give the quarterly amounts.

The contract period (for which the agreement runs) will usually be longer than the accounting period. Thus a two-part tariff consumer may have an initial agreement of one year, and subsequently be asked to give at least six months' notice of any change. The variable-block tariff has the advantage that, if operating without alternative, no question arises of notice having to be given of a change of tariff.

Example.—In the following illustration it is assumed that the floor space (external measurements) in square feet (S) is forty times the rateable value in £ per annum (V).* It is further assumed that the normal lighting consumption is given by $50 + 4V$ or $50 + 0{\cdot}1S$. These are, of course, entirely hypothetical figures, although it is believed that they are sufficiently near the mark for use in illustrating the tariff operation. As a check on the consumption per unit of floor area, it may be said that if half the area is presumed to be in active use, this consumption would give an illumination of 4 foot-candles for an average of about three-quarters of an hour a day.

As an example of a rateable value all-in tariff, the following may be cited for Derby Corporation :—

Fixed charge of 10 per cent. of the rateable value up to £20 rateable value and 7½ per cent. of the value in excess of this, plus ½*d.* per kWh. The same tariff on a floor-area basis would be 5*s.* per annum per 100 sq. ft. for the first 800 sq. ft. and 3*s.* 9*d.* per 100 sq. ft. beyond this. It will be seen that the fixed charge includes a non-proportional item of 2½ per cent. of £20 (or 1*s.* 3*d.* × 8), *i.e.*, 10*s.* a year, beyond which the charge is directly proportional. This figure is intended to cover consumer costs except that the very small connections will not pay their full share. An alternative plan, safer for the undertaking but more objectionable for the consumer, is to have a minimum fixed charge of £2 per annum for any house up to £20 rateable value or 800 sq. ft., after which the proportional figure applies.

If it were desired to have instead an all-in tariff of the variable-block form, this could be done by charging the first x units at 4*d.* and the remainder at ½*d.* The value of x would then be approximately $7V$ for the first 20 of V and $5V$ for the remainder, or $0{\cdot}17S$ for the first 800 of S and $0{\cdot}13S$ for the remainder. These tariffs would give exactly the same results as the other two for all consumptions above a certain basic figure, but below this they charge a flat rate of 4*d.* instead of a continually rising price. If the variable-block tariff were being put forward as an optional alternative, it might be advisable to increase these values slightly so as to give consumers some financial inducement to change over to the regulation two-part.

* Some degree of confirmation has since been given by the E.R.A. Sampling Survey in which the average ratio was just on 50.

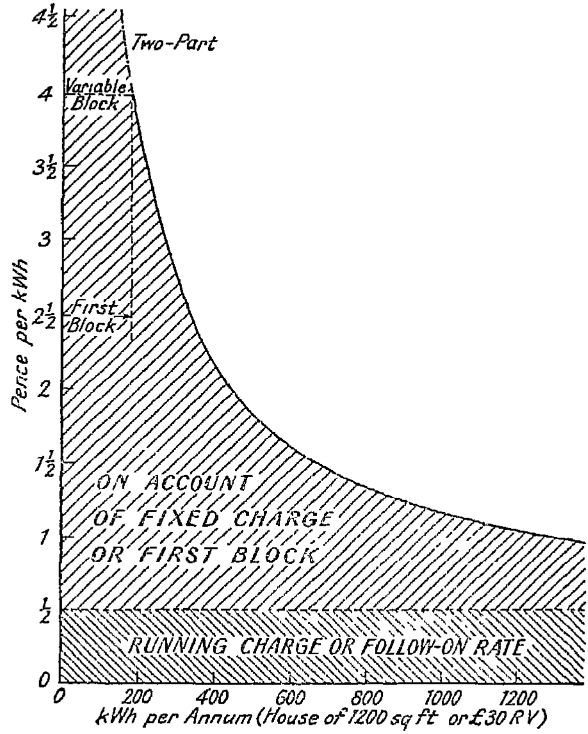


Fig. 41.—All-In Tariff.

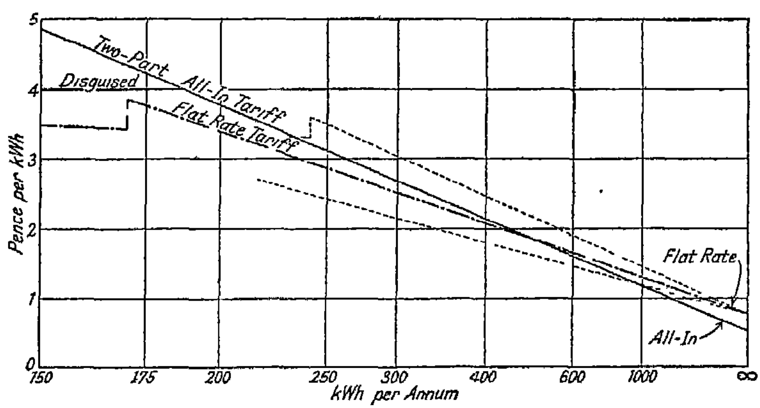


Fig. 42.—All-In and Multiple Flat-Rate Tariffs.

The operations of this tariff are shown in Figs. 41 and 42 in the form of a mean overall price plotted to a base of consumption for a house having a rateable value of £30 or a floor space of 1,200 sq. ft. In Fig. 42 the base is a reciprocal one, scaled from the right-hand end, as in previous figures of this kind. The sloping full line shows the two-part all-in tariff: the variable-block all-in follows the same line up to a price of 4*d.* (190 kWh consumption), but for smaller consumptions it remains at this figure.

On the assumption of a certain lighting consumption per unit of rateable value or floor space it is a simple matter to construct a multiple tariff (two flat rates) to give the same effect as the above. The heating rate would have to be the same as the follow-on rate of the two-part tariff, namely, ½*d.* per kWh. The lighting rate would have to be such that the difference between the two rates multiplied by the lighting consumption equalled the fixed charge. But this would have the effect of accentuating the difference between the two rates, which is not good sales policy. It is therefore usual for the heating rate of a multiple tariff to be higher than the unit rate of an all-in tariff, and this practice is followed in the illustration below.*

The nearest practical equivalent of the all-in tariffs just described would be a flat rate of 3*d.* for lighting and ¾*d.* for heating, plus a rent of 6*s.* a year for each meter. In the case of a £30-rateable-value house, if the lighting consumption followed the law given above it would amount to 170 kWh per annum. The overall price, when plotted to a base of annual consumption would then be represented by the chain-dotted line in Fig. 42. (The vertical rise represents the rent of the second meter.) It will be seen that this gives a somewhat lower price than the all-in tariff for consumptions below 500 kWh, and a higher price beyond this. If the same house were very careful with its lighting and consumed only 120 kWh the price curve would be the lower dotted line, whilst with a more generous lighting totalling 240 kWh the mean price would lie along the upper dotted line.

Variable-Block Tariff: Multiple Block.—The principle of the variable-block tariff has been described in earlier chapters, and the foregoing paragraphs show how the values of a tariff consisting of a first block of high-price units and a low follow-on rate for all subsequent consumption can be arranged to give similar results to those of any given two-part tariff. The size of the first block varies with the consumer and depends on the same factors as those which determine the fixed charge of a two-part tariff—rateable value, floor area, number of rooms, etc., in the domestic field, and M.D. or installed load in the industrial.

* An exact equivalence of all-in and two-rate tariffs is illustrated in Fig. 30 in Chapter IX. But in order to obtain this it was necessary to employ somewhat unusual values, and the present example is more representative of actual practice.

A not uncommon practice with domestic variable-block tariffs, particularly in Scotland, is to have more than two blocks. The resemblance to a two-part tariff is then less easy to establish, though the overall effect remains very much the same. The employment of more blocks makes the tariff structure more elastic and able to accommodate a bigger range of variations, either of costs or of use-values. This possibility makes the variable-block tariff *potentially* more promotional than the two-part tariff.

A three-block tariff of this type is offered by the North of Scotland Board throughout most of its areas, and the same thing with a fourth block added during part of the year applies to areas supplied from certain of their hydro-electric stations. The figures for this four-block tariff when applied to a six-roomed house are as follows :

1st block size		216	units per annum :	price $5\frac{1}{2}d.$	per unit
2nd ,,	,,	1,944	,, ,, ,,	,, $1d$	,, ,,
3rd ,,	,,	1,944	,, ,, ,,	,, $\frac{3}{4}d.$	,, ,,
4th ,,	all over	4,104	,, ,, ,,	,, $\frac{1}{2}d.$	,, ,,

The effect of such a tariff on a consumer in a given house will resemble the operations of a plain (non-variable) block tariff having the same number, sizes and prices of blocks. The effect can be shown by plotting the mean price per unit against annual consumption scaled either uniformly or inversely (see Fig. 37 on p. 211). In the former case the graph falls in a series of hyperbolæ after the preliminary horizontal portion. In the latter case these falls are all straight lines.

Yet another way of showing the operations of such a tariff is to plot the total revenue against annual consumption (uniformly scaled). The graph then consists of rising straight lines, and the slope of each line represents the price per unit in that particular block. By drawing a line from the origin to any point on the graph, the slope of this line represents the mean price per unit at this point.

Fig. 43 shows a total revenue curve for the tariff given above. Dotted lines from the origin represent mean prices of $2d.$, $1\frac{1}{2}d.$ and $1d.$ per unit. When completed by a vertical line on the right the graph becomes a closed rectilinear figure, and for this reason a tariff with these characteristics is known in some Continental countries as a "Polygon Tariff".

The argument for adding a further block of lower-priced units to any given variable-block tariff can be summarised as follows. From the use-value aspect the addition obviously makes the tariff more promotional and enables it to compete in markets which are out of reach of the higher block prices. As regards costs there may be available additional units whose incremental costs are well below the lowest existing block prices, particularly on hydro-electric systems during the rainy seasons.

The danger is that since the fixed charge (or its equivalent) of a

domestic tariff is based on non-electrical dimensions and does not vary with any features of the consumption, it is necessary for the running charge to include a contribution to the fixed costs. The only exception to this rule is where it is certain that the additional consumption will not add anything to the demand at peak periods. This points to a serious practical objection to a low-price block, namely, that it cuts away the margin on which depends the possibility of building up off-peak loads by special tariff inducements. If extra units can be had at the low rate throughout the twenty-four hours why

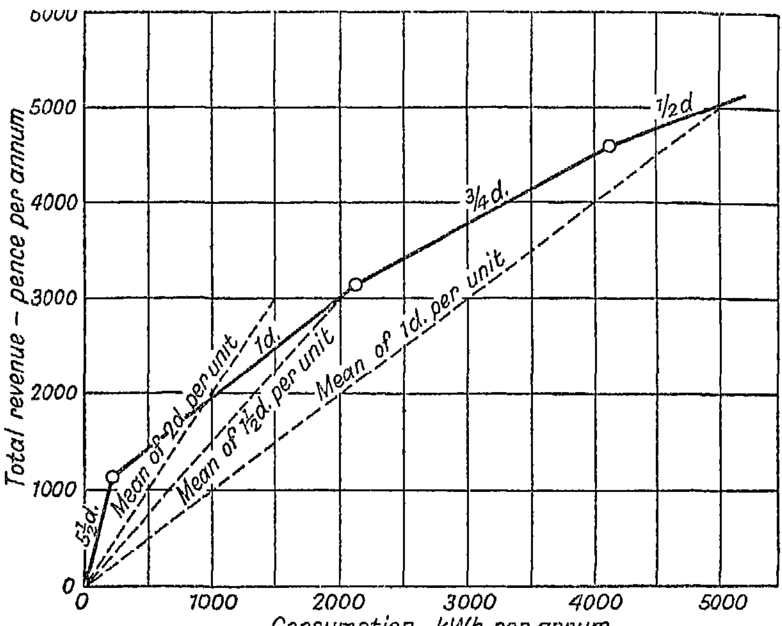


FIG. 43.—Total Revenue from Variable Block Tariff.

should consumers go to the expense of separate circuits and special storage apparatus?

The argument for adding a block *in between* two existing blocks of a variable-block tariff (*i.e.*, breaking down into smaller blocks) is quite a different one. The effect of such sub-division is to smooth out the price curve and give a more gradual transition from the dear units to the cheap. Since the net result in any case is a bill of so much for a certain total of units, it is the overall effect that matters, and this can be almost the same without the intermediate step. Thus, if in Fig. 43 the first and third lines of this graph are continued until they cross, this point will show what size of first block can be employed so as to eliminate the need for the second.

Contract-Demand System.—Various tariff systems have been developed, chiefly on the Continent, under which the consumer's standing charge is coupled with a corresponding maximum demand, *i.e.*, the consumer " opts " for a certain maximum load and pays a fixed charge accordingly. If at any time he exceeds this maximum rate of consumption the higher rate is charged at a correspondingly bigger fixed price. Frequently the limitation only operates at specified peak-periods, and at other times the consumer is free to go above his contract demand without incurring extra charges.

This system does not appear to differ in principle from the metered maximum-demand tariff and is somewhat less flexible. The consumer has to decide beforehand what rate to contract for, and will usually require instruction before he can do so intelligently, whereas on the normal M.D. system the choice is automatic. On the other hand, the apparatus required may be simpler than that for M.D. metering.

Block Tariff.—This is rarely used for domestic supplies in this country although common in America (*e.g.*, Tennessee Valley Authority, see Fig. 39). It has already been fully described under industrial tariffs, and its effect is simply that a large consumer pays less per unit than a small one. To some extent this represents supply costs, but to a far greater degree it reflects the value per unit to the industrial user. This is because the market value is measured by the cost of alternative supplies, and for a power user the cost of private generation would depend very largely on the magnitude of the load.

With the domestic consumer the alternative is not private generation but the use of some other form of lighting and heating. The market value (or alternative price) then depends upon the kind of use, being very different with lighting from what it is with heating. The aim of domestic tariff construction is that each consumer shall be charged a high price only up to the point of his essential or " high yield " consumption, and this point varies with the size or value of his house. With a pure quantity block rate, on the other hand, the price-change point would be a fixed one : consequently, a small householder would never get on to a low price at all, whilst a large householder would purchase many of his high-yield units at the low figure.

Equated Rate.—This is a flat-rate tariff in which the hire charge for wiring and/or apparatus is included in the price. It is a convenient method of collecting the two sets of costs in the case of relatively small-scale supplies to the poorer households, and it has been successful in developing general domestic consumption (*i.e.*, beyond the minimum lighting requirements) up to a point. But since large consumptions would involve correspondingly large contributions to apparatus, such a tariff cannot develop a big non-lighting load. Put in other words,

any flat-rate price per unit which includes even a small contribution to apparatus cost will be too high to encourage large-scale cooking, water-heating, etc.

Load-Rate Tariff.—This is only for domestic loads, and as already mentioned, its incidence is exactly the opposite of a maximum-demand tariff. It is designed purely to correspond to the value of the electricity to the user, and in so doing it is at complete variance with the costs formula.

The tariff is implemented by means of a special type of meter (usually pre-payment) which records at a lower rate per unit whenever the load rises above a predetermined figure, such as 400 watts. When the consumer is taking less than this power the presumption is that he is using electricity only for lighting or wireless. Its use-value to him is then high and he is charged accordingly, since his meter then records on high gear. But when his rate of consumption increases to a kilowatt or more, this can only be because he is using electricity for cooking, heating, etc., where its use-value is less. The meter then records through a different train of gearing, and he is charged at a lower price per unit.

It would be quite possible to vary the setting and make the power at which the price change operates depend on the consumer's house-size or value. This would put it more in line with the all-in tariff, either two-part or variable-block. In practice, this is hardly necessary because the difference in magnitude between the largest lighting and the smallest heating load is usually so marked that a single setting will suffice for all houses of the type likely to utilise such a tariff.

Unfortunately, this method of charging on the basis of diminishing utility is very far from reflecting the costs of supply. If the particular consumer is representative of the general load it will be obvious that the time when his price is low, owing to his load being high, will be just the time when the undertaking is likely to be incurring its peak charges and will therefore be paying at a high rate. If, on the other hand, the individual load is quite unrepresentative of the whole, this amounts to saying that the diversity is so high that individual characteristics are no guide to fixed costs whatever. The tariff then cannot be said either to follow costs or to contravene them, since no basis would exist whereby fixed costs could be allocated.

All-in Tariff Values: Fixed Charge and Electrical Demand. Whilst this section is primarily descriptive and concerned with the tariff structure, something should be said on the question of magnitudes. The selection of appropriate values for the all-in domestic tariff is a matter of considerable difficulty; for, unlike the industrial maximum-demand tariff, its form does not follow the form in which

generation and others costs are normally expressed, and the magnitudes of its parts cannot be built up logically by summating the appropriate costs incurred up to the point in question.

The all-in tariff, even in its normal two-part form, has only a superficial resemblance to the costs formula. It is impossible to identify the fixed charge with the standing costs of supply and the unit charge with the running costs, because the essence of the all-in tariff in its usual form is that the fixed charge has no direct connection with the installed load or M.D. or other electrical characteristics, and does not vary from year to year. One must, therefore, fall back on the " upper and lower limit " method of selection, as described below.

The correlation between the dimensional features of a consumer's premises (such as value, size, or number of rooms) and the electrical features (such as installed load, individual M.D., or effective system demand) can hardly fail to be significant* for the reason that *ceteris paribus*, a large house must mean more electricity consumption than a small house. On the other hand, it cannot be close enough to justify the allocation of more than a fraction of the demand-related costs.

Broadly speaking, there are three main (non-lighting) domestic services, *viz.*, cooking, water-heating and space-heating. It is true that any all-in consumer is likely to have at least one of these on some scale, and therefore that the M.D. of any one service may be very roughly proportional to the house size. On the other hand, a house of a given size may utilise one, two or all three of these services, with entirely different results as regards demand. The running charge therefore must not only cover the running costs of supply but must also contribute something towards the standing costs—otherwise, a very extensive consumer will have a maximum demand which he does not fully pay for. Unless the running charge includes a substantial contingency item, it will not deal equitably as between large and small consumers of a given house size.

Pricing Limits.—It was seen in Chapter III that there are two stages in price-fixing : first to determine what is the marginal cost, and second, assuming that this aggregates less than the total expenses, to load the remainder where it can be borne. Pricing is a process which operates between two stops or limits, a lower one representing marginal cost and an upper one representing marginal utility, *i.e.*, the use-value of the last unit sold within the market which it is proposed to reach. With the domestic tariff these stops are often farther apart than with the industrial tariff. Moreover, the process must be applied to each part of a two-part tariff since the magnitude of each part must lie between the two limits.

* A *significant correlation* means that the degree of correspondence is greater than can be attributed to mere chance.

As regards the running charge, it was pointed out in Chapter V that even under the two-part maximum-demand tariff with its resemblance to the cost formula, it is necessary to bias the cost values in order to compensate for "differential diversity". An indication of how much this bias should be is given by the E.R.A. method of allocating demand-related costs amongst groups of consumers. With the all-in domestic tariff, this procedure is even more necessary because of the tenuous connection between the fixed charge and the electricity demand.

In order to find the amount of the individual bias necessary under a domestic tariff, the E.R.A. method employed for groups could be extended to single consumers by making certain assumptions as to their average characteristics. Data and assumptions are set out below, but the calculation based thereon must be regarded as purely tentative and for illustration purposes. The data refers to the group of consumers specified on p. 148. (Consumptions are in megawatt-hours and time in 1000 hours.)

Data

Demand-related cost of group	$= £11{,}530 = C$
Effective demand of group at time of system peak . .	2,150 kW (2,200 kVA) $= P$
Annual duration of potential peak period . . .	$=$ 2,910 hrs. (*i.e.*, $T = 2{\cdot}91$)
Annual consumption of group	4,640 MWh.
Do., in potential peak period - - -	2,760 MWh $= k$

Assumptions referring to the "average" consumer :

Maximum demand (occurring in potential peak period) .	$1{\cdot}5$ kW $= d$
Annual Consumption . .	1,200 kWh, *i.e.*, 1·2 MWh

Calculations

$$\text{Number of consumers} = \frac{4{,}640}{1{\cdot}2} = 3{,}867$$

Aggregate of individual demands $= 1{\cdot}5 \times 3{,}867 = 5{,}800 \text{ kW} = \Sigma d$

Applying the formulæ $\frac{C}{P} = Tx + y$, and $C = \Sigma kx + \Sigma dy$, the value of x is £1·38 per MWh = 0·33*d.* per kWh and that of y is £1 7*s.* 0*d.* per annum per kW. With these figures, one-third of the demand-related cost is allocated to kWh consumption in the potential-peak period and two-thirds to the kW of maximum demand.

In applying this to a domestic tariff of the usual character the value of y would have no great significance since the fixed charge is not on a kW basis. But the value of x indicates that if the bulk-supply running charge plus the allowance for losses, etc., is $\frac{1}{2}d.$, the marginal running cost below which the running charge should not be fixed is $0{\cdot}5 + 0{\cdot}33 = 0{\cdot}83d.$ per kWh. This is an average figure, the allocation being based on the assumption of a uniform peak liability throughout the potential-peak period, inversely related to the load factor. Actually, the liability varies with the class of load, and the surcharge should be greater than $0{\cdot}33d.$ for peak-producing loads such as space-heating and less for cooking and water-heating.

Turning now to the marginal cost to be covered by the fixed charge, there must be a non-proportional element to cover consumer costs —say £1 10*s.* 0*d.* per annum, and a proportional element sufficient to cover the demand-related costs likely to be incurred by the average all-in consumer in that size of house, minus the portion already allocated to the running charge.

Considering next the use-value which sets the upper limit to the magnitudes of the two parts, the running charge should first be scrutinised with a view to the particular market it is desired to enter. The figure must be competitive in that field and linked with wiring and hiring facilities for the corresponding consuming apparatus. In urban areas, with gas and other alternatives at normal prices, it is probably difficult to build up a large cooking load with a running charge of more than 1*d.* (post-war value) or a large water- or space-heating load with a running charge of more than $\frac{3}{4}d.$ Probably, this upper limit will be little if any above the lower limit of (adjusted) marginal cost. The price will then have to be fixed at this point.

The upper limit to the fixed charge is set by the fact that the charge must be sufficiently low to attract (and to keep) consumers from alternative tariffs and from alternative services. If the fixed charge is high in relation to the alternative lighting flat rate, it may be difficult to obtain two-part consumers and to prevent them from returning to flat rates when their consumption declines. Probably there will be a margin between these two limiting values of the fixed charge sufficient to accommodate all the remaining expenses which have to be covered. In other words, there will be a "consumer surplus" at this point which will enable the common costs to be borne. To determine this, the total revenue from fixed and running charges can be equated to the total cost attributed to the group plus whatever share of the common cost it is found possible and expedient to include.

Besides seeing that each part is not priced higher than its market worth, it is necessary to view the tariff as a whole from the same point of view. This means that the fixed charge, plus the running charge on the basic minimum of lighting and radio consumption, must not exceed the commercial use-value of having these electrical services.

All-in Tariff Analysis.—As an alternative to building up a tariff from cost and other data, the following is an analysis of an existing one. It attempts to " rationalise " a set of tariff values into components in the light of the pricing theory which has been laid down. The values are based on the proportions given on p. 75 and the demand-related allocation derived above.

Two-Part Tariff based on Floor Area

Fixed Charge of £2 per annum (10*s.* per quarter) up to 500 sq. ft. plus 4*s.* per annum (1*s.* per quarter) per 100 sq. ft. thereafter.

This could obviously be expressed as £1 per annum plus 4*s.* per annum per 100 sq. ft., with a minimum of £2 per annum.

Running Charge of 0·75*d.* per kWh.

Variable-Block equivalent based on Number-of-Rooms

First Block, 4*d.* per kWh. *Additional Consumption*, 0·75*d.* per kWh.

Size of First block, 150 per annum up to 5 rooms : 15 per annum per additional room.

(This could obviously be expressed as £1 per annum plus 4*d.* per kWh for the first 15 kWh per annum per room, provided the annual consumption was not less than 15 kWh per room.)

The difference between the two rates in this second tariff is 3¼*d.*, and multiplying this successively by 150 and 15 gives £2 and 4*s* approximately. Hence, this will be equivalent to the first tariff when the average room-size is 100 sq. ft. (The values have been selected to give round numbers. With larger average rooms, the block sizes would be smaller to secure equivalence.)

The above may be analysed as follows :

To cover kWh costs .	0·5*d.* per kWh.
To cover consumer costs .	£1 per annum per consumer.
To cover kW costs . .	1*s.* 6*d.* per annum per 100 sq. ft. (or per room) plus 0·25*d.* per kWh.
To cover indivisible or common costs . .	2*s.* 6*d.* per annum per 100 sq. ft. (or per room).

This last is based on the value to the consumer of having electricity and being able to use it for lighting, radio, etc.

Rural Supplies.—In urban areas there is usually no need to make individual differences in the scale of charges to domestic and commercial consumers since there are no very great differences as regards the cost of making the same supply available to different premises. This is because in most built-up areas the distances from the distributor do not vary greatly, and moreover the cost of the service connection is a comparatively small part of the total cost of supply. Such costs, whether of the per-consumer or per-kilowatt variety, can be averaged with tolerable equity within the consumer group to which the tariff refers, without the need for individual discriminations. In the exceptional case, such as that of a house standing in large grounds of its own or in an isolated position, there is statutory provision for discrimination in charging ; and Area Boards can make a charge for so much of the service lines as lie on the consumer's property or exceed 20 yards from the distributing main.

In rural areas, isolation is the rule rather than the exception, and distribution costs are likely to be both larger and more variable. To some extent, these larger costs can be equitably averaged over consumers in a group, and covered by means of a special rural tariff, but there will always be isolated premises whose inclusion in such an average will result in a high tariff and will be unfair to neighbouring consumers. Their extra distribution costs must then be either made the occasion of a special charge or else spread over all consumers, urban as well as rural (and possibly industrial as well as domestic).

Which plan to adopt is a question of policy quite as much as econmics. If supplies are to be made available to every hamlet, however remote, the costs will be heavy, and to average them will be to penalise existing consumers and hamper other developments of the load which may be socially more desirable. Similar objections apply to anything in the nature of a national subsidy from outside the electricity industry. If a more moderate programme is proposed and the rate of connection of distant consumers is not too great, it may be possible to average the extra costs without serious consequences. But usually it has been thought best that some, at least, of the individual extra distribution costs should be loaded on to the particular consumer, both on grounds of equity and because this furnishes a criterion as to which extensions to undertake. Some of the methods which have been employed in levying individual distribution costs are discussed below.

An obvious objection to any individual levying is the difficulty of correct allocation and the fact that this may be upset by subsequent development. In one sense the first person to have a supply in any area, urban or rural, may be said to have necessitated enormous expenditure in distribution lines, and if he were to be charged with the whole of this cost there never would be a first consumer. Any such strict allocation must always be allayed by considerations of probable development, and the Electricity Board, like any other

retailer, must take some risks in assessing the ultimate profitability of any venture. The pioneer who has had to pay a substantial part of the cost of the line put up to supply him may well complain if within a year or two the line is used to supply other consumers who are not asked to make any such special contribution.

Broadly, the aim should be to plan the development of a rural area as a whole instead of as a series of *ad hoc* tie lines to meet the requirements of individual applicants. This will have the effect of increasing the proportion of distribution costs which is common to all consumers in the area, and decreasing the proportion which is special to the individual consumer. It will mean also that a general system of cost allocation can be adopted which will not be upset by additional consumers coming on to the line. The total costs under this plan may be somewhat higher in the early stages of the development but the final scheme will almost certainly be more economic, and the difficulty of individual allocation will be greatly reduced.

When this general plan has been made, if it is found that all consumers in a certain area, even those nearest to the mains, involve a certain minimum value of distribution expenditure, this can be expressed as an annual charge for interest and depreciation and added to the fixed charge of the tariff offered to those consumers. The only individual additions then necessary will be the additional special costs of the more distant consumers. One disadvantage of this method is that tariff uniformity throughout a Board's territory will be more difficult, and it will be almost unavoidable to employ a special rural tariff higher than the corresponding urban tariff.

Another complication in rural supply is the diversity of usage over a given area. Instead of comparatively uniform blocks of well-defined usages, domestic, industrial and commercial, there is more individual variation, and many more cases of mixed usage. The most obvious example of this is the farm and the horticultural establishment which may be at once a home, a production unit, and a shop.

Special Distribution Costs.—The following are methods of covering distribution costs special to particular consumers. They apply to outlying consumers (*e.g.*, rural) or to those whose loads involve a special line and/or transformer.

(*a*) *Capital Contribution.* Payment by consumer of lump sum representing the whole (or some given proportion) of the extra capital costs involved in making the supply available.

(*b*) *Annual Contribution* or line rental representing the interest and depreciation on (*a*). This should then continue in perpetuity, and could with advantage form an addition to the fixed charge

of the two-part tariff. Alternatively, a somewhat larger percentage can be employed and its application limited to a given period, *e.g.*, 5, 7, or 10 years, after which the initial cost can be regarded as fully amortised, and the rent payments cease thereafter.

(*c*) *Revenue Guarantee.* When the tariff offered in the area is sufficient in the agregate, *i.e.*, it covers the average of all costs incurred, it is only necessary to see that this average works out as fairly as possible in the individual case, and in particular that a consumer requiring an expensive connection consumes sufficient electricity (and therefore produces enough revenue under the tariff) to cover the extra costs. This can be ensured by obtaining from the consumer a minimum guaranteed revenue—sometimes expressed as a minimum annual consumption of electricity, since the two are linked according to the terms of the tariff.

The magnitude of the minimum guarantee may be 15 to 20 per cent. (sometimes 30 per cent.) of the special capital expenditure incurred, and it may be required for a period of 3 to 7 years. (When a special line has to be provided it is permissible under the Clauses Act to require for at least 2 years a revenue under the normal tariffs of 20 per cent. of the capital cost of this line.) Economically, this revenue should be required in perpetuity because it is not likely that under the standard tariff terms the extra profits from this consumer over the profit from a consumer not requiring special capital expenditure would be sufficient to amortise the capital in so short a period. In practice however the tendency is for individual consumptions to rise, and if the revenue were adequate for the first 3 to 7 years it would probably continue to be so.

For example, if the capital cost of an extension is £100, the minimum guarantee might be 20 per cent. of this, *i.e.*, £20 per annum. Provided a consumer's yearly bills exceeded £20, he would not be charged any rental, but if they fell below £20 he would have to pay this minimum amount. In effect, on low consumptions the user is charged a yearly rental of £20 minus his payments for electricity.

(*d*) *Part Revenue Guarantee and Part Capital Contribution.* A not uncommon plan is to cover some of the costs by method (*c*), say, on a 20 per cent. basis and the remainder by (*a*). Thus, if the capital cost in question is £100, and if the consumer guarantees a consumption which will bring in £12 a year, this represents 20 per cent. on £60; the capital contribution will thus be reduced to £40.

A serious objection to minimum-revenue guarantees is that, if in any year a consumer does not use enough electricity to cover this sum, he will appear to be in a position of being charged for electricity which he is not using. (Actually, it would be more correct to say that he is being charged for a service which he is not using as fully as he might. But since the connection-charge is in the form not of an annual fixed charge or rental, but an imaginary rental minus a rebate (the revenue), the effect is that of being charged for unused electricity.) Such an arrangement, therefore, commits a cardinal tariff fault, in that at some point on the consumption curve the consumer can use some more electricity without adding anything to his bill. In a country like Great Britain, where every extra unit means an extra pound or so of coal consumption, this is likely to promote an uneconomic use of the nation's resources. At this point of consumption the tariff in fact becomes infinitely " promotional."

(*e*) *Combined Rental and Revenue Guarantee.* One way of overcoming this difficulty within the same tariff framework is to reckon only half of the revenue as rebate of the annual rental. Thus, if the capital cost were £100 and this were regarded as the equivalent of an annual rental of £10, the annual service or connection charge could be £10 per annum less half the tariff revenue. When the revenue was £20 or over, no rental would be charged ; when the tariff revenue fell below £20 in any year, say, £16, the rental would be £10 minus $£\frac{16}{2} = £2$. The total bill would then be £18. If the full rental is denoted by R and the tariff revenue by T, the low-usage consumer pays rental less rebate plus tariff charge $= R - T/2 + T = R + T/2$, *i.e.*, the full rent plus half the tariff charge. At this point on the curve, additional consumption may be said to cost just half the tariff charge. At low consumptions the promotional element is therefore still very strong, but not such as actually to encourage waste of electricity.

A indirect method of securing something like a minimum-revenue guarantee is to insist on the installation of sufficient apparatus to ensure that the consumer will use not less than the required amount of electricity each year.

NUMERICAL EXAMPLE—CAPITAL COST £100

Method.	Payment (for Connection and for Electricity).
(*a*) Capital Contribution .	£100 plus standard tariff on each year's consumption.
(*b*) Annual Rental 10% .	£10 p..a, possibly only for a limited number of years, plus standard tariff.

Method.	Payment (for Connection and for Electricity)
(*c*) Guaranteed Minimum Revenue of 20%.	Standard tariff with a minimum of £20 p.a., possibly only for a limited number of years.
(*d*) Sum of 40% (*a*) and 60% (*c*).	£40 plus standard tariff, with a minimum of £12 p.a.
(*e*) Combination of (*b*) and (*c*) on Low Consumptions.	£10 p.a. plus half standard tariff.

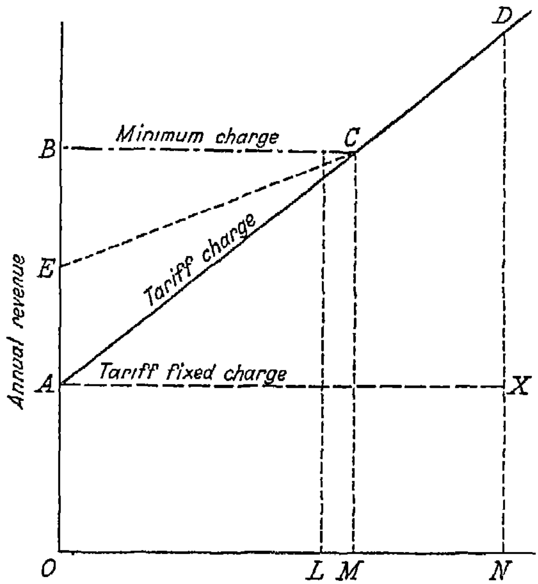


FIG. 44.—Rural Charges.

Graphical Illustration.—The graph in Fig. 44 represents the annual revenue from a consumer plotted against the annual consumption. The full line *ACD* shows the charge under the normal two-part tariff, and at any consumption *ON* this is made up of the fixed charge *NX* and the running component *XD*. If, however, (under method (*c*)), the consumer guarantees an annual minimum payment of *OB* corresponding to a consumption *OM* his payment will follow the line *BCD*. Even if his consumption is something less than *OM* (say *OL*) he must pay the minimum charge, and at this point he could consume the additional *LM* units for nothing.

N.B.—For the same revenue requirements the standard tariff in cases (*c*) to (*e*) must be higher than it is in (*a*) and (*b*).

If, instead of a minimum guarantee, there is a line rental added to the tariff charge with a rebate dependent on the amount of the consumption (method (*e*)), the payment will follow some intermediate line *ECD*. The values shown on the graph are those given above (rental equals half the normal revenue guarantee and rebate equals half the tariff revenue), and point *E* is then midway between *AB*. At low consumptions the tariff is still highly promotional (since extra units only cost half the tariff charge), but it does not actually invite waste of electricity.

Naturally the only satisfactory solution is to secure such utilisation that the consumer is always working on the portion *CD* of the curve, but until this is achieved the operations lower down are of importance.

Cost Aspect of Revenue Guarantees.—The idea underlying methods (*c*) and (*e*) is to use a fixed proportion (*e.g.*, 15 to 20 per cent.) of the special capital expenditure as a criterion of the minimum revenue which can justify a connection. This implies that the minimum cost of giving a supply is a fixed ratio of the capital expenditure incurred, and can be expressed as an annual percentage. The justification for this is that probably nearly half of this minimum cost is the interest and depreciation on the extra capital expenditure, and other items, such as rents, rates, taxes and possibly management, may also be taken as a function of this expenditure. But the remaining items making up the cost at low consumptions, namely, bulk supply, distribution, and consumer costs, are not really proportional to the expenditure on the extension.

Moreover, even when it is useful as a guide to minimum requirements the revenue-guarantee basis is structurally faulty as a costs expression, as is well shown when it is applied to the calculation of rental rebates. The extra costs resulting from a special connection do not differ in character from the other distribution expenses and should be levied by means of the normal tariff elements, *e.g.*, by an addition to the fixed charge, thus affecting the revenue at all consumption levels. The point could be put another way by saying that the rule for abating the line rental by some fraction of the revenue is based on the assumption that on larger consumptions the normal tariff yields a surplus sufficient to pay the extra capital charges on special connections. Whether this is so or not depends on the magnitude and proportions of the tariff, but clearly it should not be so.

Assuming that all those who desire a supply and are fairly close to distribution mains have been connected up, and that the standard tariff covers the marginal costs of supply and an appropriate share of the common expenses, it follows that a new consumer (more distant than the average) incurs costs not provided for in the tariff and should make a special contribution to the extra capital cost incurred in making his connection. The question of whether he is a large or small

consumer is (or should be) irrelevant : if the tariff is correctly proportioned it should recover the costs (with a suitable margin) equally on all consumptions, *i.e.*, the running charge should cover running costs and the standing charge should include capital charges on the average length of connection.

When a consumer on the route of a main and paying only the tariff charge increases his load, his extra bill is supposed merely to pay for the extra cost of supplying more electricity, yet when a consumer on the end of a long line does the same thing, his extra bill (under the same tariff) is supposed to yield a surplus which will justify rebating some of his line rental. If, in fact, this is so, the normal consumer may well claim that on a large consumption he is being overcharged.

Probably the truth is that it is politically almost impossible to charge the small consumer and the more distant consumer the real costs of supplying him : *i.e.*, the standing charge (particularly the non-proportional consumer-cost element) is less than it should be, and the running charge is correspondingly greater. Hence, the large consumer yields a disproportionate profit to the undertaking. Besides desiring, for political or other reasons, not to deal harshly with the small outlying consumer, the undertaking has also in mind the tendency usual in electricity for the small consumer to become a big one and for the outlying consumer to be joined by others.

CHAPTER XII

TIME-VARIABLE TARIFFS AND RESTRICTIONS

Positive Load-Factor Improvement.—One of the chief aims of a tariff, in so far as it directs consumption, is to encourage the use of electricity at such times as it is cheap to supply, and to discourage it at other times. These other times will be the periods of maximum demand upon the generating station and distribution system, since it is then that the plant is fully loaded and that the bulk of the standing costs are incurred. The tariffs described in the last chapter have only done this very indirectly, by encouraging the class of consumption least likely to come into the peak period. It is possible, however, to aim more directly at the matter by bringing the time of day into the tariff. Such a scheme will generally involve the use of a time switch or remote-control relay, and the cost of hiring and operating this must be put against the gain to the consumer in other directions. It is therefore more likely to be advantageous on a large installation than on a small one.

The devices studied in the present chapter chiefly concern domestic and, to a lesser extent, commercial supplies. As regards industrial loads, one of the tariffs already employed (the Hopkinson M.D.) may be said to aim directly at the improvement of the individual load factor: in so far as the improvement is not neutralised by reduced diversity the system load factor will be correspondingly improved, although the effect cannot be so positive as when time-of-day is brought directly into the tariff.

In general the industrial load cannot be regarded as so susceptible to modification by tariff inducement as is the domestic load. Motive power has none of the storage characteristics which are possible in thermal applications. Moreover, when electricity is a very small element in the total cost of production, its elasticity of demand will be low, and a large price difference will be necessary to affect its use. The most amenable elements in the industrial load will be those where the consumption is large in proportion to the labour, *e.g.*, furnaces; and here the off-peak tariff, or even the normal M.D. tariff, may succeed in shifting load from day to night. Apart from such cases, the main application of time-varying tariffs will be in the non-industrial field, and their importance can best be seen by examining the present domestic situation.

Domestic Development.—The outstanding feature of the last 25 years in this country's electricity supply has been the domestic development. Although we were pioneers in the earliest days, 50 or more years ago, our development in the first half of this period lagged behind that of many other nations. Taking as a criterion domestic sales of electricity per head of population, the units sold for lighting, heating and cooking were only 38 per head in 1927, but they had grown to 330 in 1947, a nine-fold increase in 20 years. Even since the beginning of the war the figure has doubled.

This development was the combined result of a number of factors: sound engineering, enthusiasm for the electrical idea, promotional activities in hiring and wiring, and many another. But probably the greatest single element in the whole campaign has been the all-in tariff. Introduced comparatively recently and having to overcome various legal disabilities and considerable public opposition, it is now the basis of sale for something like 80 per cent. of the domestic consumption.

Like all rapid developments this one needs guidance as well as encouragement, particularly at the present time. There are, in fact, signs that this particular phase of domestic electricity salesmanship has reached its mark, if not indeed overshot it, and that some new line of development is called for. Probably the first considerable occasion on which undertakings "saw the red light" was in the cold snap of December 1938, which resulted in distribution undertakings having to pay many unanticipated millions of pounds in bulk-supply charges for additional kilowatts of demand on which they had collected inadequate amounts of revenue. Undertakings suddenly realised how large was the seasonal heating load they had inadvertently built up.

Domestic Diversity.—The very title "all-in tariff" implies the absence of restrictions, and this equally suits the consumer (by simplifying installation) and the supplier (by encouraging diversity). In the pioneering days it must have required no little courage for an undertaking paying £3 10*s*. per annum per kW for bulk supplies, plus heavy capital charges on distribution, to connect a 8-kW cooker at a tariff with a fixed charge of £2 or £3 a year, and a running charge of ½*d*. per unit. The fact that so many undertakings did this and, so far from being ruined, made it a commercial success is the best possible proof that the potential diversity was really there, and that the all-in tariff was the way to evoke it.

There are however limits to everything, and even domestic load diversity is not an inexhaustible source of profit. Its limits will be more apparent if it is dissected, and this can best be done by taking a single typical case. Measurements at a sub-station supplying an estate of fully electrified dwellings* gave the following results :—

* These were not *all*-electric : they had solid fuel space-heating equipment, but they used electric cooking, electric water-heating, and some electric space-heating for intermittent and supplementary purposes.

(*a*) Average installed load per consumer - - - - 11 kW

(*b*) „ M.D. per consumer - - - - - 8 kW

(*c*) „ simultaneous M.D. per consumer (A.D.M.D.) 2·1 kW
(at 1.30 p.m. to 2 p.m. on a Sunday).

(*d*) Average system demand per consumer (A.D.D.) - 0·5 kW
(at 8.30 a.m. to 9 a.m. on a cold weekday).

Referring to these figures, (*b*) gives the average individual M.D. of each consumer. This was not actually measured but was estimated from his equipment on the assumption that there would be occasions when he was using his cooker at its maximum rating of 6 kW plus some additional kW in water- and room-heating. This maximum usage, however, is infrequent and short-lived, and there is an enormous diversity between the aggregate of the individual M.D.'s and the combined simultaneous M.D. on the estate. This latter figure, divided by the number of consumers, is described as the after-diversity maximum demand. It is shown at (*c*) and discloses a diversity on the distribution network of $\frac{8}{2{\cdot}1} = 3{\cdot}8$.

Whilst the above is the important figure for distribution costs, the figure affecting generation and main-transmission costs is the diversity at the time of M.D. on the main system. This occurs at 8.30 a.m. to 9 a.m. on a cold weekday morning, and the reading at this time discloses a further diversity of $2{\cdot}1/0{\cdot}5 = 4{\cdot}2$. The overall diversity effective for generation costs was therefore $3{\cdot}8 \times 4{\cdot}2 = 16$.

On the basis of some such figures as these, the costs of supplying the domestic load have been calculated, and the two-part tariff magnitudes have been worked out. The question is how far can one continue to go on these lines, and what limits are there to what has been called the " indirect " method of improving the system load factor.

In the earliest days of electricity supply, the " direct " method of load-factor improvement was often pursued, even in the domestic field. A M.D. tariff on the present industrial model was frequently employed, with a standing charge based on metered M.D. Consumers using electricity only for lighting (as the majority then did) had an annual load factor of 10 per cent. or less. This was very expensive to supply, and under such a tariff the consumer paid a correspondingly high figure.

Let us suppose the same plan had been followed in dealing with later developments of electrical service, such as the vacuum cleaner. The individual load factor of the electric cleaner is far less than that of the electric lamp ; with a usage averaging only two to three hours a week, its load factor will be something under 2 per cent. Yet it would hardly be suggested that the vacuum-cleaner load is expensive to supply in the

way that the lighting load is, for the obvious reason that the vacuum-cleaner usage is spasmodic and may occur at almost any hour, summer and winter, whereas lighting is associated with certain times of the day and year throughout the whole area of supply.

Diversity Limitations.—The limit to the diversity factor may therefore be very simply expressed in these terms. Diversity exists between different times of usage of the same load (*e.g.*, vacuum cleaners) and between different loads (*e.g.*, heating and power, or domestic and industrial). The former diversity tends to disappear when the load is associated with particular times, as are the lighting and space-heating loads. When a load of this character becomes a major component of the total load, marked peaks will occur traceable to that particular load, diversity will flatten out, and the standing costs will increase. In order to maintain the diversity on which reducing costs are based, it is therefore necessary to have as wide a range of usage as possible, and above all not to let any seasonal load get out of hand —using the word "seasonal" to indicate any load likely to be associated with particular hours in the year throughout the whole area.

The obvious sinners in this respect are lighting and space-heating. Lighting is much less of a problem because of its economy and value. To supply the basic minimum of dark-period lighting requires a relatively small quantity of electricity per consumer, and its sales value is so high that there is no great difficulty in securing adequate payment for the costs incurred. Moreover, where surplus lighting is employed on a scale much beyond the minimum requirements this will often overflow beyond the peak-hour danger period, as in the case of architectural and shop-window lighting.

Space-heating is a much more serious matter, because its potentialities are far greater than those of lighting and because its usage is more erratic and unforseeable. The figures of the E.R.A. Sampling Survey suggest that there may be something like fifteen million kilowatts of space-heating equipment on consumers' premises—nearly 50 per cent. more than the total generating capacity. This is a threat (however unlikely to materialise) to the very existence of the supply industry, and through it the industry of the country.

Space-heating usage is of many different kinds, some admirable in every way, others less easily defended. Electricity is by common consent an ideal source of intermittent heating. Wherever relatively small amounts of heat are required for short periods, the rapidity and convenience, portability and labour-saving characteristics of electric heating make it supreme. Furthermore, such heating requirements are likely to be distributed over a large part of the day and to extend over most of the year, since an electric fire is as useful to save lighting a coal fire in the chilly autumn and spring weather as in the depths

of winter. The use of electricity in place of a solid-fuel-fired central-heating system for long-period base-load heating is on less sure grounds, whilst the use of electric fires purely in very cold weather to make good the deficiencies of other systems may be a sin against the national economy.

It has been estimated that at the time of year at which the system peak occurs there may be a difference in the total load of the country of as much as a million kilowatts between a moderately cold and an extremely cold winter day. This extra million kilowatts cold-snap load is of only a few hours' duration a year, and in a mild winter may not occur at all. Yet plant must be provided to meet it, and the overhead cost of this plant is almost certainly not covered by the revenue from the sale of these few extra units. The question is how to discriminate against this load without a host of uneconomic contrivances and without unduly restricting the freedom of use which is so great an attraction in the present domestic-tariff system.

Dangers of All-in Tariff.—The above notes show that there are certain risks in our exclusive reliance on the all-in tariff and the "indirect" method of load-factor improvement (encouragement of an expanding and diversified load), as distinct from the "direct" method (*e.g.*, by time-of-day tariffs). The fixed-charge calculation, and with it the justification for the whole all-in tariff development in this country, is built up on a highly problematical factor, that of diversity. Usually, it works out all right, but occasionally it breaks down. The diversity relied upon does not materialise, and there is a "run on the bank".

The all-in tariff in fact is not consumer-proof, or, more precisely, apparatus-proof. Its success in recovering the costs of the load it attracts depends upon the character of the consumption: water-heating is excellent; cooking is good; space-heating not so good. Much depends, also, upon the magnitude of the load in proportion to other loads on the system. A balance is needed between the rates of development of the different classes of load, and future trends may make it necessary to re-examine tariff structures which have served excellently in the past.

Emergency Conditions.—So far, only the long-term economic factors have been mentioned, but a new situation has arisen through the crisis conditions following the war. In the United Kingdom before the war the installed capacity of generating plant connected to the Grid was some 20 per cent. in excess of the largest demand falling upon the Grid. By the winter of 1948–49 this margin of reserve capacity had fallen from + 20 per cent. to about − 20 per cent. under comparable conditions. (In round numbers, if the effective output capacity is put at 10,000 MW, the potential demand at the time of

the annual peak is of the order of 11,000 MW. in normal cold weather, and may rise to 12,000 MW in excessively cold weather.) For a number of reasons, such as load-spreading and various peak-avoidance endeavours, conditions are not exactly comparable, and the figures are only rough ones. In one respect the comparison is worse than it appears, for before the war none of the installed capacity exceeded twenty years of age, whereas now much of it is twenty-five years or older.

These emergency conditions, as evidenced by an alarming gap between national capacity and national demand, have existed ever since the war and seem likely to continue for a number of future years. Moreover, the general impoverishment resulting from the war means that we cannot for many years have anything like the range and quantity of goods we desire, and must choose much more narrowly between them.

Summing up, the short-term or emergency situation means that, however much we need electricity and though we value it above all other requirements, we have not at the moment the plant required to meet the demand, and the necessary new plant cannot be installed for several years. The long-term situation is that we are poorer than we were and have not sufficient steel, labour, dollars, etc., for all our requirements.

We can have as much electricity (or any other one thing) as we like, provided we are willing to go short of sufficient other things. We must, therefore, evaluate our electricity need and see how much we desire more kilowatts as compared with more cigarettes or more anything else. For this, it is essential that the electricity be as accurately costed as possible, so that if, for example, particular sorts of electric heating are unduly expensive to provide, the consumer shall be aware of the fact and in a position to make his choice in full view of the economics of the situation.

Peak-Load Devices.—Since the immediate gap between capacity and potential demand is a species of *force majeure* it may be necessary to examine *force-majeure* devices. If plant is insufficient, someone must go short, and ideally the less-essential load should in such circumstances be forcibly prevented so that the national economy shall be least impaired.

This is where electricity, by its very nature and composition, is of all things the least amenable. Built up as a service available at all times, places and seasons, its principal technical feature is a constant voltage closely maintained up to the extremest tips of its distribution system. As a consequence, a consumer anywhere can put what apparatus he likes across the line and immediately has the whole power-station energy to draw upon. This feature of electricity supply makes anything like rationing or restrictions very difficult. When

goods are running short they can be doled out in smaller quantities: even with gas supply the pressure can be reduced, and with a telephone the consumer's calls can be controlled from the exchange, and when the lines are busy he may have to wait. But with electricity it is all or nothing, and once a consumer has access to the mains there is little that can be done to control him, short of cutting him off again.

It is no part of the present work to review the many restrictive devices and suggestions that have been considered : ranging from a full-blooded heating and lighting rationing-system for the household, down to such minor irritations as requiring water-heaters to be fitted to an alternative circuit. The 1948 (" Clow ") Committee considered a number of these and gave reasons for rejecting most of them. Since this book is concerned with tariffs, the only restrictive measures here examined are those associated with changes in electricity price. Broadly, these are all methods of charging more or less at certain times, but they may be split up into the following :—

Restricted-Hour Tariffs—for supplies which are only available outside prescribed hours.

Off-Peak Tariffs—in which a lower price is charged for electricity consumed outside prescribed peak-load hours.

Time-of-Day (or Two-Rate) Tariffs—in which different prices are charged for electricity used at different times of day.

Seasonal Tariff Variation—in which different prices are charged at different times of the year.

Controlled-Period M.D. Tariff—a two-part M.D. tariff in which the demand record is only in operation during specified periods.

Control Systems.—Reference should be made to the methods by which different rates are operated or apparatus switched out during peak periods. The methods can be classified as follows :

Local Control . . .	Time Switch.
Telecontrol . . .	By Pilot Wire.
Telecontrol superimposed on power networks . .	Ripple Control.
	D.C. bias.
Mains Frequency Operation.	

Most of these are self-evident, but a few comments should be made.

Local Control by time switch in its simplest form is the cheapest method, but the least flexible. A timepiece, either spring-driven (hand- or electrically-wound) or frequency-driven, opens and closes a switch at particular times each day and thus controls the circuit either to the apparatus or to the meter. If different times are required at week-ends the clock is more complicated, and if a seasonal change is to be made or adjustment for " Summer Time " there are greater

complications still. If the clock is frequency-driven a reserve wind is advisable to carry it over periods of supply interruption. Errors are likely to accumulate, and if the timing gets out of step for any reason it cannot be put right from the centre and individual visits become necessary.

Telecontrol. Systems of remote-control from appropriate radiating centres such as sub-stations have the advantage of flexibility. Control can be varied as desired throughout the year, and it is possible to operate thereby a system of purely unprogrammed switching out, *e.g.*, of water-heaters, to suit the peak-load exigencies of the moment. The most popular systems are those in which electrical messages are superimposed on the existing power network, using either high-frequency ripple or D.C. bias.

In the former case a ripple generator is coupled to the network and emits at an audio frequency, usually some 700 or 800 cycles per second. This excites tuned relays using the same principle of resonance as is employed in wireless reception, and this relay carries out the desired operation. There is no difficulty in transmitting a dozen or more different signals, and picking them up selectively without interference or over-spill. Several systems of proved reliability have been developed, employing robust and well-nigh operation-proof relays.

The system sponsored by another firm employs a D.C. difference of potential or bias between the star point and earth of a three-phase distribution network. The emission apparatus is simple, but the system lends itself chiefly to local operation.

Mains Frequency. Mains frequency operation is one of the latest suggestions. When the load on an interconnected A.C. system such as that in Great Britain approaches and threatens to exceed the available capacity, the first thing that happens is that the speed of all the sets (and therefore the frequency) falls slightly. This is just as automatic and inevitable as is the fall of speed in a car on a hill once the limit of power-increase by pressing the accelerator is reached. To some extent this very fall eases the situation by reducing the load somewhat; but, if the speed continues to fall, a critical value is approached and the control engineer has no alternative but to call for some load to be shed.

The least harmful way of doing this would be to cut out thermal-storage loads and anything else capable of "free-wheeling" for a while without serious loss of service. The logical step from this is the development of a relay sensitive to changes in the supply frequency. This can be attached to the appropriate consuming apparatus and will automatically switch it out if the frequency falls, and in again when the normal frequency is restored.

Such relays have been developed, and have the obvious advantage over telecontrol that they save the cost of the emission plant. The saving is less than it might appear because on a large telecontrol

system the relays represent much the greater part of the cost. A more important point is that the mains frequency system is much less flexible and controllable. There may well be times when a different load reduction is desirable in different parts of the country, whereas the frequency is the same throughout. In general, the supply engineer would prefer to have all such switches under his direct and voluntary control.

In examining the economics of any of these systems *vis-a-vis* the corresponding tariff concession, it is evident that the capital charges and maintenance costs of the relay itself must be allocated to the individual installation which it controls. The cost of the emission plant can, however, be spread not only over the special tariff consumers, but also over any other usages. With twelve or more demand signals available it may be possible to use some for, say, street-lighting operations, and thus share the costs.

Restricted-Hour, Off-Peak and Non-Firm Supplies.—As regards restricted-hour and off-peak tariffs, in the former a special rate is quoted for supplies which are only made available for specified periods, *e.g.*, from midnight to 6 a.m. for thermal-storage heating installations. In the latter case, there is continuity of supply but a lower price is quoted for supplies during certain hours, *e.g.*, from 6 p.m. to 7 a.m.—and possibly again from 12 p.m. to 2 p.m.—with a higher price during the remaining hours. In practice, there is no great distinction between these two cases ; it must be presumed that there is always a continuous supply to be had at some alternative rate or other.

The above are essentially endeavours to fill up the valleys in the load curve rather than to shift the peaks, *i.e.*, they can be used to build up a load in the off-peak period which would otherwise be supplied non-electrically, but they are not likely to make any substantial change in existing loads. They are usually somewhat special in their application, appealing to a comparatively small number of consumers, and whilst they are economically sound and to be encouraged they are unlikely to contribute materially towards solving the peak-load problem.

Since the times are agreed beforehand, the necessary operation can be by means of time switches, although of course telecontrol can be used if preferred. With restricted-hour services, the supply can be interrupted and restored by a time switch according to a pre-arranged schedule. When two or more rates apply, the same sort of switching device can be employed to change the metering without interrupting the supply.

Somewhat akin to an off-peak tariff is the system whereby water-heaters are supplied with a small tariff concession on condition that the undertaking can cut them off for occasional, relatively short, periods

at times of system peak. This cannot be done by time switches and must employ telecontrol. References to experiments carried out on such a system are made in an earlier chapter (p. 145). The 1948 Committee recommended further investigation into the giving of "non-firm" supplies of this sort.

The system has the advantage that the annual hours of interruption are likely to be far less than under the normal restricted-hour and off-peak arrangements, and its application is therefore not confined to apparatus specially designed for night consumption. It might, for example be used with tubular or panel heaters (with minor modifications) in commercial or large domestic installations. Another advantage is that the arrangement is very flexible: provided a number of different signals are available, loads can be switched out and back in stages, or even in relays when a long interruption becomes necessary.

The economics of the arrangement are somewhat complex. At first sight, a comparison of capital costs leads to a highly favourable conclusion. The first cost of a complete supply system, extending to the L.T. consumer's terminals, will exceed £100 per kW at present-day prices; whereas a telecontrol system plus the second meter only costs £10 to £15 a point, and each point can control several kilowatts. The gain is, however, reduced in the ratio of the diversity, based on the unlikelihood of the load being on at that particular moment. Most of the loads lending themselves to such control are not specifically peak-producing, and could be made still less so by appropriate encouragement on other lines.

Moreover, a comparison of capital costs is misleading because the kilowatts made available through a non-firm supply are purchased at the consumer's inconvenience, and must be paid for. The maximum possible tariff concession would be the difference between the normal two-part tariff running charge, and the bulk supply running charge increased to cover distribution losses and to provide a small margin of profit or contribution to general expenses. This difference—possible 0·2*d.* per kWh—has to recompense the consumer for the cost and inconvenience of the special apparatus and usage, and the second wiring circuit.

Another arrangement which may be combined with a tariff concession is the alternative circuit operated by the consumer by means of a change-over switch. In this case, one piece of apparatus such as a storage water-heater is put on a different circuit from either another piece of apparatus (*e.g.*, a cooker) or the whole remaining heating installation. Both circuits cannot be employed simultaneously, and this limits the possible maximum demand of the individual consumer. Provided there is sufficient storage the consumer need suffer no deprivation so long as he remembers to operate the control correctly. Owing to diversity, this device has little effect on the system demand though it may reduce the M.D. on the distribution network. The reason for

the small overall effect is, in short, that (presumably) the same total amount of cooking and water-heating is done as before but the individual consumptions run consecutively instead of concurrently. If there are hundreds of domestic consumers going through the same performance at various times throughout the period 6 a.m. to midday, the aggregate curve will have almost the same height as before though it may be displaced slightly (*e.g.*, half-an-hour later). Depending on the shape of the system load curve, this may or may not be a good thing but the gain is unlikely to be material.

Time-of-Day Tariff.—The following is an example of a pure time-of-day tariff which charges three different flat rates according to the time of day of the consumption. The proportions and general construction are based on a French tariff which was available for general domestic and commercial purposes other than lighting. The times have, however, been altered to accord more nearly with United Kingdom load curves.

The three rates with their distinguishing names are as follows :—

"Peak" (*i.e.*, high)	3*d.* per kWh
"Day" (*i.e.*, medium)	1*d.* " "
"Night" (*i.e.*, low)	½*d.* " "

The following is a schedule of the times for these three rates :-

	"Peak."	"Day."	"Night."
Winter .	8 a.m. to 12 noon ; 3 p.m. to 6.30 p.m.	12 noon to 3 p.m.; 6.30 p.m. to 11p.m.	11 p.m. to 8 a.m. —
Summer	—	8 a.m. to 6 p.m.	6 p.m. to 8 a.m.

Probably the most economical way of operating such a tariff is to have three standard energy meters in parallel to read the "peak," "day" and "night" consumptions. A remote-controlled three-way switch or triple contactors direct the supply through the appropriate meter at specified times during each day.

A pilot lamp can be connected to indicate to the consumer which meter is recording at any given moment. By making the peak price the sum of the other two, one of the meters can be dispensed with. The three-position switch then sends the current through one or other of the meters or through both in series. There are two objections to such an arrangement. Firstly, the possible values may be unsuitable : if the "night" rate is low, its addition to the "day" rate is unlikely to give a sufficiently high "peak" rate. Secondly, there are legal and commercial objections because, although the two

meter readings provide an exact basis for the electricity bill they do not enable the kWh consumption to be determined.

Since the time-of-day tariff appears to enable the demand-related costs to be allocated to the individual consumer more accurately than any other method it is natural to ask whether a " pure costs " tariff on these lines could be successful, provided, of course, that the necessary apparatus could be economically supplied. By " pure costs " tariff, is meant one which distributed none of the expenses on a use-value basis. The fixed charge in such a tariff would then be the same for everybody and would cover only consumer costs. Supposing this to be £1 per annum, the tariff in its simplest form might consist of £1 per annum per consumer plus 4*d.* per kWh during specified (peak and near-peak) hours and ¾*d.* per kWh during all other hours.

Experience in Continental countries which have come nearest to such a tariff seems to indicate that it is impossible to find values within the above framework such as to enable satisfactory development to take place.* In other words, if the fixed charge includes no contribution representing the special use-value of *having* an electricity supply (of any kind and size) the tariff will either be too low to bring in sufficient revenue or else too high to promote the vigorous development that can profitably occur under other types of tariff. A pure costs tariff in fact does not achieve the economic target or optimum development in which each consumer takes every unit for which he is prepared to pay marginal cost.

Scientific Tariff.—If a *really* scientific tariff is desired (regardless of complexity) and not one that merely regards cost science, one should boldly face the fact that some of the costs and expenses are to be regarded as indivisible and therefore to be spread on a market-bearing basis. Such a tariff would be a combination of those outlined on p. 230 and p. 248. Each of the necessary functions would be recognised, and a separate tariff element would cater for each. For illustration purposes, the following values may be given.

Fixed Charge of £1 12*s.* 6*d.* per annum up to 500 sq. ft. plus 2*s.* 6*d.* per annum per 100 sq. ft. thereafter.

This could be expressed as £1 per annum plus 2*s.* 6*d.* per annum per 100 sq. ft. with a minimum of £1 12*s.* 6*d.* per annum.

Running Charge, 3*d.* per kWh during specified (peak and near-peak) hours and ½*d.* per kWh during all other hours. (An intermediate step of 1*d.* for the potential-peak periods as in the previous example would make the tariff still more scientific but unduly complicated.)

* It will be noted that the French example cited above is not an all-in tariff, but a supplement to another which caters for the basic domestic consumption.

The fixed charge is made up of a non-proportional element of £1 per head to cover consumer costs plus a *pro rata* charge of 2*s*. 6*d*. per 100 sq. ft. which spreads the common costs on a use-value basis.

The running costs are covered by the uniform running charge of ½*d*. per kWh, and the demand-related costs are covered by the surcharge of 2½*d*. per kWh during potential-peak periods.

Difficulties and Effectiveness.—Apart from any theoretical advantages of cost representation, there are a number of practical difficulties in the way of an extensive use of time-of-day tariffs in this country. In the first place, there are some twelve million domestic and commercial consumers, three-quarters of whom contribute revenues of less than £6 per annum, and one-third less than £3. To equip them all with remote-operated meter-changing would cost as much as a considerable addition to the generation capacity, and the kW of demand saved by many of the smaller consumers would probably not be worth the cost of their installations. If the device were applied only to the larger domestic and commercial consumers, it would have to take the form of an optional alternative and therefore coupled with a tariff inducement.

Probably the psychological difficulties are even more serious than the technical and economic ones. Much has been done on voluntary lines during the emergency period to make consumers " peak-load conscious " and use electricity by the clock. It is extremely doubtful whether they would take kindly to a permanent regime of this kind, motivated by financial rewards and penalties. It must be remembered that, regarded merely as a revenue-collecting device applied to a static pattern of consumption, no great fault is to be found with the existing tariff system : it satisfies the consumer, and it brings in the revenue to the undertaking. Unless the tariff can go beyond this, and induce the consumer to modulate his consumption, there is little point in making a change from present methods. In economic parlance, if all the demand is inelastic it is relatively immaterial how the costs are spread.

The question is, will the consumer alter his consumption habits or will he merely grumble and pay up ? This question suggests another, *viz.*, *can* he change. How far is it within his power to modify his consumption at the bidding of the tariff-maker, " charm he never so wisely ".

Usage and Apparatus.—Such questions throw one back to something very like the " indirect " approach to load-factor improvement, though not in quite the *laisser-faire* manner of the "All-in" tariff. Can one (whether by tariff inducements, control of apparatus sales or by other means) aim at encouraging the *type of installation* likely

to improve the system load factor? When the aim was merely all-round development, both the electricity industry and the gas industry frequently operated hire or hire-purchase terms which, in effect, subsidised apparatus likely to promote supply sales. The same kind of deliberate policy can be directed, with greater discrimination, at encouraging apparatus least likely to build up system peaks. Discriminating charges can be applied to the same end, *i.e.*, not so much to induce consumers to use their existing apparatus outside peak hours as to induce them to go in for apparatus and installations appropriate to functioning outside these hours.

Clearly there is a great difference in this respect even between different kinds of installation for the same purpose. Consider, for example, immersion-type water-heater units. These are commonly employed, particularly in the south of England, as an adjunct to a coke-fired boiler: as such, they will be very largely a summer load, and even when "topping up" in the winter, this will be largely off-peak. Exactly the same equipment is extensively used in the Midlands and North with a back-fired boiler, which is let out at night. As such, it will be a substantial winter load and will almost certainly come on to the morning peak. (The experiments described on p. 154 show this.)

There is a very great difference in the cost of supplying these two sorts of usage, yet in probably not 1 per cent. of areas is any difference made in the charge for electricity. If it were technically and economically feasible to make a tariff differentiation and it raised no administrative difficulties, there were be every justification for it on economic grounds. If this differentiation ultimately resulted in a somewhat bigger proportion of the preferable type being installed than would otherwise have been the case, it might be as good as an additional turbine.

Water-heating is an obvious example of a load which is, at least potentially, susceptible of modification by tariff inducements. Its load can be shifted to different times of day owing to its storage characteristics, and to some extent to different times of year by suitable inter-running with solid fuel. The cooking-load is less easily moved about, although a development of storage cookers might improve the daily load distribution. On the other hand, the cooker load is already well distributed as between summer and winter, and within the daily and weekly cycle it has an excellent diversity, both with other cookers and with the system load.

The space-heating load, both because of its character and its magnitude, is clearly the heart of the problem. (The E.R.A. Sampling Survey indicated an average installed load per consumer surveyed of 1·4 kW for space-heating, as against 0·9 kW for cooking.) The space-heating load is emphatically *not* uniformly distributed through the year, and what is perhaps even more serious, it has a poor diversity with

other loads of the same sort. Hence, even when the space-heating load is well spread over the less-cold months it still tends to " bunch ", and a chilly spell, even at an easy time of year, may yet embarrass the station engineers' overhaul programme since it operates simultaneously on all space-heating users.

The most obvious method would be a differential rate for each particular usage, *e.g.*, a higher flat rate for space-heating than for cooking or water-heating. This would be a retrogade step involving a reversion to the multiple tariff with its complicated wiring. Moreover, it would penalise the many legitimate and nationally-useful heating applications merely because certain particular ones had got temporarily out of hand and were not paying their full cost quota.

Failing the multiple tariff, there are two other possible weapons in the tariff-maker's armoury, one of which (the seasonal tariff) may be described as a " blunderbus " whilst the other (the time-of-day tariff) is by comparison a " precision rifle ". The time-of-day tariff has already been discussed : it aims directly and unequivocally at a redistribution of the load between night and day, and (if desired) between winter and summer, working-day and week-end, or whatever is required. The objection to it is the cost of the apparatus, and its economic application is probably limited to the larger domestic and commercial consumers. The former device, the seasonal tariff, only aims at a winter/summer redistribution.

Seasonal Tariff Variation.—A number of undertakings in this country prior to 1948 had seasonal tariff variations. The year was divided into two halves for the purpose of price change, which may be called " winter " and " summer ", though the precise timing would vary from consumer to consumer through the operations of continuous meter reading. To quote one example, the all-in tariffs, both domestic and commercial, of the London and Home Counties Joint Electricity Authority had a running charge of ½*d.* summer and ¾*d.* winter in the inner zone, and ¾*d.* summer and 1*d.* winter in the outer zone (the zones were geographical expressions corresponding roughly to urban and rural areas).

The 1948 (" Clow ") Committee to study methods of controlling the non-industrial electricity demand recommended *inter alia* that a seasonal tariff be introduced, and they also supported the suggestion to concentrate the tariff increase into a single winter quarter and to spread the rebate over the remaining three quarters of the year. At the request of the Minister of Fuel and Power, the British Electricity Authority and the Area Boards put this recommendation into effect in the running charges of two-part domestic tariffs (excluding prepayment consumers) for the year 1948–49. The figures employed were a winter increase of 0·35*d.* per kWh and a summer rebate of 0·1*d.* per kWh on an existing charge of 0·75*d.* (0·875*d.* in one area).

The reasons for using three-month and nine-month periods instead of two of six months are both psychological and arithmetical. If the tariff variation is to be successful the consumers must be well-informed and thoroughly impressed with the importance of making a seasonal variation of their consumption. This involves publicity and propaganda, which is much more effective and likely to have better results if the aim is concentrated into a three-months period. The reason for the differential can be explained in terms of climatic variations, and it is easy to show that almost all the cold spells in Great Britain occur in the three months from mid-December and to mid-March.

The second reason concerns the values that can be employed. There are obvious objections to employing a running charge at any time of the year less than the running cost of supply, and the maximum summer rebate is therefore limited to the difference between the existing running charge and the basic running cost, say 0·2*d.* per kWh. Since the summer rebate is limited, the winter increase is also limited if the total revenue is to be unaltered : but if the winter increase covers only three months and the summer rebate covers nine months—instead of six for each—the winter increase can be very much greater without swamping the summer rebate.

One difficulty arises from the very general practice of continuous meter-reading. The dates of the changes will then vary from consumer to consumer, and if the meter-reading is absolutely continuous without any gap whatsoever there will be only one moment in the year at which everybody in the country will be on the high rate. The day after the last consumer has had his tariff raised, the first consumer will have his tariff lowered : thus the three " winter " months will vary between, say, October to December and January to March, with a mean of mid-November to mid-February. Technically, the result may not be objectionable since it will give a gradual build-up of economy-inducement reaching a maximum, say, on December 31st and then gradually tapering-off again. But from the publicity standpoint it is unfortunate.

Seasonal Tariff Values and Operation.—Estimates suggest that the " average " domestic consumer takes about one-third of his energy during the three " winter " months, and two-thirds during the remaining nine months. (For the reasons stated above relating to continuous meter reading, there can be no precise figure, since even with a given consumption distribution it would vary from consumer to consumer according to when the meters were read.) At first sight it might appear that if the magnitude of a surcharge was double that of the rebate the undertakings' revenues would not be effected. But this is only true if consumers as a whole make no difference either in the amount or in the distribution of their consumption.

It must be presumed however that consumers *will* change their consumption (if the differential has any effect whatever) and the change is likely to act to the disadvantage of the undertaking in two ways. In the first place, consumers who are able to divert consumption from winter to summer (*e.g.*, by using up a bigger proportion of other fuels in the high-price quarter) will expect to reduce their annual bills thereby, and this means less revenue for the undertaking. In the second place there may be a net reduction in consumption over the year. It is true that most of this consumption is in two-part tariffs in which the undertaking is sure of its fixed charges, but, even so, the loss in revenue from the running charges is likely to be greater than the saving in costs through non-supply of these units.

Hence it is a difficult matter to forecast the various effects and to calculate values for the differential which will leave the undertakings' revenue balances undisturbed. Assuming, however, that the values *have* been correctly assessed* the effect on consumers' pockets will be somewhat as follows : Consumers who take more than one-third of their year's consumption in the winter quarter, *i.e.*, those whose ratio of winter to summer consumption is higher than the average, will pay more than before. Consumers who take substantially less than the average winter ratio will find their annual bills reduced.

The effect can also be described in terms of services. The cost of a cooking service, being distributed fairly uniformly throughout the year, will be less than before. The cost of a water-heating service, particularly of the type which is grafted on to a continuously burning coke stove will be quite appreciably less than before, but the cost of cold-weather space-heating, and the cost of lighting, will be increased.

Summing up, the seasonal tariff differential has the great advantage that it can be easily applied to the whole body of consumers (except pre-payment) without any additional apparatus and with no great administrative difficulties. Whilst making no specific discrimination, it does broadly pick out and charge more for the peak-producing loads. It has the great disadvantage that it fails to distinguish between peak and off-peak times of day.

If successful it chiefly results in a shifting of loads from winter to summer, and therein makes for a better utilisation of distribution plant. On the generation side, with the present small margin of plant available for the summer overhaul the gain is more doubtful. Moreover, like all devices dependent on sales elasticity, it is useless by itself. Its effect is not mechanical and automatic but operates through personal reactions, and this requires very thorough publicity and enlightenment.

The above conclusion can be broadened by saying that possibly the chief value of any time-varying tariff lies in its long-term effect in

* It is easy to be wise after the event, but in the case of the three-month surcharge of 0·35*d.* applied in the autumn of 1948, a figure of 0·15*d.* would have been more appropriate for the nine-month rebate.

encouraging the type of installation (storage heaters, summer water-heating, etc.) least likely to come on to the peak. Its short-term effect as a peak deterrent is limited because, in the absence of suitable apparatus, consumers cannot *defer* their consumption, they can only restrict it. When the really dangerous time arrives, on a cold winter morning, a very considerable price differential is necessary in order to be effective. The result is likely to be a reduction of kWh over the year with but little reduction in the maximum demand. Moreover, a really substantial price differential is arithmetically impossible whenever (as in the seasonal tariff) the differential operates over a much wider time belt than the actual danger period.

Controlled-Period Maximum Demand System.—Two-part M.D. tariffs are almost invariably used for costing bulk supplies and are general for large industrial consumers. They are fairly common for commercial consumers, and are almost unknown for domestic supplies. The reason for this last can very well be seen in the figures given on page 240. The M.D. of any individual domestic consumer may occur at any hour of the day—possibly when he is doing some evening entertaining: but the only demand that matters from the point of view of distribution costs is that which occurs when the majority of the domestic consumers take their maximum, say 1 p.m. on a Sunday; and the only demand that matters for the costs of the power station is that which occurs at the time of the system peak, usually the early morning or late afternoon on a cold weekday.

A metered demand system for such a consumer would mean making a high charge for payment at times when it was most unlikely to matter. To some extent, the same thing is true of many commercial loads, and even of some industrial ones, and to meet this point the controlled-period M.D. system is sometimes employed. This is the arrangement whereby the demand-recording mechanism is cut out during certain periods of each day (off-peak times) either by time switch or remote control. This is an obvious improvement in cost representation over the normal, continuously recording, M.D. system, particularly for loads with unconventional hours.

An extension of the same system to the domestic field was one of the suggestions examined by the 1948 Committee. One objection is that the idea of M.D. is little understood by domestic consumers, who would probably resent the idea of their bills being adversely affected by a single overload. Moreover, those who did understand it might be tempted, once a certain overload had been reached, to be careless for the rest of the year, knowing that only a still larger overload could now add to their standing charge. It was therefore thought that a better restraint would be exercised by way of a time-of-day variation, affecting every kWh consumed in the peak period, than by a penalty depending on a single occurrence.

PART IV

POWER-FACTOR COSTS AND TARIFFS

This section deals with power factor in relation to industrial supplies. The first chapter deals with the causes and costs of low-power factor, the second with power-factor tariffs, and the third with the economics of power-factor improvement.

CHAPTER XIII

POWER FACTOR: MEANING AND COST

Definition.—When the pressure applied to, and the current in a circuit are both steady, the power in it is found from the product of the pressure and the current; but when the pressure and current are varying, the power cannot be calculated in this simple manner. Thus, when the pressure and current are alternating they can each be represented by their virtual (Root Mean Square) values, but the power will usually not be found from the product of the two R.M.S. values, but will be less than this product.

The power in the direct-current circuit could be likened to a perfectly trained boat crew or a perfectly made cord—all the rowers or all the strands are "pulling together," so that the effectiveness of the arrangement is 100 per cent., *i.e.*, the composite result is the maximum that can possibly be extracted from all the elements making it up. In an alternating-current circuit, the current either does not follow the exact shape of alternation which the pressure maps out, or it does so at a slightly different time, or both; so that the two are not pulling together perfectly, and occasionally one is actually pulling against the other.

The power at any instant is, of course, the product of the pressure and current at that instant, and if all these instantaneous powers are summated, their average value over any period gives the true or effective power (watts or kilowatts, kW). When the R.M.S. values of pressure and current are multiplied together, this product is called the apparent power (volt-amperes or kilovolt-amperes, kVA). The ratio — true power $\div$ apparent power is called the power factor.

The pressure on most modern supply systems follows very closely a sine wave in its alternations, and the current follows the same shape more or less closely according to the character of the circuit, but usually lagging behind the pressure. When pressure and current both follow sine waves they can be represented by the projections, upon a straight line, of vectors of constant length, rotating at the same constant speed but with a definite angle between them. This is called the angle of lag, when the current is behind, or lead when it is in front, and is uaually denoted by the symbol $\varnothing$. In such cases the power factor is equal to the cosine of the angle of lag or lead, and true power (kW) $=$ apparent power (kVA) $\times \cos \varnothing = E_{R.M.S.} \times I_{R.M.S.} \times \cos \varnothing$.

When the wave shapes are irregular, or when there is a polyphase circuit in which the different phases are not balanced, the power factor

is not always easy to find or even easy to define. But it is believed that the above definition covers the case as well as a simple one can, and will be sufficient for the present purpose. Furthermore, it will be assumed in what follows that the pressure and current both follow sine waves.

Causes.—The fundamental reason for bad power factors is that in most A.C. plant the current has two functions to perform, namely, to do the work and to provide the field. The current (or that component of it) which does the work—*e.g.*, transforms electrical energy into mechanical or high voltage into low—is necessarily in phase with the pressure. The two " pull together," and if this were all, the power factor would be unity, or 100 per cent. But the current (or component) which supplies the field is at right angles to the pressure ; so far from pulling together, the two are entirely at cross purposes, so that they result in no true power at all.

The actual current which flows is therefore the resultant of two components in quadrature or at right angles to one another—the power or work-doing component and the reactive or field-making component. As a result, the current is larger than it otherwise would be, since it forms the hypotenuse of a right-angled triangle, and equals $\sqrt{P^2 + R^2}$ where P and R are the power and reactive components. The current required to produce a magnetic field is 90° behind the pressure, so that with electro-magnetic apparatus the resultant current is a lagging one. The current required to produce an electric field is 90° in front of the pressure so that with electrostatic apparatus the current is a leading one.

Although there is actually only one (resultant) current flowing at any time, it is often convenient to resolve this into its two components, just as a distance at sea may be spoken of as so many miles to the north and so many to the east of a certain point. These can then be treated as though they had separate existences in the form of a power current and a magnetizing (or electrifying) current. Moreover, at any one point all the power currents or components can be added numerically to form a single whole, and all the reactive currents can be similarly added or substracted.

The following are some notes on the chief pieces of apparatus affecting power factor.

Induction Motors.—Probably a third of the total energy generated in the country is consumed in induction motors. They constitute far the simplest machine developing mechanical power and their use shows no sign of diminishing. Their fields have all to be supplied from the mains and this forms the largest single cause of bad power factors.

Transformers.—Almost all the energy is generated in the form of high-tension A.C., and this must all pass through a transformer at least once,

and usually several times, before utilisation at a low voltage. It is true that the magnetizing current of a transformer is only a small fraction of that of an induction motor of the same output, owing largely to the fact that there is no airgap. Nevertheless, in the aggregate the effect is very considerable.

In both the above cases the magnetizing current depends only upon the applied pressure and not upon the load or main current. The harm done to the system therefore depends upon the amount of apparatus connected and not upon the extent of its load. But since the power factor and angle of lag depend on the ratio between the magnetising current and the load current, it follows that the power factor gets worse as the load (on existing apparatus) goes down, but can be restored by switching such apparatus out of circuit.

Electric Furnaces.—When the station serves electro-chemical and metallurgical industries the power factor is often low for two reasons. In an arc furnace the arc itself has a lower conductivity at the commencement of the cycle, so that the current is delayed with respect to the voltage wave, and in addition it is often necessary to employ reactance so as to minimise the dangers of short circuits. A plain resistance furnace may have quite a good power factor, but in furnaces of the induction type it is usually bad, particularly when iron or steel forms the secondary.

Current Limiting Reactors.—Apart from their use in connection with furnaces, choking coils are frequently employed in central stations to minimise the current in the event of faults, and these are a further cause of bad power factors.

Transmission Lines.—Although on light loads these act as condensers and take a leading current, this is more than neutralised on load by the inductive drop caused by the load current. The effect is small on multicore cables, but relatively large with an air line.

Static Condensers.—As was seen above, apparatus utilising an electrostatic field takes a leading current, and this by itself would also result in a bad power factor. But owing to the enormous predominance of lagging currents due to other apparatus, condensers merely serve to neutralise some of this, and in that way they improve the power factor.

Synchronous Motors.—Since the field in this case is separately excited (with D.C.), such a motor does not require to be supplied with a magnetising current from the A.C. mains. If the correct amount of field is supplied, such a motor therefore runs exactly at unity power factor. If the field supplied from the D.C. source is less than this amount a magnetizing component will be drawn from the A.C. mains, and the resultant consumption will be a lagging one, exactly as with an induction motor. Conversely, if the field is *over*-excited from the D.C. side the motor will take a *leading* current from the mains. It will

then serve to improve the power factor by injecting a component into the mains capable of supplying the needs of other (lagging) plant. When operated in this way it is frequently called a "synchronous condenser" by analogy with the static condenser mentioned above.

Phase Advancers.—The term "phase-advancing plant" may be applied to any piece of apparatus such as a static or synchronous condenser which (because it takes a leading current) helps to neutralise other lagging currents and so push forward the current (relative to the pressure) and improve the power factor. But the name "phase advancer" is generally reserved for a piece of apparatus used in conjunction with a slip-ring induction motor. By means of this gear the field of the motor can be separately supplied, and this saves the need for taking a lagging current from the mains. Moreover, by over-exciting the field, the stator current can be made to lead slightly, though not to the extent that this can be done with the synchronous motor.

Power-Factor Analogy.—One of the hardest tasks the consumers' engineer has to face in connection with power loads is to justify a kVA or power-factor tariff and to explain its precise incidence. Many of those who have tried to expound the matter to their less-technical clients have been driven to the conclusion that the less said about power factor the better. But whilst there is no need for giving gratuitous technical instruction there is a three-fold reason why the industrial consumer should have some real understanding of the nature of power factor. He should know in the first place what phase-displacement is due to, secondly how it may be lessened or eliminated, and finally how he may be called upon to pay for it. The following paragraphs are an attempt at a simplified explanation such as may appeal to those who, although they may be technically experienced, are not primarily electrical engineers. It employs no specialised terms except such as are defined on the spot, and it aims at leading to an understanding of the three aspects mentioned above, namely, the cause, the cure, and the cost of phase-displacement.

The essential element in the understanding of any A.C. problem is the *vector*—a straight line having length and direction. Strictly speaking, an alternating quantity could only be represented by a vector which rotated fifty times a second (or whatever were the frequency) and so could neither be drawn nor seen. But by taking any convenient fixed line to represent the pressure direction, the current can be represented by another fixed line starting at the same point and making an appropriate angle with the first line. What this angle is will depend, as already stated, on what duties the current has to perform.

In the case of a pure resistance the only effect of the electric current is to produce heat—*i.e.*, electrical energy becomes converted into

thermal energy. No magnetic or electric fields are needed for this particular conversion, and there is nothing for the current to do except to develop true power and energy. This current is therefore wholly in line with the pressure ; and in any apparatus which only develops heat the current is necessarily in phase, and the power factor is unity.

All plant employing electro-magnets (*i.e.*, motors, transformers, etc.) require current for two entirely different purposes. In the first place, current is required to drive the plant and thus pass on power from the point of supply to the point of use. This current is in phase with the pressure, and their product gives the true power consumption. In the second place current is required to magnetize the field. This current

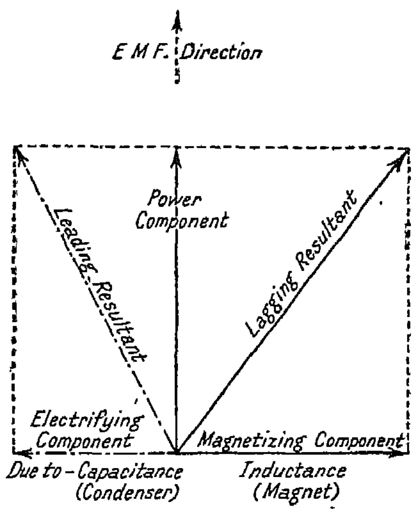


FIG. 45.—Current Components.

is at right angles to the pressure : it represent no power, since no power is required to maintain the field once it has been created. It is called the reactive, wattless or idle current, although it is far from being idle since it provides the field which is an essential link in the chain by which power is passed on.

The above state of things is represented vectorially on the right of Fig. 45. Actually, there is only one current, namely, the resultant of the two right-angle components. But, as explained previously, it is often convenient to regard the components as though they were actual currents existing independently, namely, a *power current* and a *magnetizing* current. This is because all the power components of all the plant connected to a point will add numerically to form the total power current drawn from the line, whilst all the magnetizing components will add to form the total magnetizing current.

In the case of a condenser an *electrifying* current is required in order to provide the electric field. Again, it is at right angles to the pressure and so represents no power, but it is on the opposite side to the magnetizing current. A mathematical convention has decreed that the positive direction of travel for angles is opposite to that of the clock, and the three currents are represented as in Fig. 45, the condenser current " leading " by 90° and the magnetizing current " lagging " by 90°. These last two are therefore diametrically opposed, and help to cancel one another. A condenser and a choke coil of suitable magnitudes connected side by side may each take a reactive current (but in opposite directions) yet collectively they may draw no such current from the line, and the power factor of the whole arrangement will be unity.

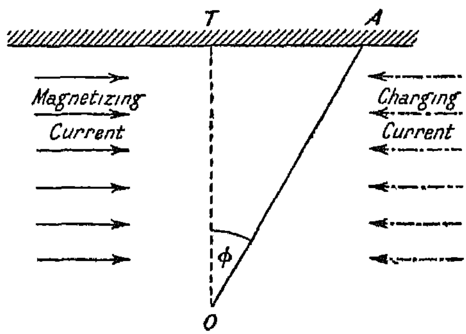


FIG. 46.—Power-Factor Analogy.

In Figs. 45 and 46 the pressure is regarded as being in a direction due north. The process of taking power from a line may then be likened to the task of a boatman at O (Fig. 46) trying to reach a coast-line due north of him. If the power-consuming plant is purely rheostatic, having neither magnetic nor electric field, the boatman is enabled to proceed due northwards and so reach the coast in the shortest possible distance, OT. This distance of travel represents vectorially (*i.e.*, in magnitude and direction) the current drawn from the line. But usually the water is not so calm and tideless as this, and with any apparatus employing a magnetic field there is a magnetizing current represented by a water current flowing from west to east. During the progress of his northward travel our boatman, all unbeknown, is being carried eastward so that he makes a longer journey than he otherwise would. His actual course is represented by OA, and this gives a measure of the current taken from the line to do the same job as before.

The ratio $\frac{OT}{OA}$ or minimum (power) current divided by actual current is called the power factor. It will be noted that his actual journey

OA may be regarded as made up of two journeys or components taking place simultaneously — the intentional or power journey from O to T and the drift or magnetizing current journey from T to A.

When the apparatus employs an electric field there is a similar drift or current but in the opposite direction (chain-dotted arrows). The results are similar except that the boatman is carried westward instead of eastward. It should again be emphasised that from the point of view of power transference it is only the northward travel that counts, and the aim is to reach the north-lying coast in as short a distance as possible. Lateral travel avails nothing from a power point of view, but it is a necessary concomitant of magnetic or electric fields. Moreover, it is (in a sense) a defect in the boat rather than in the medium of travel that causes it to depart from its ideal course.

The foregoing is an attempt to clarify the power-factor issue by concentrating attention on to the root causes, namely, the reactive (magnetizing or charging) currents. In many ways it is unfortunate that this vague, almost metaphysical, concept of "power factor" is ever mentioned in connection with tariffs or to non-technical persons. Power factor is not a quantity but a consequence—it is merely a statement of the trouble, not a measure of it; and it would be far more defensible as well as more explicable to omit the term entirely and to levy a fixed charge for all magnetizing current drawn from the line.

Magnitude and Effect.—The effect of power factor on the carrying capacity of a conductor is easily seen. Since a bad power factor means a bigger current for the same amount of power, and since all current produces the same heating and copper loss whether it is in phase or not, it follows that the power-carrying capacity of such conductors is reduced in the ratio of the power factor. The excess current necessitated by a bad power factor has been likened to the "froth on the beer" (a more exact analogy has been given above) and it is evident that if plant were pumping beer along a pipe the effective capacity of the transmission would be reduced by the presence of gas bubbles in the liquid.

Unless phase-advancing plant is installed, the power factor of a station supplying a mixed load will usually be less than 80 per cent., and it may be as low as 65 per cent. when the load is predominantly industrial. Even at the higher of these figures this means that a very appreciable portion of the capacity of the alternators, cables, etc., is wasted in carrying wattless current—current which performs no mechanical work, but which heats up the conductors just the same. Thus if the power factor could be improved from 80 per cent. to unity, the capacity of the whole of the electrical equipment could be raised by 25 per cent. without the addition of a single machine, switch or cable.

The effect of a bad power factor is very similar to the effect of a bad

load factor considered in previous chapters. In each case the consumer is demanding apparatus which he is not utilising to the full energy output of which it is capable. So that if metered and charged only on energy consumption, he will be putting the station to an unrecompensed capital expense which will be greater the poorer his load factor or power factor. There are, however, the following differences. With power factor there is comparatively little in the nature of diversity factor, since a leading power factor on a large scale is almost unknown. A bad power factor demands bigger cables and electrical plant for a given energy consumption just like a bad load factor, but it does not demand bigger steam or prime-mover plant. On the other hand a bad power factor causes heating losses, so that, in addition to the standing cost, it affects to some extent the running cost also. It also has a serious effect upon the voltage regulation.

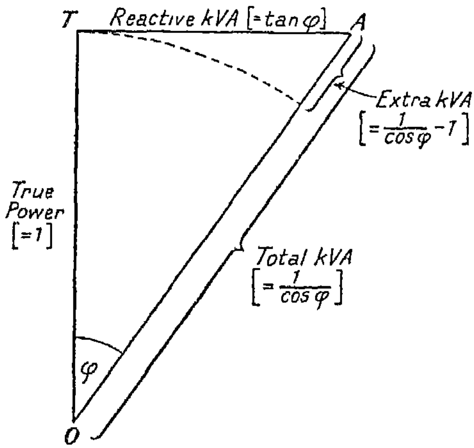


FIG. 47.—Power-Demand Vectors.

Basis of Reckoning.—Before considering the costs of bad power factors, it is necessary to decide on what basis the costs are to be reckoned. At any particular moment (*e.g.*, the time of maximum demand) the power taken by a consumer can be represented by a vector diagram (Fig. 47). A similar diagram can be used to represent the energy over a given period, but only the power aspect need now be considered. It will be seen that there are three or four quantities concerned in the problem, namely, the true power OT, the apparent power OA (variously referred to as the " demand," " lagging " or " total " kVA) and the wattless component or reactive kVA TA. These are all functions of the angle of lag ϕ, and if OT is made equal to unity, $OA = \frac{1}{\text{cost } \phi}$ = the reciprocal of the power factor, whilst $TA = \tan \phi$.

Power-factor costs could therefore be related to any one of these varying quantities, and the relation between them is discussed in a later section. The size of the current-carrying portions of the system will be determined by the total kVA *OA* ; but in matters of regulation, or the installation of improvement plant, the reactive kVA *TA* is the more important quantity. It also forms a somewhat more precise and scientific basis, since all reactive kVA are in the same direction and cannot give rise to phase diversity.

On the whole, however, total kVA is probably the most significant factor on the supply side, and it also forms the basis of one of the commonest power-factor tariffs. It will therefore be employed in the discussion of costs below, but other possible bases will be subsequently examined.

Cost of Low Power Factor.—In order to obtain a clear idea of the supply cost resulting from a bad power factor, it will be well to start with a purely theoretical situation, namely, a station and supply system in which every item can be varied at will. If this is considered first as supplying a load at unity power factor, and then as supplying the same power, but at a lagging angle, it will be possible to make some estimate of the extra cost involved. It may be supposed that the tariff in the first case is a two-part one of an annual charge per kW of demand plus a uniform price per kWh consumed. In what follows, the numerical difference between the apparent and the true power demand will be referred to as the " extra kVA," and the numerical difference between the apparent and the true consumption as the " extra kVAh." The object of the present section is then to discover what each extra kVA of demand costs as compared with the true kW of demand, and what each extra kVAh of consumption costs as compared with the cost per unit of the true energy.

The cost of giving a supply to a consumer may be divided up into four groups : fixed cost of generation, fixed cost of transmission and distribution, running cost of generation, and running cost of transmission and distribution. The mean costs of supply in 1948 under these four heads have already been estimated as follows :—

At the generating station or bulk-supply point, £4 10*s*. per annum per kW plus 0·38*d*. per kWh.

To the consumer, £7 1*s*. per kW plus 0·43*d*. per kWh (excluding consumer costs). These figures, of course, cover small as well as big stations, and since the load is *not* at unity power factor the standing charge would be considerably less if measured per kVA. Moreover, as already explained, these figures are an average covering both domestic and power consumers, and differ both in size and ratio from the average two-part industrial tariff. They are quoted here merely as a guide to the proportions of the costs in question.

Fixed Costs.—As regards the first two groups, before considering the question of the rate at which extra kVA are to be charged, it is necessary to decide when they are to be measured, since the instantaneous kVA will change with every change in the magnitude or character of the load. But since the largest connection of apparatus and therefore the largest consumption of magnetizing current is likely to occur when the true power is a maximum, it may be assumed that maximum kW and maximum kVA will synchronise. It is reasonable also to suppose that the effect of the consumer's power factor upon the fixed costs of supply will be chiefly felt at the instant he is taking his maximum power. Hence the effect of a bad power factor as distinct from a bad load factor can be assessed by examining his maximum demand over the period in question and seeing how much more this costs to supply than if he demanded the same power at unity power factor.

Taking the first group of costs only (*i.e.*, fixed costs of generation), this will include capital expenses, running expenses, rent and rates, management, etc. If the cost of generation in the example depicted in Fig. 13 is examined, it will be found that the capital charges are in three roughly equal parts, of which two, civil work and steam plant, may be considered as proportional to the true power, and the third, electrical plant, will be proportional to apparent power (total kVA). The other fixed expenses can be similarly split up, but a much greater proportion will probably be found dependent upon apparent power, and as regards some items of attendance and management an apparent kVA will cost more than a true kW. As a rough approximation it may be supposed that between one-third and one-half of the total expenses in Group (1) will vary with the kVA of demand, the remainder being proportional only to true power.

Taking the second group, namely, the fixed costs of transmission and distribution, the plants employed are of two kinds—those in which the output is limited by heating considerations (underground transmission lines, switchgear and transformers) and those in which regulation is the limit (overhead lines and distribution networks). The size of the former is a direct function of current, whether that current is in phase or not. In the case of the latter the position will depend on the value of the line constants, but in general the cost of a lagging kVA will be at least as high as the cost of a kW. The same remarks will apply to the operating and management expenses, which usually rank entirely as fixed costs. For the present purposes it will be sufficiently accurate to regard all fixed costs of transmission and distribution as directly proportional to kVA.

It will be seen from the figures for fixed costs quoted above, that the transmission and distribution costs in this country make a total (allowing for increased diversity) somewhat less than the generation costs, so that combining Groups (1) and (2) it may be tentatively

suggested that to the final consumer the fixed cost per extra kVA is of the order of two-thirds to three-quarters that of a true kW. Taking the latter figures, if there are two consumers, each of whose maximum demand is 100 kW, one at unity and the other at 71.4 per cent. power factor (140 kVA), their fixed charges should be in the ratio 100 : 130.*

Phase Diversity of Demand.—There is another point which must be considered. In the above argument it has been implied that a given addition to a consumer's maximum demand increases the station M.D. by the same amount, but this is only true if the consumer's maximum coincides with the station maximum not only in time, but also in phase angle. It is, of course, obvious that even in a D.C. system an addition to the consumers' M.D.s will produce a smaller increase in the M.D.

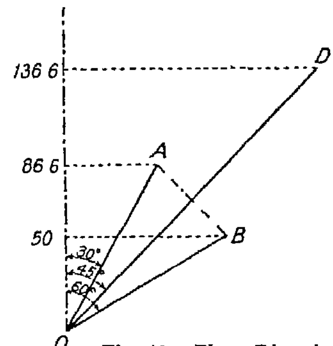


Fig. 48.—Phase Diversity.

on the station in the ratio of the diversity factor ; so that the charge levied per kW of consumer's maximum can be less (except for the effect of losses) than the cost per kW of station plant, in this ratio. Diversity factor may thus be regarded as the aggregate effect of this non-coincidence in time of the consumers' maxima, and its existence is fully realised and easily allowed for. What is not generally recognised is that in an A.C. system there is a diversity in phase-time as well as in time-of-day, so that when a charge is made on the consumer's maximum kVA instead of kW, such a charge should be less on this ground than the corresponding cost per kVA at the station. This item is called by the author " diverse phase factor," to distinguish is from the term " diversity factor " in its generally accepted meaning.

* It will be noted that instead of levying the extra fixed charge for bad power factor on the difference between the kW and the kVA (here called the extra kVA) it can be levied on the full kVA with a corresponding reduction in the charge per kW. Thus an annual charge of £8 per kW plus £6 per extra kVA can be (and would be) more simply expressed as a charge of £2 per kW plus £6 per total kVA. The other method is here used in preference to this because it is desired to show exactly what addition would be necessary to a simple two-part tariff in order to cover the extra cost resulting from a bad power factor.

Referring to Fig. 48, if the M.D. on the station (direction OD) lags 45° behind the voltage (*i.e.*, power factor approximately 70 per cent.), and if there are two consumers, OA and OB, each with a M.D. of 100 kVA, but lagging 30° and 60° respectively, then even if their demands coincide as regards time of day with the station demand they would not swell the latter by 200 kVA, but only by 193 kVA. The diverse phase factor in such a case would be 200/193 = 1·035.

Diverse phase factor can be measured in the same way as diversity factor, but by imagining all demands to coincide in time. The sum of all consumers' simultaneous kVA demands divided by the kVA demand coming on the station would then measure the diverse phase factor; or it could be defined as the numerical sum of these simultaneous demands divided by their vectorial sum. On a kVA tariff it is the former which is measured and charged for, and the latter which represents extra costs, so that the consumers are to this extent overcharged. The difference is not great in proportion to the whole power, but as it only occurs when kVA are charged for instead of kW, its incidence is entirely on the "extra kVA," and in proportion to this it is quite an appreciable item.

In the case cited above, the extra kVA's of the two consumers are 13·4 and 50, making a total of 63·4, whereas the extra kVA coming on the station is only 56·6. With the figure suggested above for the cost of extra kVA on the system, namely, 75 per cent. of the cost of a true kW, the correct price to charge a consumer would then be 67 per cent. Moreover, the factor becomes greater the bigger the phase divergence between the consumers.

The effect of phase diversity (like that of time diversity) is that the price charged to the consumer for lagging demands should be rather less than would appear from purely cost considerations. Taking the system as a whole, the position as regards fixed costs may then be summarised by saying that the cost of supplying a kVA of lagging demand (expressed as a ratio of the cost per true kW) will be about one-third at the station busbars,* and will rise to about two-thirds at the consumer's terminals.

Selected-Station Charge Adjustment.—Reference may be made here to the power-factor clause employed for stations selected for service under the 1926 Act.† When the owner of such a station purchased supplies from it, the fixed kW charge was multiplied by $\frac{0{\cdot}8 + 0{\cdot}2/B}{0{\cdot}8 + 0{\cdot}2/A}$, where B was his power factor and A was the power factor

* Reference may also be made to the grid tariff for bulk supplies, which was essentially a generation-costs tariff. This had a power-factor clause which was equivalent (at a working power factor of 70 per cent.) to charging extra kVA at one-third the price of true kW. (*See* footnote, p. 283.)

† *Statutory Rules and Orders, 1929*: No. 1016. H.M. Stationery Office.

of the station, both expressed as decimals. In other words, four-fifths of the fixed charge was independent of power factor, and the remainder was proportional to kVA, so that extra kVA were charged for at one-fifth of the price per kW. But this calculation makes no allowance for either time- or phase-diversity, and since A and B are not metered simultaneously, the charge per extra kVA *falling on the station* was higher than this. It should be explained that A was metered at the time of maximum demand upon the station as a whole, whereas B was metered at the time of the owner's M.D. upon the station.

Running Costs.—Taking the third group of costs, namely, running expenses of generation, these will be almost independent of power factor ; whilst as regards the fourth group, covering the running cost (*i.e.*, losses) of transmission and distribution, these will be proportional to the kVAh. Hence the question of the cost of the extra kVAh will turn on the proportions which Groups (3) and (4) bore to each other originally. In the case of public supply in 1948, the losses in transmission and distribution amounted to 11 per cent. of the units sent out. If only these losses were affected by power factor the price of one extra kVAh should be just under one-eighth that of a true kWh.‡ But it must be remembered that there are also the alternator losses which are proportional to the kVAh ; and, moreover, owing to bad power factors, sets have frequently to be kept running which could otherwise be closed down. When to these factors are added various smaller ones, such as the cost of the extra excitation, etc., it is probable that a figure of one-fourth is more representative.*

Summary and Modifications.—The above may be summed up as follows : If the supply costs (to the low-voltage consumer) at unity power factor are represented by £q per annum per kW of demand plus p pence per kWh of consumption, the additional costs incurred through lagging loads can be approximately covered by £ $\frac{2}{3}q$ per annum per extra kVA of demand plus $\frac{1}{4}p$ pence per extra kVAh of consumption. Alternatively, the whole fixed costs could be expressed as £ $\frac{1}{3}q$ per annum per kW of true power demand plus £ $\frac{2}{3}q$ per annum per kVA of total demand. The running charge could be similarly expressed as $\frac{3}{4}p$ per kWh plus $\frac{1}{4}p$ per total kVAh.

‡ If the station supplies one extra kVAh without increasing its true energy or its fuel cost, this will increase the kVAh to the consumer by $1 - 0{\cdot}11 = 0{\cdot}89$, and will decrease the kWh to the consumer by 0·11. Hence on this basis one kVAh should cost $0{\cdot}11/0{\cdot}89 = 0{\cdot}124$ times as much as one kWh.

* In a paper by E. V. Clark (*Journal I.E.E.*, **64**, p. 627), it is suggested that on a system working at 70 per cent. power factor, all kVAh might be charged at one-ninth the price of true kWh. This would correspond to a price for extra kVAh of just under one-sixth that of true kWh. On the other hand, in some of the Continental systems employing composite reading meters (such as that of Professor Arno, of Milan) the tariff is true kWh plus *one-third* of extra kVAh.

Turning now to actual situations, there are several reasons why the above theoretical considerations require modifying. As regards the fixed cost, when the generating station is actually in existence and designed for some particular power factor, its component parts cannot easily be varied. A common practice is to design a station on the assumption of a power factor of about 80 per cent., *i.e.*, the electrical end will be made capable of carrying 25 per cent. more (apparent) power than the steam end. When the actual power factor is below the designed figure, any improvement will virtually increase the capacity of the *whole* station in the ratio in which the kVA are reduced : hence the cost of a kVA at this point is not two-thirds, but fully as much as the cost of a kW. (But under these circumstances it would be more economical to install phase-improvement plant rather than let the whole station capacity suffer.) For power-factor improvements beyond this point there will be much less economic justification, and the "cost" of a kVA (*i.e.*, the amount saved by its extinction) will be much less.

It will be appreciated that the power-factor costs estimated above are the costs to the supply undertaking, not the costs to the consumer. What the latter pays for power factor will depend upon the tariff, *i.e.*, on the extent to which the actual costs are reflected in the price charged. This difference in viewpoint, as between undertaking and consumer, is an important factor in the economic choice of phase-advancing plant.

Regulation.—Regulation can be defined as the drop in voltage resulting from a load, or the rise when that load is removed, expressed as a percentage of the rated voltage. Thus if a consumer nominally supplied at 100 volts finds that at the time of maximum load the voltage has fallen to 90, the regulation is 10 per cent., and naturally it is extremely important to keep this voltage drop down to as small a figure as possible.

The drop may be due to a variety of causes. In an isolated station the prime mover may fall in speed very slightly, and the alternator will then generate slightly less voltage owing both to the speed change and to loss in magnetic-field strength. This drop will be further increased by the fall in voltage due to impedances, first in the machine itself, and then in the cables and other gear supplying the consumer. In a large station or interconnected system, the voltage drop in the machines themselves can be made extremely slight by means of suitable regulators, and it is chiefly the subsequent impedances in the distribution gear which effect the total drop.

In order to illustrate the effect of power factor upon regulation, it will be well to take actual figures. In the case of an overhead line whose resistance and reactance drop at full load and unity power factor are 10 per cent. and 20 per cent. respectively, in order to have 100 volts at the receiving end it is necessary to start with the voltage

of 112. With the same line and the same kW loading, but at 80 per cent. power factor, it is necessary to start with 125 volts. When the same load is taken at 60 per cent. power factor the starting voltage would have to be 137, so that a boost of 37 volts would be necessary if no power-factor improvement were attempted.

Thus a bad power factor damages the regulation for two reasons. In the first place it necessarily increases the voltage drop in the cable, since it means a larger current for the same power. Secondly, when the line possesses reactance as well as resistance the extra kVA will actually produce a bigger drop than if the same number of true kW were added, since the addition is a vectorial one. In the above case the currents are in the ratio 1 : 1·25 : 1·67, whereas the voltage drops are in the ratio 1 : 2 : 3.

It is impossible to say except in very general terms how regulation considerations will affect the foregoing estimate of power-factor costs. No change need be made in the generation estimate, but on the distribution side it can be said that wherever regulation is a serious factor the cost of lagging kVA will be somewhat greater than that estimated above. In many cases the installation of phase-advancing plant by the supply undertaking will be necessary on regulation grounds even when it is not needed on grounds of current loading and cable heating.

CHAPTER XIV

POWER-FACTOR TARIFFS

Types.—A considerable number of supply authorities base their charges only upon true power and energy consumption, making no extra charge for bad power factors. This may be because their load already approaches the power factor for which the station was designed or because of the cost and unsatisfactory character of the metering equipment. They then have to rely only upon verbal encouragement and advice to their clients. The weakness of such a practice is that the undertaking has no hold whatever over the type of consumption taken from its mains, although one type will put it to a much greater expense than another.

Where tariffs taking account of power factor are employed in this country these apply only to industrial consumers,* and they can generally be divided into two groups.

(*A*) A two-part tariff consisting of a standing charge per kVA, plus a running charge per kWh.

(*B*) A true power and energy tariff, which may be either of the two-part or single-part type, with in each case a bonus/penalty if the power factor is above or below some datum figure such as 75 or 80 per cent.

A third type of power-factor tariff, rather different from either of these, is common on the Continent. Usually it is of the single- rather than the two-part type, and the charge varies with power factor, but not to the full extent of the total apparent consumption. Not infrequently it is in the form of a flat rate per complex unit, the latter being perhaps two-thirds of the true kWh plus one-third of the apparent kVAh (*i.e.*, extra kVAh at one-third the price of true kWh). Special integrating meters have been developed for registering such complex units directly on the dial, so that consumers can at any time see the readings on which their bill is reckoned.

kVA Tariffs.—As regards tariffs in class (*A*) outlined above, the energy is measured by an ordinary watt-hour or cosine meter. The maximum demand is recorded over the integration period (*e.g.*, half-an-hour) on either a compensated or a combined sine and cosine meter. Another method is to estimate it from a maximum-reading ammeter, which when multiplied by the line voltage ($\div$ 1,000) gives the maximum

* Domestic and commercial consumers whose usage is largely heating have naturally good power factors although the fluorescent lighting of shops and offices is sometimes a source of anxiety.

kVA demand during the metering period. Unfortunately this thermal-demand meter is far from accurate, and as its integration period is indeterminate its readings cannot be precisely defined. Moreover, unless voltage-compensated, its correctness will entirely depend on the constancy of the supply voltage.

Whichever type of meter is employed, the kVA of demand then recorded is usually all charged for at one definite figure (extra kVA ranking the same as kW), which (pre-war) was usually some £4 to £5 per annum per kVA. When such a tariff operates it is easy to show that a good return can be obtained on capital invested in phase-improvement plant in bringing the power factor to a point little short of unity.

In the published tariffs of authorised undertakings this is much the commonest form of power-factor penalisation. In the survey described in an earlier chapter, approximately half of the industrial maximum-demand tariffs based their fixed charge upon kVA and half on kW. The chief criticism which can be levied on such a tariff from the point of view of representing actual costs is that in it the extra kVA are charged for at the full rate, whereas they do not in fact cost as much as true kW, but only in the neighbourhood of one-half to three-quarters as much ; and, on the other hand, the extra kVAh are not charged for at all, whereas they do cost something, which has been estimated above as about a quarter that of true kWh.

There are several justifications for this procedure, the chief of which is the convenience of a simple tariff and the comparative ease of measurement. It will be noted, moreover, that the two errors are in opposite directions, thus tending to cancel, though the extent to which they succeed in doing this will, of course, depend on the proportions of the two charges and the load factor of the consumer.

Actually, on the basis of the mean cost figures quoted above, the fixed cost is so large compared with the running cost that an excess rate on the former would be very far from balanced by a failure to charge on the latter. But in actual tariffs, the fixed charge is smaller and the running charge larger than the cost-figures indicate. Hence, an excessive rate of penalisation on the fixed charge is not so serious as it seems. There is, moreover, another reason which makes the above tariff less unfair to the low-power-factor consumer than at first appears. If the demand is measured by a maximum-reading ammeter, this means that the charge is virtually based on the peak-load power factor, which is usually better than the mean power factor owing to apparatus being then more fully loaded.

The objection to over-penalisation of power factor in the kVA tariff could, of course, be overcome by charging the kVA at a lower rate than the kW, as suggested in a footnote on p. 283. This, however, would sacrifice the simplicity of the tariff and is rarely or never done.

Another criticism of the class (*A*) tariff is that it gives unequal rewards to different consumers for providing equal services to the supply authority. This can be seen by comparing two adjacent consumers, each of whose M.D.'s has a true power value of 100 kW, but whose power factors are 70 per cent. and 89 per cent. respectively. If the former installs phase-improvement plant injecting 50 kVA leading by 90° he will bring his power factor to 88.7 per cent. and reduce his kVA of demand from 143 to 113, a saving of 30 kVA, but if the second consumer installed the same plant, thus bringing his power factor almost to unity, his saving would only be 11.6 kVA. Yet the gain to the supply authority would be the same in either case.*

Of course, if all the consumers did the same thing, the gain to the undertaking would be in the same ratio as the reduction in the consumers' demand, *i.e.*, it would be at a decreasing rate the nearer the power factor came to unity ; and a tariff based on demand kVA would then be equally fair to both sides. The difference in the economics of the case to the two parties arises from the fact that when the individual improves he brings his demand much nearer to unity, and so *decreases the rate of change of the improvement (see p. 280 and Fig.* 50), whereas he does not change the lag on the station materially and so does not affect the degree of profitableness to the supply authority. While station power factors remain bad it would therefore be worth while for the authorities to pay at a level rate for all reactive kVA injected into the system, or else to charge for kVA at a slightly lower rate to consumers taking it at or nearly at unity power factor.

Summing up this point, it may be said that for a fixed total power factor on the generating station, all leading reactive kVA have the same value, whereas if paid for on the basis of consumers' kVA a bigger saving will accrue to an individual who improves his power factor from bad to medium than one who improves from medium to good. But as regards the gradual improvement of the station power factor as a whole, leading kVA installed later, *i.e.*, as the power factor approaches unity, are of less value to the supply authority in the same way that they show a smaller saving in the consumers' electricity bills.

Power-Factor Indicator.—Reference may here be made to an indicating device developed by the author. The object of this is to give a visual explanation of the meaning of power factor and to demonstrate the incidence of the kVA tariff. It is in the form of a two-dimensional working model as illustrated in Fig. 49. The arm on which the index finger is resting hinges about its lower end and carries a scale in black followed by a scale in red. The black portion is scaled 0 to 100 kVA;

* It will be noted that if the consumers are not adjacent, but each is situated at the end of a separate transmission line, the gain to the supply authority in respect of the improvement in *regulation* would be greater in the first case than in the second, and would in this respect be more in harmony with the rewards given to the consumers.

after this, the red figures start and indicate the "extra kVA" due to a lagging demand. The inclination of the arm represents the angle of lag, and it can be set to any power factor from unity to 0·5 lagging. The arm will then indicate the amount of extra kVA for a true power demand of 100 kW.

Along the top of the indicator is a scale of reactive kVA, and from it the effect of a given condenser bank can be quickly shown. This scale is also an index to the total magnetizing component. Thus, every 100 horsepower of small induction motors connected to the line at the

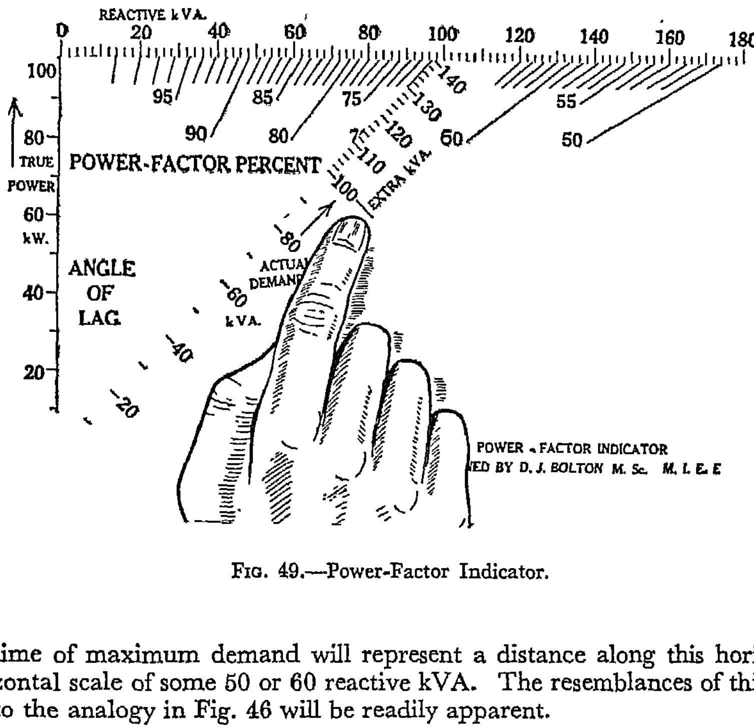


FIG. 49.—Power-Factor Indicator.

time of maximum demand will represent a distance along this horizontal scale of some 50 or 60 reactive kVA. The resemblances of this to the analogy in Fig. 46 will be readily apparent.

Bonus/Penalty Tariffs.—Turning now to the second group of tariffs outlined above (class (*B*)), the bill is first computed on a true power and energy basis, and then subject to a series of proportionate allowances or additions dependent on the power factor. Unlike the kVA method of power-factor penalisation (which is only possible with a two-part tariff), the bonus/penalty method is applicable either to a two-part or to a flat-rate tariff. When applied to a two-part tariff,

it may vary the fixed charge only or it may vary the whole bill. In a flat-rate tariff it can only apply to the bill as a whole.

Since the costs of bad power factor are almost confined to the fixed expenses, it is wrong in principle to impose a penalty on the whole bill, and will inevitably be unjust in its application, particularly to consumers having a good load factor. This can easily be seen by taking two consumers with the same maximum demand and the same power factor, one having 20 per cent. load factor and one having 40 per cent. On a flat-rate tariff, with a bonus/penalty clause, the latter consumer will pay twice the power-factor penalty of the former consumer, since his units are twice as great. Yet the expenses involved in his bad power factor (since they are almost entirely fixed costs and therefore a function of M.D.) will be very little greater than those of the other consumer. With a two-part tariff having a bonus/penalty on the whole bill the disparity between their power-factor penalties will still exist, although not so great as with a flat-rate tariff. On a two-part tariff in which the bonus/penalty only applies to the fixed charge the disparity will not occur, and no injustice will be done as between different load factors.

These remarks would appear to indicate an insuperable objection to any power-factor penalty applied to a flat-rate tariff, whether the penalisation takes the form of a bonus/penalty for power factor or a charge for kVARh as suggested later in the chapter. This objection is substantially the same as the objection to flat rates in general, namely, that they cannot represent the fixed costs owing to differences of load factor. But it was shown in the chapter on diversity that this objection is not so serious as it appears, owing to the operations of differential diversity. And if the individual demands are sufficiently staggered over the consumers' use-period to make a flat rate of charge tolerably representative of costs, then by the same token a whole-bill power-factor penalty will be sufficiently representative of power-factor costs.

One objection to the bonus-penalty tariff lies in the difficulty of metering power factor, and of explaining its meaning to the consumer. On the other hand, it is more flexible than the kVA tariff, since the penalty can be of any desired magnitude, independently of the rest of the charge. Also, as will be seen later, the reactive kVA are approximately proportional to changes in power factor. Hence, charging for power factor amounts to very much the same as charging for reactive kVA, and there are several advantages in charging on such a basis.

Examples are difficult to give, since a bonus/penalty clause seldom appears among the published tariffs of undertakings in this country, although it may be embodied in the terms of private contracts. The best example of this type of power-factor charging was to be seen in the C.E.B. Grid tariff for bulk supplies. The magnitude of the fixed charge varied slightly, but taking a rough average and making some allowance for the reductions in fixed charge with growth of load, this

charge may be put as £3 per annum per kW, with an addition of 4*s*. 6*d*. per kW for each 0·1 power-factor reduction below 0·85. The running charge was not affected by power factor and may conveniently be left out of the calculation entirely.

No power-factor penalty is embodied in the B.E.A. bulk-supply tariff to the Area Boards, and this may be taken as indicating that the average power factor at the various bulk-supply points is sufficiently nearly the same in all areas to make an adjustment of charges unnecessary. Moreover, should the power factor of any Area Board's load prove unsatisfactory, the Central Authority could direct the Board to rectify the matter. This does not mean that power factor is no longer important or that Area Boards may not find it necessary to vary their charges so that industrialists bear their fair share of the costs involved. It does, however, mean that the extra costs in question will be chiefly those of distribution.

Comparison of Types.—It is important to compare the incidence *of a bonus/penalty power-factor clause with that of a plain kVA tariff.* It will be assumed that the bonus/penalty applies to the standing portion only of a two-part tariff, as in the case of the Grid tariff cited above. Otherwise it will evidently be impossible to make a direct numerical comparison with the two-part kVA tariff. (Only the standing charge will be brought into consideration, the running charge being assumed the same throughout.) It will also be assumed for the moment that the magnitude of the bonus-penalty as a ratio of the initial standing charge is the same as the magnitude of the power-factor change as a ratio of unity power factor. This is the same as saying that a change in power factor of 0·01 involves a 1 per cent. change in the initial standing charge. This may be described as a " 100 per cent." bonus/penalty clause. In the case of the Grid tariff the penalty ratio is 4*s*. 6*d*. divided by £3 = 0·075 for a 0·1 fall in power factor, which is therefore a 75 per cent. penalty clause.

With this assumption, the relationship between the two types of power-factor tariff is easily established. In Fig. 50, OA represents, as before, the magnitude and direction of the actual kVA at the time of maximum demand, lagging by an angle ϕ behind the true power direction OT. (Attention, for the moment, should be directed to the full lines and the large initials.) If p is the power factor and k the value of OA when OT has the value unity, then

$$p = \cos \phi = \frac{OT}{OA} = \frac{1}{OA} = \frac{1}{k}$$

Let p increase by a small amount δp and let δk be the corresponding change in k. Then

$$\delta k = \delta p \times \frac{dk}{dp} = \delta p \times \frac{1}{-p^2} = -\frac{\delta p}{\cos^2 \phi}$$

In words, when the angle of lag increases, the increase in kVA (for 1 kW true power) equals the decrease in power factor divided by the square of the power factor (or the square of cos ø). It follows that the extra kVA resulting from a given power-factor decrease becomes greater as the power factor gets less. At unity, a drop in power factor of 0·1 will increase the demand by approximately 0.1 kVA for each 1 kW of true power. At 0·85 power factor a drop of 0·1 increases the demand by 0·138 kVA. At 0·7 power factor a drop of 0·1 increases it by 0·204 kVA, and so on.

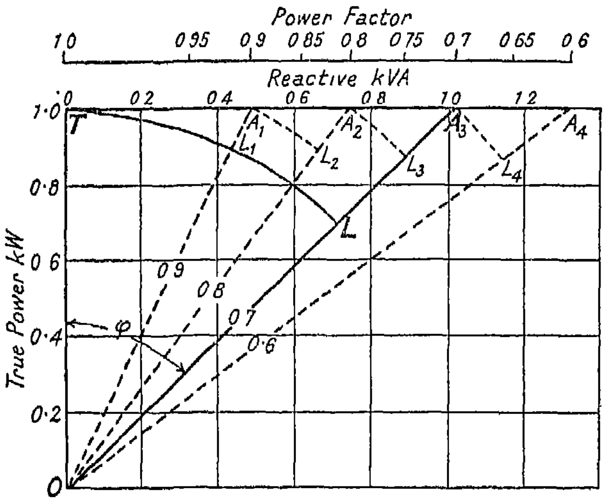


FIG. 50.—Rates of Change.

Referring to the figure, as ø increases and OA swings round to the right, the length of OA (*i.e.*, the actual kVA demand) increases at a greater rate than cos ø decreases. A power-factor scale is shown at the top, and the four radii OA are drawn for equal differences of power factor. By drawing arcs from each point A to the next radius line, it is easy to see that the lengths L_1A_1, L_2A_2 are getting greater.

It follows that the two types of tariff are not in a fixed ratio to each other, but vary according to the particular consumer's power factor. This can be very well seen by comparing a kVA tariff and a "100 per cent." bonus/penalty tariff of the same initial magnitude. Thus compare two tariffs, one of which charges £10 per kVA and the other which charges £10 per kW with a penalty of £1 (*i.e.*, one-tenth of the fixed charge) for each 0·1 by which the power factor falls below

unity.* At power factors bordering on unity the two tariffs will charge the same, for at this point the change in kVA is numerically equal to the change in power factor. At power factors about 0·85 a change of 0·1 will make (for each 1 kW of load) a difference in the kVA tariff of £1·38, and at power factors about 0·7 the same change will make a difference of £2·04, whereas in the second tariff the difference will always be £1 at all points. Hence, a very-bad-power-factor consumer will have more inducement to improvement on the kVA tariff than on the bonus/penalty one.

In the case of the Grid tariff, which had a 75 per cent. penalty clause, it will be seen that at power factors in the neighbourhood of 0·85 the charge made for the extra kVA due to falls in power factor was $0{\cdot}75 \times 0{\cdot}85^2 = 0{\cdot}54$ times as much as the charge per kW. At power factors of 0·7 it was $0{\cdot}75 \times 0{\cdot}7^2 = 0{\cdot}37$ times as much: extra kVA at this point cost one-third as much as true kW. It will also be noted that in the Grid tariff no charge was made for the first 0·18 extra kVA per kW (corresponding to 0·85 power factor).

Reactive kVA.—In the above sections, two quantities—namely total kVA (i), and power factor (ii)—have been compared together, and their corresponding tariffs have been reviewed. There is, however, a third important variable in the problem, namely, reactive kVA (iii), and in the following paragraphs the two previous quantities are compared on the basis of this third one. (The phase angle ø could itself be used, but this is more of a mathematical conception than a practical basis for power-factor charging.)

Reactive kVA, represented by the distance *TA* in Figs. 47 and 50, has several advantages as a basis of power-factor reckoning. In the first place, it seems a somewhat more tangible quantity and one more directly measurable than, say, power factor or phase angle. In the second place, it forms a more logical basis for charging the consumer. Both a *power* current (or component of current) and a *magnetizing* current are needed to operate most electrical machines, and both may reasonably be charged for. Moreover, the magnetizing current is the most constant and easily predictable element in the consumption, since it depends on what apparatus is connected and does not vary much with the load. Finally, this basis avoids the difficulty of phase diversity, already referred to. The reactive kVA's of different consumers (if simultaneous) can be added numerically to give the total reactive kVA on the line. Total kVA's can only be added vectorially, and power factors cannot be added at all.

* It will be noted that the point selected for the median power factor does not affect the question. A tariff of £10 per kW with £1 penalty per 0·1 for all power factors below unity is exactly the same as a tariff of £11 with a bonus/penalty for power factors above or below 0·9, or a tariff of £12 with a bonus/penalty centred round 0·8. The variation in the rate-of-change concerns the consumer's actual power factor, not the median power factor.

In Fig. 51 the lower curves plot the total kVA (i) and the power factor (ii) to a base scaled *uniformly* in reactive kVA (iii). A power-factor scale is also shown along the base line. The full portions of the curves are drawn over what may be called the useful working range of power factor, namely, from 0·95 to 0·65. The two curves coincide at unity power factor, but over the useful range curve (i) becomes progressively steeper, whilst curve (ii) remains almost straight. Over this range the reduction of power factor is very nearly proportional to the increase in reactive kVA, the power factor change being approximately one-third of the change in reactive kVA per kW of load.

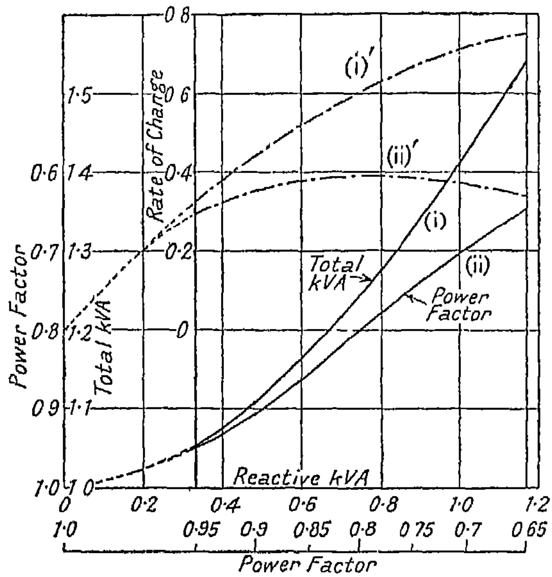


FIG. 51.—Power-factor Quantities.

This is more clearly shown in the upper curves of Fig. 51, which plot the slopes or rates-of-change of the lower curves. Curve (ii′) only varies from 0·3 to 0·38 with an average of about 0·33.* This leads to the very important conclusion that, *over the normal range* of power-factor variation, a bonus/penalty method of charging corresponds very closely to a flat rate per reactive kVA. In figures, a 100 per cent. power-factor penalty is equivalent to a 33 per cent. levy on reactive kVA. The same thing can be seen from Fig. 50, where the distances A_1A_2,

* Mathematically, the rate-of-change of (i.) with (iii.) which is represented by (i.′), equals sin ø. (This is proved in the next chapter.) The rate-of-change of (ii.) with (iii.), represented by (ii.′) equals sin ø cos² ø. and this function is almost constant over this particular range of ø.

A_2A_3, A_3A_4 are tolerably equal, and have a value of about 0·3 on the reactive kVA scale.

The Grid tariff can then be related to reactive kVA by saying that 4*s*. 6*d*. for 0·1 power-factor reduction (per 1 kW true power) is equivalent to approximately $\frac{4s\ 6d.}{3} = 1s.\ 6d.$ per 0·1 reactive kVA = 15*s*. per reactive kVA. Reactive kVA are therefore charged for at 15/60 = one-quarter the price of true kW.

Summary.—It will be clear from the foregoing that there is a third way of charging for power factor—in many ways more scientific, although rarely practised—namely, to charge according to the amount of magnetizing current or reactive kVA drawn from the line, and represented in Figs. 47 and 50 by the distances *TA*. This gives three possible power-factor tariffs, namely:

(i) Charging for the extra kVA of demand (distances *LA*).
(ii) Charging for the reduction in power factor (horizontal scale above *TA*).
(iii) Charging for the reactive kVA (distances *TA*).

It cannot be said that any one of these three exactly represents the supply costs resulting from bad power factors. The first of them represents the costs of the extra copper sections necessary to carry the lagging currents, whilst the third represents the costs of the phase-improvement plant necessary to restore the power factor. The third one is rarely employed, but the second gives very similar results over the working range. One or other of these two is certainly preferable technically to a kVA tariff, because the penalty can be made of any desired magnitude, and because it gives equal rewards to the consumer for equal services to the supply system. In the case of generation or bulk-supply charges, a plain kVA tariff represents an excessive over-penalisation of power factor, and it was no doubt for this reason that a power-factor penalty was employed in the Grid tariff.* The advantages of a plain kVA tariff are its simplicity, and the fact that for distribution costs it gives a fair measure of the extra costs incurred.

If one were to recommend a power-factor penalty for general adoption it would certainly take the form of a charge for magnetizing current or quantity (kVARh). A single flat rate per kilo-var-hour could be used as a supplement either to a two-part maximum demand or to a block tariff. (In the former case, a standing charge supplement (kVAR)

* Another way of getting over this difficulty would have been to have employed a differential kVA tariff, *i.e.*, to charge the extra kVA, not at the full rate, but at some fraction of the rate charged for true kW. For example, the Grid tariff could have charged £3 per kW plus £1 per extra kVA, and this would have given approximately the same results in the region of 0·7 power factor as the penalty actually employed. The tariff could then have been expressed in the form "£2 per kW plus £1 per (total) kVA."

would be even better.) Such a charge can fairly represent the cost of supply and the alternative cost of phase advancement. Both kVAR and kVARh can be easily and accurately metered, and they are not attended by any ambiguity or difficulty of definition even in an unbalanced system. Moreover, the magnetizing current is just as real an element of consumption as the energy current, and the numerical sum of the consumers' readings measures what the system must supply, in kVARh just as in kWh.

CHAPTER XV

POWER-FACTOR IMPROVEMENT

The two previous chapters have dealt respectively with the costs of bad power factors and the tariffs designed to cover them. The cost data forms the necessary basis on which the supply engineer can act, and can construct a tariff which will pass these costs on to the consumer. But this in its turn prompts a further set of actions, since the tariffs which are a finishing point for the supply engineer are the starting point for the consumer. A third chapter is therefore appended to show how the consumer can make the best of the situation in which power-factor penalties are imposed. This chapter will give him some assistance in either preventing the trouble or in curing it at the minimum expense.

Cause and Prevention.—In order to understand how lagging currents may be reduced or prevented, one may refer to the remarks at the commencement of this part, on the meaning of power factor. It will be recalled that phase-displacement is the result of taking from the line either a magnetizing-current or a charging-current component: it is therefore necessary to see what these two components of current are dependent upon.

The two largest users of magnetizing current are transformers and induction motors. In each case a magnetic field is employed, and the greater this field, the longer its path, and the bigger its air gap, the greater will be the ampere-turns required to magnetize it. The value will also greatly depend upon the point on the $B : H$ curve at which the machine is working. The slope of the curve at the usual point of operation on both these machines is about one-third—*i.e.*, a 1 per cent. increase in field requires a 3 per cent. increase in magnetizing current. This shows the extreme importance of not working too far up the curve: together with the avoidance of air gaps in butt-ended transformer cores and the reduction of gaps in induction motors (use of ball bearings, high speeds, etc.), it constitutes the chief contribution that the designer can make to the power-factor problem. Where a large constant-speed drive is required, a synchronous motor can be used instead of an induction motor, which will not only obviate lagging consumption in itself but will neutralise the lagging currents of other apparatus. The same is true to a lesser extent of an induction motor with phase advancer.

When, as is usually the case, the apparatus is connected across the line, the magnetizing current will be approximately constant at all

loads; so that every motor, transformer, etc., left on the line (whether loaded or not), will draw its particular magnetizing component from the system all the time it is connected. (It is important to concentrate on these individual magnetizing currents which add up to give the total reactive demand rather than on the individual power factors which cannot be added and tell one nothing of the effect on the system.) When the apparatus is in series with the line, as with current-limiting reactors, overhead lines and series motors, the field is a function of the load so that the magnetizing current varies with the load current.

In the case of static condensers, the reactive current is required for an electric field, not a magnetic field, and is a function of pressure rather than current. Condensers left connected therefore draw a constant charging current dependent on the line voltage. Moreover, the permittivity (unlike the permeability) is a constant, so that the curve of electric field against charging current is a straight line through the origin, quite different from a $B : H$ curve. A condenser, therefore, unlike an iron-cored choke coil, gives a straight-line response in its performance as a reactor.

A combination of both types of reactor can be seen in the case of an overhead line. Visualising the fields which exist between the conductors of a three-phase transmission line it will be seen that there is a magnetic field in the form of concentric rings round each conductor. Since the apparatus is series connected and there is no iron, this field is proportional to the line current and varies directly with the load. There is also an electric field, in the form of lines stretching between conductors, and always at right angles to the magnetic lines. This is proportional to the line voltage and is therefore constant. Reactive components must therefore be supplied to the line in order to provide both these fields, and these two components are in phase opposition and help to cancel one another. Normally and at heavy loads the magnetizing component is much the greater, and the line is predominantly inductive; but at light loads the reverse may be true. Thus inductance during the daytime may become capacitance at night, and a voltage drop be replaced by a voltage rise.

Cure.—The utmost that can be done by good design and by switching off unnecessary plant will usually still leave the current lagging by an appreciable amount. Thus the load of an ordinary factory, unless there is considerable lighting or heating consumption, will probably have a power factor of not more than 75 per cent. at the time of maximum demand, and still less at other times. This can only be cured by the installation of apparatus taking a leading current of sufficient magnitude to neutralise the outstanding lag. Moreover, whenever the tariff includes power-factor penalties it will usually prove to be a paying proposition to install such apparatus, and thus to improve the power factor to unity or nearly so.

The work of the present chapter covers only the case in which apparatus is installed for the sole purpose of power-factor improvement. Such apparatus will therefore be either a static or a synchronous condenser (*i.e.*, a lightly-running synchronous motor). The economics of installation will of course depend upon the basis on which the lagging costs are expressed or the type of penalty imposed.

kVA Basis.—This covers all cases in which the power-factor costs are expressed as an annual charge on the total kVA (or on the extra kVA) of demand. It assumes that the running charge is unaffected by power factor. It therefore represents the usual case of a consumer on a two-part tariff in which the standing charge is based on total kVA. It also represents in many cases the economic position of the supply authority when installing phase-improvement plant, since it has been seen that their power-factor costs are largely in this form.

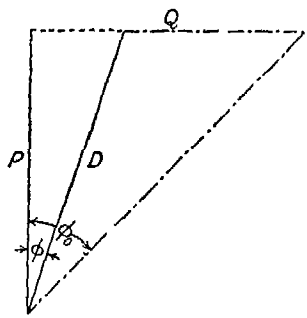


Fig. 52.—Power-factor Improvement.

Fig. 52 represents the demand vectors at the peak period. P is the true power demand in kW corresponding to the mechanical output required from the motors. This is presumed to remain constant and unaffected by the phase-improvement operations. The original load, before power factor was improved, lagged by an angle ϕ_0 so that the demand from the mains was represented by the length of the inclined chain-dotted line (in total kVA). Condensers were then installed of negligible losses, and their contribution is represented by the length Q (reactive kVA) at right angles to P. As a consequence, the phase position is improved to a new angle ϕ, and the total demand from the mains is shown by the length of the line D.

The fixed costs of supply can be represented in the most general form by $XP + YD$ where X is the cost per kW of true power and Y is the cost per kVA of lagging demand. (It has been seen that, for an undertaking, about two-thirds of the fixed charges come into the Y group. For a consumer on a plain two-part kVA tariff, X is zero ; and in any

case, since P is constant, the value of X is immaterial. This item of cost could therefore have been omitted entirely, and is only included for the sake of completeness.) The fixed costs of the condenser installation can be similarly represented by ZQ, where Z is the cost per reactive kVA. *N.B.*—It is essential that Y and Z be expressed in the same units, preferably as £ per annum per kVA.

The total fixed costs are then given by $XP + YD + ZQ$. As phase improvement proceeds, Q gets greater and D gets less, but this latter reduction occurs at a slower pace as D approaches P. The economics of power-factor improvement will therefore depend on the relative rates-of-change of Q and D. Phase improvement should be carried up to the point where the cost of D is being reduced at the same rate as the cost of Q is being increased. A small additional improvement at this point only just saves what it costs, and any further increment will save less than it costs.

Expressing the above items in terms of ø, the total fixed cost becomes $XP + YP \sec ø + ZP (\tan ø \quad - \tan ø)$. It is easy to show that the minimum value of this total (*i.e.*, the marginal point where the two rates-of-change are equal and opposite) occurs when $Z/Y = \sin ø = \sqrt{1 - (\text{p. f.})^2}$. In words, the most economical angle of lag to which phase improvement can be carried is that angle whose sine is the ratio—improvement plant cost per reactive kVA divided by supply cost per lagging kVA.*

This confirms what was seen from the figure, that the amount of lagging kVA saved by 1 kVAR of improvement plant gets less as the power factor approaches unity. Numerically, the rate-of-change of lagging kVA with condenser kVA at any point equals the sine of the angle of lag at that point.

Value of Ratio.—It was stated above that in finding the value of the ratio Z/Y the two quantities must be given in the same units, *i.e.*, either capitalised costs or else annual (fixed) charges on the necessary plant. When the supply authority is installing the phase-improvement plant, it may be possible to make a direct comparison of first costs. Thus, if the supply plant affected by power factor costs £10 per kVA and condensers cost £2 per kVAR the ratio will be $2/10 = 0{\cdot}2$. The most economical angle of lag will then be $11\frac{1}{2}°$ (p. f. = 98 per cent.).

A safer plan, however, is to compare annual charges, since the fixed costs of supply are not merely the interest and depreciation on the capital employed but must include every item varying with the kVA and independent of the consumption, *i.e.*, the annual cost of the buildings and land, all rents and taxes levied on them, maintenance,

* If the losses of the phase-improvement plant are not negligible and its angle of lead instead of being 90° is something less, say ø′, minimum total costs occur when $\sin ø / \sin ø' = Z/Y$.

upkeep and management. The same capital expenditure on improvement plant may not involve the same floor space, maintenance, etc.; even the depreciation may be less owing to a longer working life.

As an example, it may be supposed that the capital costs are as suggested above, namely, £10 per kVA for supply plant and £2 per kVA for improvement plant. Then, if the annual percentages necessary to cover the various items detailed above were 15 per cent. and 12 per cent. respectively, the ratio would then be $\frac{2 \times 0{\cdot}12}{10 \times 0{\cdot}15} = 0{\cdot}16$ instead of 0·2. Hence, even this small difference between the two percentages will make an appreciable change in the size of the phase-advancing plant which it would be economical to install.

Naturally, when some of the items mentioned above, such as floor space, attendance, etc., are already available or would be used to the same degree by either type of plant, it is quite legitimate to leave such items out, and let the annual percentage cover only the pure interest and depreciation. Thus, if the interest were at 6 per cent. throughout, and the useful lives of the generating and improving plant were assessed at 20 and 25 years respectively, the total percentages would be 8·7 and 7·8. This would therefore make only a small change in the ratio which determines the economical angle of lag.

When the improvement plant is being installed by the consumer, the only supply cost to be considered is the tariff, which is here presumed to have an annual standing charge levied on the total kVA. The first cost of the condenser must then be converted into an annual figure for the purpose of comparison. In such cases every possible extra expenditure entailed by the proposed installation, including such things as added responsibility and worry, must be scrupulously assessed and included in the percentage before the cost of such plant is compared with that of the supply kVA.

Hence, a higher percentage will frequently have to be employed than when comparing the installation of alternative types of plant; and an allowance of at least 12 to 15 per cent. per annum of the capital cost will usually be necessary to cover all these items in addition to interest and depreciation, the lower figure applying more particularly to static condensers. Moreover, the industrialist will usually have attractive alternative uses for additional capital expenditure in his own business and will therefore put the hypothetical rate of interest proportionately high, corresponding rather to "profits" than strictly "costs."

As an example of the above, let it be supposed that the consumer is charged £4 per annum per kVA of demand, and improvement plant costs £2 per kVA with an annual allowance (to cover all the above items) of 18 per cent. The value of the ratio will then be $2 \times 0{\cdot}18/4 = 0{\cdot}09$, giving an economical lag of 5 degrees and power factor 99.6 per cent. To take another case, if the supply charge were £6 per kVA

and the annual allowances were 13½ per cent., the value of the ratio would then be just half the above figure.

Graphical Representation.—The above results can usefully be amplified by means of graphs. Figs. 53 and 54 show the cost of lagging loads in the form of the percentage addition which they make to the annual fixed cost or "charge." Referring to Fig. 53, the ordinates give the annual cost of the bad power factor, expressed as a percentage of the fixed charge on a unity power-factor load of the same magnitude.

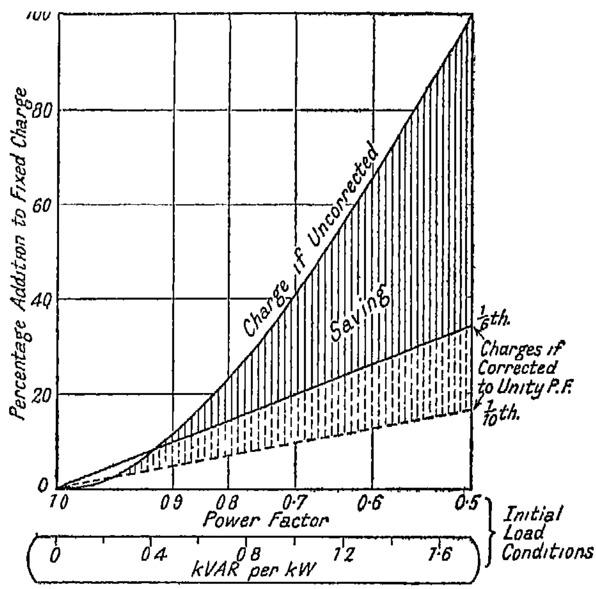


FIG. 53.—Correction to Unity Power Factor.

The base of the graph represents the initial conditions, and it has two scales. One scale shows the initial power factor, and the other (which is evenly divided) shows the corresponding kVAR per kW. The upper curve gives the extra annual fixed cost of a two-part tariff in which the standing charge is based on the total kVA of demand. It shows an addition to the fixed charge of 0 to 100 per cent. as the power factor falls from unity to 0·5.

The middle curve (labelled 1/5th) shows the annual costs of condensers if the power factor is raised to unity by this means. The assumption for this curve is that the annual charges on the phase-improvement plant (per reactive kVA) are one-fifth as much as the standing charge of the tariff (per total lagging kVA). Thus, if the tariff is £4 per annum per kVA of demand, the condensers are presumed

to cost 16*s*. per annum per kVAR, which is 40 per cent. on a capital cost of £2 per kVAR, *i.e.*, it would cover the first cost in 2½ years.

The lowest curve shows the condenser costs for a ratio of one-tenth, *i.e.*, for a £4 per kVA tariff the condensers must cost 8*s*. per annum, or 20 per cent. on a capital cost of £2 per kVAR. It will be appreciated that even this lower figure represents a much bigger percentage than is necessary to cover normal interest and depreciation at pre-war plant costs. On the other hand, the figure has to cover the cost of losses (which have been neglected), housing, insurance, etc., of the extra

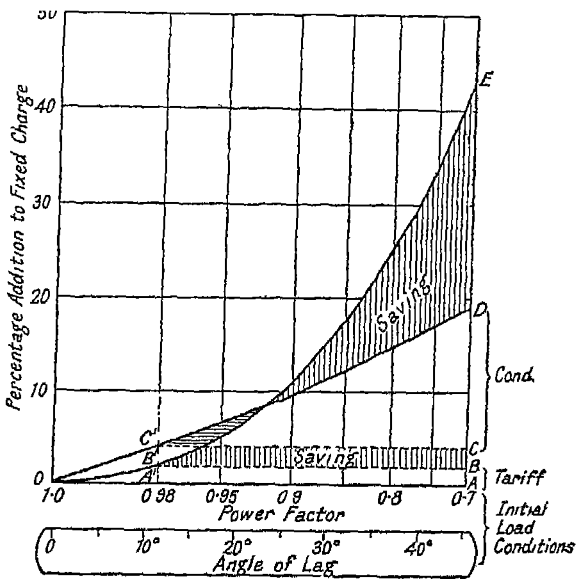


FIG. 54.—Correction to Economic Power Factor.

plant, since the alternative (an increase in the supply bill) does not involve any such incidental expenses.

The shaded areas in the figure represent the saving effected by phase improvement when this is carried up to unity power factor. It will be seen that for any initial power factor, say 0·7, the extra fixed charges before improvement amount to 43 per cent. of the basic fixed charge, whereas after improvement to unity they are only 20 or 10 per cent.

There is a further point that can be shown by this type of curve. Fig. 54 shows the same graph redrawn for the one case only (ratio 1/5th). As before, the base represents the initial conditions, but in this case it is evenly divided in angles of lag so as to enlarge the high-power-factor end. Consider any example such as that in which the initial power factor is 0·7 and represented by the vertical line *AE*. If

uncorrected, the extra fixed charge at this power factor is represented by the height AE as a percentage of the fixed charge at unity power factor. If corrected to unity, the fixed charge becomes AD (paying for the condensers) giving a saving of DE. But suppose the power factor is corrected, not to unity but to 0·98 per cent. The extra fixed charge of the tariff is now reduced not to zero but to $A'B' = AB$. The fixed charge on condensers necessary to do this is now represented by AD minus $A'C' = CD$. Thus, the total fixed charge due to an initial power factor of 0·7 is now given by AB (for tariff) plus CD (for condensers), *i.e.*, by the two bracketed portions.

Moreover, whatever the initial power factor, provided it lies to the right of $A'C'$ there will always be some net saving in improving the power factor up to 0·98, namely, the sum of the two shaded areas. It will be seen that the upper shaded area becomes negative after about 0·92 and must be subtracted from the other. It will further be noted that the position of the economic power factor (in this case 0·98) will depend on the ratio of the annual charges on tariff and condenser. When this ratio is $\frac{1}{10}$th it will occur at 0·995.

Running-Charge Modification.—So far, the decision has been made purely on the basis of standing charges rated on maximum demand. But the introduction of phase-improving plant will, in general, lower also the running charges, since it will save I^2R losses in cables and generating plant to a greater extent than it adds losses of its own. This, of course, applies more particularly when static condensers are employed, and in this country it is chiefly of interest when the supply authorities are installing the improving plant. For although the extra kVAh due to bad power factor cost money, this cost is not usually passed on to the consumer in the form of a tariff.

By assigning symbols to all the different variables, it is easy to build up a formula taking account of both sets of costs, but in the author's opinion this is too complicated to be of much practical service. Since the case rarely arises, when it does so it is better dealt with individually, using actual figures rather than formulæ. All that can usefully be said in general terms is that when the phase-improvement plant saves more losses than it introduces, the most economical angle of lag will be somewhat smaller (and the saving somewhat greater) than appears when only fixed costs are reckoned with.

Size and Saving on Reactive kVA Basis.—Up to this point, all estimates have been made on the assumption that supply costs are dependent on the magnitude of the total kVA and kVAh. This assumption will be strictly correct where the phase improvement is being carried out by a consumer on a kVA tariff, and it will usually also be approximately correct when the supply authority does the work. But in some

cases, the costs of bad power factors can be better expressed on the basis of reactive kVA, particularly with distribution costs in places where regulation rather than heating is the limiting factor. A very similar situation arises when the person installing the apparatus is purchasing supplies on a power-factor penalty clause, since, as was seen in the last chapter, power-factor changes are approximately proportional to reactive kVA over the normal range.

The solution of such a problem is fairly simple provided the running costs can be neglected. Since phase-advancing plant is rated in reactive kVA, its cost is a direct function of this quantity. Hence, when the supply (power factor) cost is also proportional to reactive kVA the two are in the same vector direction and can be directly compared. The method will be sufficiently clear from the following worked example.

The supply costs at a certain point are represented by a two-part tariff, the fixed portion of which is increased by 10*s.* per annum per kW (and *pro rata*) for each 0·1 drop in power factor below 0·85. If phase-advancing plant can be purchased for £2 per kVAR, with an annual charge for interest, depreciation, housing, etc., of 15 per cent. per annum, find the annual saving (if any) on a demand of 1,000 kW, in improving the power factor from 0·7 to 0·85. Neglect any effect due to losses. If the fixed charge had included an equal *pro rata* bonus for power factors above 0·85, to what point would it pay to improve?

First Part approximately.—Over the normal range, the power-factor change is approximately one-third of the change in reactive kVA per kW of load (p. 282). On a demand of 1 kW, a power-factor improvement of 0·1 (saving 10*s.* per annum) will therefore need a reactive kVA of 0·3 (costing $0{\cdot}3 \times 0{\cdot}15 \times 40s. = 1{\cdot}8s.$ per annum). It is therefore evident that improvement will be well worth its cost. (*N.B.* This ratio of one-third does not hold right up to unity power factor, as will be evidenced by the second part of the question.)

First Part exactly.—Let D_0 and D be the initial and final values of the total kVA of demand.

$$D_0 = \frac{1{,}000}{\cos \phi_0} = \frac{1{,}000}{0{\cdot}7} = 1{,}429 \text{ kVA.}$$

$$D = \frac{1{,}000}{\cos \phi} = \frac{1{,}000}{0{\cdot}85} = 1{,}178 \text{ kVA.}$$

$$\text{Initial reactive kVA} = \sqrt{1{,}429^2 - 1{,}000^2} = 1{,}021 \text{ kVAR.}$$

$$\text{Final reactive kVA} = \sqrt{1{,}178^2 - 1{,}000^2} = 621 \quad ,,$$

$$\text{Difference} = 400 \quad ,,$$

Cost of plant to effect this change

$$= £2 \times 0{\cdot}15 \times 400 = £120 \text{ per annum.}$$

$$\text{Saving on tariff} = £0{\cdot}5 \times 1{,}000\left(\frac{0{\cdot}85 - 0{\cdot}7}{0{\cdot}1}\right) = £750 \quad ,, \quad ,,$$

$$\text{Net saving} = £630 \quad ,, \quad ,,$$

Second Part.—The economic limit to phase advancement is reached when the last small increment of power factor improvement only just pays for its cost. For 1 kW true power the reactive kVA $= \tan \phi$, and the power factor $= \cos \phi$. For any small increase in the phase angle, the change in bonus (per unit change in angle) is

$$\frac{10s.}{0{\cdot}1} \times \frac{d}{d\phi} \cos \phi = -\ 100s.\ \sin \phi.$$

The cost of effecting the change (per unit change in angle) is

$$40s. \times 0{\cdot}15 \times \frac{d}{d\phi} \tan \phi = 6s.\ \sec^2 \phi.$$

When these two are equal and opposite, $100s.\ \sin \phi = 6s.\ \sec^2 \phi$, or $\sin \phi \cos^2 \phi = 0{\cdot}06$ (*cf.* footnote, p. 282). This occurs when ϕ is very small and $\cos^2 \phi = 1$ approximately, so $\sin \phi = 0{\cdot}06$ or $\phi = 3\frac{1}{2}°$, and $\cos \phi = 99{\cdot}8$ per cent.

GLOSSARY

I. TARIFF TERMS

(Extracted from *British Standard* 205, 1943,* with the substitution of " kWh " for " units.")

5601. **Tariff.**—The formula determining the price of a supply of electricity.

5602. **Maximum Demand.**—The highest value of the power, kilovolt amperes, or other quantity such as the current, taken within a demand assessment period ; the appropriate quantity being averaged by integration over each of successive demand-integration periods, or alternatively by the sluggishness of the measuring device. In some cases for tariff purposes the M.D. is measured during part only of the demand-assessment period, *e.g.*, during prescribed winter months and/or prescribed hours of the day.

5603. **Demand-Integration Period.**—The duration of successive equal intervals of time (*e.g.*, 30 minutes) during which the power, kilovolt-amperes or other quantity is averaged by integration for the purposes of determining the maximum demand.

5604. **Demand-Assessment Period.**—The prescribed period (*e.g.*, a year, a quarter or a month) within which the maximum demand is measured.

5605. **Installed Load** (Connected Load). The sum of the rated inputs of the consuming apparatus installed on a consumer's premises for connection to the system of electric supply. In assessing the installed load for tariff purposes, apparatus serving only as standby to apparatus of like nature is usually omitted.

5606. **Service Charge.**—A charge per consumer, levied once or periodically, which is independent of the amount of electricity actually supplied, but is usually related to the capital cost of making the supply available to the consumer.

5607. **Fixed Charge.**—A charge appertaining to a prescribed period of supply, but independent of the number of kWh supplied and of fluctuations in the demand. It is usually based on the size or rateable value of the premises, or on the capacity of the apparatus installed.

* British Standards Institution, 28 Victoria Street, London, S.W.1.

5608. **Kilowatt Charge** (Kilowatt-Demand Charge) (Fixed Kilowatt Charge, deprecated). A charge appertaining to a prescribed period of supply, based on the maximum demand in kilowatts.

5609. **kVA Charge** (kVA-Demand Charge).—A charge appertaining to a prescribed period of supply, based on the maximum demand in kilovolt-amperes.

5610. **Standing Charge.**— A general term covering fixed charge, kilowatt charge and kVA charge.

5611. **Unit Charge** (Running Charge). A charge per kWh supplied.

5612. **Fuel Cost Variation** (Fuel Correction) (Coal Correction). —A variation of the charge per kWh, based on the cost and, frequently, the calorific value of the fuel consumed at a prescribed generating station or group of generating stations.

5613. **Standby Charge.**—A charge appertaining to a prescribed period, made for the availability of a supply of electricity whether a supply is actually taken or not.

5614. **Flat-Rate Tariff.**—A tariff comprising a single charge proportional to the number of kWh supplied.

5615. **Two-Part Tariff.**—A tariff comprising a standing charge and a unit charge.

5616. **Maximum-Demand Tariff.** (Hopkinson Tariff).—A two-part tariff comprising a kilowatt or kVA charge and a unit charge.

5617. **All-In Tariff.**— A tariff making no distinction between the purposes for which the supply is used : usually two-part for domestic supply.

5618. **Block Tariff.**—A tariff in which the price is based on a diminishing series of rates per kWh, applying to successive blocks (of fixed sizes) of kWh supplied during a prescribed period.

5619. **Step Tariff.**—A tariff similar to a block tariff except that when each block is exceeded *all* the kWh, not merely the additional ones, are charged at the lower rate.

5620. **Variable-Block Tariff.**—A tariff similar in form to a block tariff, but in which the sizes of the blocks are variable, usually according to the maximum demand, the capacity of apparatus installed, or the size or rateable value of the premises.

5621. **Multiple Tariff.**—A tariff in which different prices, usually flat rates, are charged for electricity used for different purposes.

5622. **Two-Rate Tariff.** (Time-of-Day Tariff). A tariff in which different prices are charged for electricity used at different times of the day.

5623. **Seasonal Tariff.**—A tariff in which different prices are charged for electricity used during different parts of the year.

5624. **Load-Rate Tariff.**—A tariff comprising alternative unit charges, the lower of which operates while the load (current or power) exceeds a predetermined magnitude.

5625. **Equated Rate.**—A tariff (usually a flat rate for domestic supply) in which the price charged for electricity includes the hire, or hire-purchase, and maintenance of apparatus.

5626. **Off-Peak Tariff.**—A tariff in which a lower price is charged for electricity consumed outside prescribed peak-load hours.

5627. **Restricted-Hour Tariff.**—A tariff for a supply of electricity which is only available outside prescribed hours.

5905. **Load Factor.**—The ratio of the number of kWh supplied during a given period to the number of kWh that would have been supplied had the maximum demand been maintained throughout that period : usually expressed as a percentage.

5907. **Diversity Factor.**—Of a group of consumers or loads connected to a common system. The ratio of the sum of the maximum demands of the several consumers or loads to their maximum simultaneous demand.

II. COST AND OTHER TERMS

(Largely extracted from *Electrical Research Association Technical Report* K/T 109.)

Cost Formula.—The formula expressing the cost or some defined portion of the cost of a supply of electricity at a stated point or under stated conditions.

Running Cost.—That component of the total cost which is related to the energy, usually by direct proportionality, and which is independent of the passage of time. *Typical Unit :* Pence per kWh.

Standing Cost.—A general term covering all components of the total cost which are independent of the energy and which are proportional to the passage of time. (Usually comprising demand-related cost, consumer cost, and residue cost.)

Demand-Related Cost.—That component of the standing cost which is related to the maximum demand. *Typical Unit :* £ per annum per demand unit.

Consumer Cost.—That component of the standing cost which is independent of the amount of electricity actually supplied but is proportional to the number of consumers or connections. *Typical Unit :* £ per annum per consumer.

Residue Cost.—The residue of the standing cost not included in the two previous categories. *Typical Unit :* £ per annum.

Cost Classification.—The identification, arrangement in order, and grouping, of cost components by the methods of accountancy, the aim being to sectionalise cost according to the purpose or to the functional elements of the supply system.

Cost Relating.—Grouping or sub-division of cost components on the basis of relationship to energy, time, power demand, number of consumers, or other reference not directly amenable to the methods of accountancy.

Cost Allocation.—Sharing a cost among parties on behalf of whom it is deemed to be incurred.

Marginal Cost.—The amount, at any given volume of supply, by which aggregate costs are changed if this volume is increased or decreased by unit amount. (*N.B.* It is necessary to be clear as to what elements compose the volume of supply.)

Joint Cost.—The cost related to two or more different classes of product or service produced together in the same process.

Component Load.—The load caused by a user or group of users of electricity sharing with others in the supply from a given system or part of a system.

Collective Load.—The effective total of the component loads in respect of a given supply system or part of a system.

Load Curve.—A curve showing power, kilovolt-amperes or current plotted to a base of time. If preceded by a time adjective, *e.g.*, daily, this indicates the extent of the base line.

> *Note.*—In many cases, load curves are not plotted from instantaneous readings, but from successive demand values. The graph should then consist, not of a smooth curve, but of a rectilinear figure in one or other of the following two forms : (*a*) A point-to-point graph, *i.e.*, the plotted values joined by straight lines ; (*b*) a stepped graph composed of vertical lines and horizontal bars, each of the latter representing one value. In case (*a*) it is recommended that the points be plotted against base positions representing the *middle* of each demand-integration period. The practice of putting the dot at the end of these periods, though common, is to be deprecated.

Load-Duration Curve.—A curve connecting the ordinates of a load curve arranged in order of magnitude. The curve therefore shows the duration of each magnitude of load.

Load Model.—A three-dimensional model, one dimension of which represents days, another, time-of-day, and the third, load. It is usually constructed by cutting out daily load curves in contour and stacking them with their time bases side-by-side.

Load Map (Contour Plan).—A load representation in the form of a contour map, in which the contours signify magnitude of load, the longitudes days, and the latitudes time of day.

Peak.—A relatively high salient of a load curve.

Absolute Peak.—The highest salient of a load curve during a specified period, usually one year.

Potential-Peak Periods.—Periods in which a reasonable possibility is deemed to exist of an absolute peak developing.

Off-Peak Periods.—Periods in which no likelihood is deemed to exist of an absolute peak developing. Should strictly be termed " off-potential-peak."

After-Diversity Demand (A.D.D.).—The average load or demand per consumer or per connection of a group of similar consumers or connections.

After-Diversity Maximum Demand (A.D.M.D.)—The highest value of the A.D.D. during the demand-assessment period. Thus, an individual cooker may have an M.D. of 5 kW but 100 of such cookers may only have a collective M.D. of 200 kW. The A.D.M.D. per cooker is then $200/100 = 2$ kW. The average individual M.D. divided by the A.D.M.D. equals the diversity factor.

Differential Diversity.—Applied to a group of similar loads. The tendency for a low load factor to be associated with a high diversity factor. An inverse relationship between individual load and diversity factors.

Saturation Factor.—Expressed numerically (usually as a percentage) and in reference to a specified appliance. The proportion of a given "population" possessing one or more of these appliances. Thus, if the "population" concerned is the total number of consumers, an electric-cooker saturation factor of 30 per cent. indicates that 30 per cent. of consumers have electric cookers. The term is sometimes, but erroneously, employed to describe the ratio between the total number of appliances of a given type and the total number of consumers. (*Note.*—The inclusion of the word "factor" indicates that the word "saturation" is being used quantitatively and not in its absolute hundred-per-cent. sense as in the phrase "electric cooker development is approaching saturation" or "shows signs of saturation".)

Effective load Factor.—Load factor in relation to the demand at the time of system peak. The load factor as defined above (No. 5905) is the ratio between the kWh over a given period and the maximum demand in kW multiplied by the length of the period (in hours). The *effective* load factor is obtained from the same ratio by substituting, in place of the M.D., the demand at the time of peak on the supply system (or some stated part of the system). The load factor cannot exceed unity, but the effective load factor can have any higher value than the load factor, up to infinity, since the demand at the time of system peak may have any value from the M.D. down to zero. In the case of a group of similar connections, the effective load factor bears the same relationship to the load factor that the A.D.M.D. bears to the A.D.D. at the time of system peak.

Promotional Tariff.—A tariff in which increased annual consumption reduces the overall mean price per kWh. The term is particularly applied to tariffs having a fixed charge or block sizes not varying from

year to year. It is not usually applied to tariffs having a standing charge or block size based on maximum demand or installed load, in which a reduction in mean price is only obtained if the load factor increases. Tariffs with sliding-scale reductions in price for increased consumptions would not normally be described as promotional unless the reductions are substantial.

THE END

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